6-15-2021

Letter to Harold Davenport from Paul Erdos

Budapest, 26. VI. 1934.

Dear Mr Davenport,

Excuse me for my long silence, but I was fully occupied with my examinations. At last I had my final examination on the 23. inst. and so I am completely and definitively free. My promotion will take place on 3. VII.

In one of my previous letters I proved that the number of integers until n^2, that are the product of two factors $< n$ is $o(n^2)$. My prove had unnecessary complications. In its new form it runs as follows.

Let $z = xy$; $x < n$; $y < n$.

We assume two different cases.

1. At least one ~~one~~ of the factors e.g. x has less than ~~$\frac{2}{3} \log\log n$~~ prime factors. In this case we get for z, $o(n^2)$ integers, for x may take only $o(n)$ different values.

2. Both x and y have more than $\frac{2}{3} \log\log n$ prime-factors. Thus in consequence of the Hardy-Ramanuyan theorem, we get for z also here $o(n^2)$ integers. In this case z has more than $\frac{4}{3} \log\log n$ prime factors

In connection with this Privatdocent Dr L Kalmár/Szeged/ announced. The conjecture that the number of integers until n^2 that are the product of two integers $< n$ is $c\frac{n^2}{\log n} + O\left(\frac{n^2}{\log^2 n}\right)$

Presently I am concerned with the conjecture that almost all all integers have two divisors d_1 and d_2 so that $d_1 < d_2 < 2d_1$, but till now without any result.

I should be very glad to know how you are and what are your projects for the sommer.

Waiting for your kind answer, I am very sincerely Yours

P. Erdös