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**COST AND CO₂ OPTIMIZATION OF REINFORCED CONCRETE
FRAMES USING A BIG BANG-BIG CRUNCH ALGORITHM**

by

Farah Ishtiwana Huq

A Thesis

Submitted in Partial Fulfillment of the

Requirements for the Degree of

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ABSTRACT

Farah Ishtiwana Huq. M.S. Civil Engineering. The University of Memphis, April 2011. Cost and CO₂ Optimization of Reinforced Concrete Frames using a Big Bang-Big Crunch Algorithm.
Charles Camp, Ph.D.

A computational procedure is developed that automatically generates structural designs for reinforced concrete frames using Big Bang-Big Crunch (BB-BC) optimization. The objective of the optimization is to minimize the total cost or the CO₂ emissions associated with construction of reinforced concrete frames subjected to constraints based on the specifications and guidelines prescribed by the American Concrete Institute (ACI 318-08). BB-BC optimization is an iterative population-based heuristic search method that has a numerically simple algorithm with relatively few control parameters as compared to other evolutionary methods.

Designs for several reinforced concrete frames that minimize the cost and the CO₂ emissions associated with construction are presented. In the first frame example, low-cost designs developed using BB-BC optimization are compared designs developed using genetic algorithms. In the second set of frame designs, both low-cost and low-CO₂ emission designs using BB-BC optimization are compared to designs developed using simulated annealing. In both cases, the BB-BC algorithm generated designs that reduced the cost and the CO₂ emissions of construction for example frames.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The main purpose of a structure is to provide safety against collapse while providing serviceability to the client. Reinforced concrete structures are very reliable in fulfilling the basic requirements of safety and serviceability. Due to its strength, durability, long service life, fire protection, low cost, and above all, energy efficiency, reinforced concrete is widely used for many of the most common buildings, bridges, dams, retaining walls, and water tanks. Concrete has been used for thousands of years starting with lime mortars from 12,000 to 6,000 BCE in Crete, Cyprus, Greece, and the Middle East (Nilson et al. 2010). The modern form of reinforced concrete has been developed over years of use and also with the demands of society. Today reinforced concrete structures are a major part of the construction industry. Safe and economic designs of reinforced concrete structures are essential to our modern society.

The growing concern about increased CO₂ emission from various sources has spread to all sectors of industrial production. The construction sector of the world economy is a significant source of CO₂ emissions. The cement industry is responsible for 5% of total global emissions of CO₂ (Worrell et al. 2001). The United States was the third largest cement producer in 2001 (Hanle 2010). Cement is the major constituent material of concrete, and the CO₂ emission from concrete production is directly proportional to the cement content used in a concrete mix. The embodied CO₂ of one ton of concrete is around 100 kg. Various research efforts have focused on the production of environmentally friendly cement. However, another useful way of reducing CO₂ emissions is through optimization of structural design.

To meet the increased demands of civilization, demand for reliable reinforced concrete structures is increasing every day. Since cost is one of the most important factors in design, the objective is to use as little material as possible and still meet strength and durability requirements. Structural optimization is used to comply with this objective. The traditional approach to design

does not fully optimize the amount of materials. In many cases, the prior experience of the designer rather than analysis is employed to select cross sections, material grades, reinforcement, and other parameters necessary for designing a reinforced concrete structure. After a rigorous trial-and-error procedure, design variables (cross sectional dimensions, reinforcement, material grades etc.) are determined which satisfy limit states prescribed by standard codes of practice. While this process leads to safe designs, the amount of materials used is not necessarily optimized. The amount of redundant materials has been estimated to be as much as 10% (Paya et al. 2008). The optimum design of a structure not only reduces the cost, but also reduces CO₂ emissions through efficient use of materials.

Due to the potentially large number of variables and complexity of the analysis of framed structures, evolutionary optimization algorithms can be efficient tools for performing structural design. Evolutionary optimization methods have been extensively used in the field of structural engineering. Big Bang-Big Crunch (BB-BC) is a newly developed heuristic algorithm that is numerically simple with few control parameters. Recent studies (Kaveh and Talatahari 2010) show that BB-BC methods are computationally efficient for structural optimization.

1.2 Objective of the Study

The objective of this study is to apply a BB-BC algorithm to the design of reinforced concrete frames to develop low cost and low CO₂ emission designs. In addition, BB-BC designs will be compared to reinforced concrete frame designs published in previous studies.

Reinforced concrete structures usually have numerous design variables. Another objective of this study is to show the efficiency of the BB-BC approach in handling complex optimization problems and to compare the performance of the BB-BC method to other heuristic algorithms.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

When an optimization statement is formulated for a particular problem, the solution can be obtained by an appropriate algorithm via mathematical programming, optimality criteria, or various evolutionary algorithm approaches. A detailed review of structural optimization methods was presented by Cohn and Dinovitzer (1994). Typical structures that were optimized in earlier studies were plane trusses, beams, columns, shafts, plane frames, arches, space trusses, and plates and shells. Most of the optimization examples were steel structures, with only a few reinforced concrete structures. Another significant trend of earlier research was the use of deterministic methods rather than probabilistic methods. Mathematical programming was widely used for moderate structures where constraints could be linearized. Optimality criteria methods were preferred for solving large and complex problems. Finally, heuristic algorithms showed great potential for optimization of complex problems. A common objective of many structural optimization problems is weight or cost minimization.

2.2 History of Optimization by Deterministic Methods

Some earlier works on reinforced and prestressed concrete members using deterministic methods (linear programming, direct search technique etc.) were done by Goble and Lapay (1971), Kirsch (1972), Friel (1974), Brown (1975), and Namaan (1976). Their work revolved around concrete beams and slabs. Other researchers' works also included designs of reinforced concrete beams, slabs, and frames using different deterministic methods according to the type and size of the problem. Chou (1977) used the Lagrange multiplier method for cost minimization of a singly reinforced T-beam. Gunaratnam and Sivakumaran (1978) presented minimum cost designs of reinforced concrete slabs. Kirsch (1983) outlined a simplified three-level iterative procedure for cost optimization of multi-span continuous reinforced beams. Cohn and Macrae (1984) showed minimum cost designs of reinforced concrete beams using a feasible conjugate

direction method. Abendroth and Salmon (1986) did a parametric study on the sensitivity of the optimum cost of partially or fully restrained reinforced concrete T-beams in which they used a quasi Newton-Raphson method. Prakash et al. (1988) performed minimum cost designs of singly and doubly reinforced rectangular and T-shape reinforced concrete beams using Lagrangian and simplex methods. Kanagasundaram and Karihaloo (1990) optimized continuous L-shape and T-shape reinforced concrete beams using two different methods: sequential linear programming and sequential convex programming. Chakrabarty (1992) performed cost minimization of rectangular reinforced concrete beams using geometric programming and the Newton-Raphson method. Moharrami and Grierson (1993) minimized cost of reinforced concrete frame designs subjected to lateral and vertical loading conforming to ACI Building Code (1989) using the optimality criteria approach. Adamu et al. (1994) described a continuum type optimality criteria approach for minimum cost designs of singly reinforced beams. Adamu and Karihaloo (1995) outlined a discretized continuum type optimality criteria method for minimum cost design of two-dimensional multi-bay and multi-story reinforced concrete frames. Fadaee and Grierson (1996) used optimality criteria approach for minimization of three-dimensional reinforced concrete frames with members subjected to biaxial moments and shear forces. Fadaee and Grierson (1996) focused on formulating the constraints for the combination of the axial load, biaxial bending moment and biaxial shear. They presented an example of a one-bay one-story space frame. Balling and Yao (1997) outlined a comparative study of optimization of three-dimensional reinforced concrete frames with rectangular columns and T or L shaped beams. Their example had one, two and four-story frames subjected to lateral and vertical loads. They used sequential quadratic programming for the designs of one, two, and four-story frames subjected to lateral and vertical loads.

The studies cited here are examples of efforts made to apply deterministic methods to find an optimum solution to structural design problems. The current trend of research in this field is to employ various heuristic algorithms to develop optimum designs.

2.3 History of Optimization by Heuristic Methods

Goldberg (1989) was one of the first researchers to use a genetic algorithm (GA). He showed that it is possible to solve an engineering optimization problem with a GA. After that, Jenkins (1991) used GA to optimize plane frames. Rajeev et al. (1992) expanded the application of GAs into discrete design variables to obtain the optimum weight of trusses subjected to stress constraints. After that, Koumousis and Georgio (1994), Adeli and Cheng (1994), and Rajan (1995) used GAs to develop the optimum design of truss structures. Another work on truss structures was done by Rahimi et al. (2008) where sizing, geometry, and topology optimization of trusses were done by the force method and a GA. Lagaros et al. (2008) performed the optimum design of steel frames with web openings using an evolutionary algorithm. Zielinski et al. (1995) used a GA to optimize reinforced concrete short tied columns with applied axial forces and bending moments. Another work in using GAs for reinforced concrete beam optimization was done by Coello et al. (1997). Rafiq and Southcombe (1998) presented the optimal design of reinforced concrete biaxial columns using a GA. They attempted to show how a GA conducts a global search to identify the optimal reinforcement bar sizes and bar detailing arrangements. Koumousis and Sejonah (1998) used a GA to design the reinforced concrete members of a multi-story building. Rajeev and Krishnamoorthy (1998) optimized a two-dimensional frame using a GA based methodology.

Ceranic et al. (2001) presented designs of reinforced concrete retaining structures subjected to earth and hydrostatic pressure. A modified simulated annealing (SA) algorithm was used for this design which improves convergence to a minimum cost. The objective function included the cost of concrete, the reinforcement and the formworks.

Leps and Sejnoha (2003) proposed a new approach to the design of reinforced concrete continuous beams using an augmented SA method in conjunction with a GA.

Lee and Ahn (2003) proposed a GA based design of reinforced concrete frames subjected to gravity and lateral load. In this formulation, difficulties in finding optimum sections

and reinforcement from a large solution space are reduced by constructing data sets of section properties and reinforcements. Construction practices were also implemented by linking columns and beams by groups. The cost objective function included both the cost of rebars and concrete in beams, rebars and concrete in columns, and formwork in both beams and columns. Constraints are based on bending moments in beams and interaction diagrams for columns. They performed optimization of three-bay three-story, three-bay nine-story, and three-bay twenty-story frames.

Camp et al. (2003) presented works on optimization of reinforced concrete frames, simply supported beams, and uniaxial columns. The objective of the research was to design light-weight reinforced concrete frame structures which fulfill the strength and serviceability requirement of the ACI Building Code (1999) using a GA. The cost objective function included the cost of concrete, reinforcement and formworks. Low-weight designs were developed for a two-bay six-story reinforced concrete frame.

Sahab et al. (2005) showed the cost optimization of reinforced concrete flat-slab buildings. The objective function included the cost of floors, columns, and foundations. The optimization process is accomplished in three different steps. In the first step, the optimum column layout is found by an exhaustive search. In the second step, a GA is employed to obtain the column dimensions and slab thicknesses. In last step, an exhaustive search is used to find optimum number and size of reinforcing bars in each member. Designs for a one-story and a four-story reinforced concrete flat-slab building were developed.

Govindaraj and Ramasamy (2005) worked on the application of a GA to the optimum design of reinforced concrete continuous beams. The cross-sectional dimensions of beams were the only design variables. The areas of longitudinal steel were converted into a least-weight detailing of steel reinforcements by generating a database of reinforcement templates containing different reinforcement bars with pre-specified patterns. The objective function included the cost of concrete, steel, and formworks.

Zou et al. (2007) presented a multi-objective optimization for performance based design of reinforced concrete frames. In formulating the total life cycle cost of a reinforced concrete frame, the initial material cost was expressed in terms of the design variables. The damage loss was described as a function of seismic performance levels. The life cycle cost of reinforced concrete frames was minimized. The best solution was found by a Pareto optimal set based on optimality criteria approach.

Yepes et al. (2007) outlined a method for optimum design of earth retaining walls by a SA. The formulation of the problem included 20 design variables. The objective function included the cost of concrete, reinforcement, formwork and excavation fill.

Perea et al. (2007) presented work on the optimization of reinforced concrete frame bridges using a parallel GA and a mimetic algorithm (MA). The structural optimization had 50 design variables. There were three types of concrete as variables and 44 other variables defining reinforcement bar diameters and bar lengths. A comparison between the algorithms showed that a parallel MA is more efficient than the GA.

Kaveh and Jahanshahi (2008) described a procedure for the plastic limit analysis of frames using ant colony optimization (ACO). In their work, an ant colony system was employed to optimize the process for finding the collapse load factor for two-dimensional frames. Three different variants of ACO algorithms were developed, and their relative performances were compared.

Barakat and Altoubat (2008) presented evolutionary-based optimization procedures for designing conical reinforced concrete water tanks. The objective function included the cost of concrete, reinforcement, and formwork required for walls and floors. The wall thicknesses (at the bottom and at the top), base thickness, depth of water tank, and wall inclination were considered as design variables. Three optimization techniques were used to obtain the optimum solution: shuffled complex evolution (SCE), SA and GA. After several tests, the SCE technique proved to be superior to the other two algorithms.

Paya et al. (2008) described a methodology to design reinforced concrete building frames using a multi-objective SA algorithm. The objective functions were the economic cost, constructability, environmental impact and the overall safety of reinforced concrete framed structures. The SA methodology was applied to a symmetric two-bay four-story building frame. Pareto results of multi-objective SA algorithms provided more practical, easier to construct, more sustainable, and safer solutions than the lowest cost solution.

Paya et al. (2009) presented a CO₂ optimization of reinforced concrete frames by SA. Two objective functions were examined: CO₂ emissions and the economic cost of reinforced concrete framed structures. The SA methodology was applied to six typical building frames with 2, 3, and 4-bays and up to 8 stories. The results showed that the lowest CO₂ solution is not the lowest cost solution.

2.4 History of Optimization by BB-BC Algorithm

In this study, a BB-BC algorithm is used to design reinforced concrete frames. The BB-BC algorithm was proposed by Erol and Eksin (2005). In their work, they described how BB-BC modeled the beginning of the universe by generating random points in Big Bang phase and then shrinking those points to a single representative point, the center of mass, in Big Crunch phase. The performance of BB-BC method showed improvement over GAs.

Camp (2007) applied BB-BC to the design of low-weight space trusses. The objective of the optimization was to minimize the total weight or cost of the structure subjected to material and performance constraints. In this study, low weight design and performance comparisons for several benchmark-type truss structures were presented.

Kaveh and Talatahari (2010) presented the most recent work on the BB-BC optimization method. They used the BB-BC algorithm for optimal design of skeletal steel structures. More importantly, they proposed a hybrid BB-BC algorithm which improved the computational efficiency of the algorithm.

CHAPTER 3

REINFORCED CONCRETE

3.1 Introduction

The word concrete originally came from the Latin word *concretus* which means compact or condensed. Concrete is a composite construction material composed of cement (mainly Portland cement) and other cementitious materials such as fly ash and slag, fine and coarse aggregates, water, and chemical admixtures. Concrete solidifies and hardens after the addition of all the constituent materials and water. The cement reacts with water, (the process is known as hydration) which bonds the other components together. Admixtures may be added to the mixture to change some characteristics such as durability, workability, and curing time. Concrete is strong in compression as the aggregates efficiently carries the compression load. However it has a low tensile strength as the cement holding the aggregates in place can crack. The tensile strength of concrete is approximately ten percent of the compressive strength (Nawy, 1996).

Reinforced concrete is obtained when steel is placed in concrete to improve the tensile strength of the concrete. The steel can also be used as compression steel in both beams and columns. Reinforced concrete structures are rigid and easy to maintain. In addition, reinforced concrete members are heavy because of the low strength-to-weight ratio (Nawy, 1996)

3.2 Characteristics of Reinforced Concrete Beams

When the load on a reinforced concrete beam is gradually increased from zero to a magnitude which causes the beam to fail, the beam goes through several different stages. When the tensile stress in the beam is less than the modulus of rupture (tensile stress at which concrete begins to crack), the entire beam section is effective in resisting the bending moment stresses. The embedded reinforcement deforms the same amount as the adjacent concrete. Stress in the concrete is proportional to strain at this stage. When the load is increased and the modulus of rupture is exceeded, hairline cracks start forming at the tension face of the beam. The bending

moment that produces these cracks is known as the cracking moment, M_{cr} (Figure 3-1). The hairline cracks are so small that they are hardly visible. The formation of the hairline cracks changes the behavior of the beam significantly. Since the beam section is cracked, the concrete no longer resists the tensile stresses. The entire tension force is transmitted to the steel placed in the section. If the concrete stresses do not exceed approximately $f'_c/2$ (f'_c is the ultimate compressive stress in concrete) at moderate loading, stresses and strains still remain proportional. If the load is increased further after the hairline cracks are formed, the corresponding stresses and strains increase and are no longer proportional. However, the variation of strain is still assumed to be linear. As the load is increased further, the capacity of the beam is reached. There are typically two types of failures. When moderate amount of reinforcement is provided, and the steel reaches its yield point, a large amount of stretching in the reinforcement occurs which produces subsequent widening of tension cracks in the beam. These cracks propagate upward and the increase in strain in compression zone resulting in the crushing of concrete. This is referred to as a secondary compression failure. On the other hand, if a large amount of reinforcement is provided, the crushing of concrete occurs well before the steel starts to yield. While the exact criteria for this is not yet established, it has been found that the rectangular beam fails in compression when the concrete strain reaches a value of about 0.003 to 0.004 (Nilson et al. 2010). Figure 3-2 shows a typical moment-strain diagram.

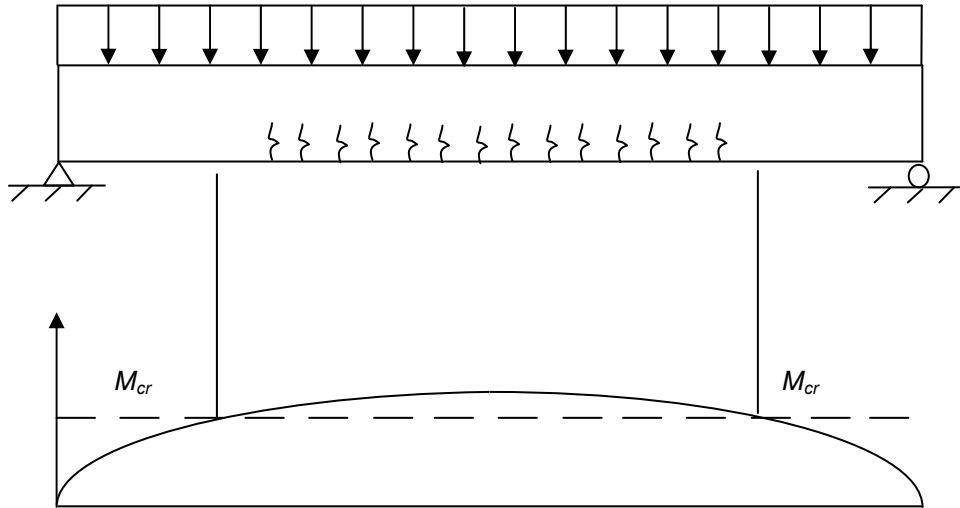


Figure 3-1. Development of Cracks in a Simply Supported Beam

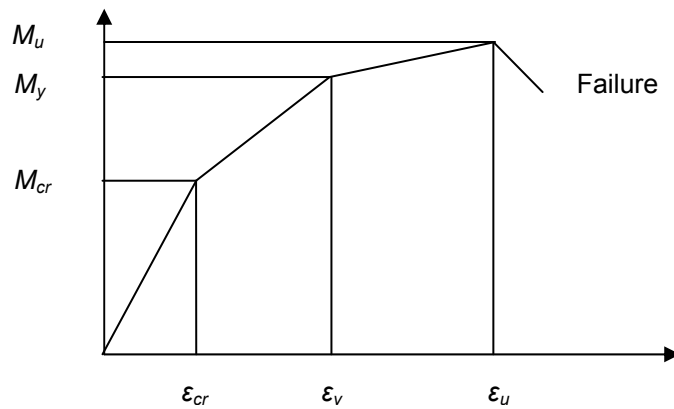


Figure 3-2. Moment-Strain Diagram

3.3 Simply Supported Reinforced Concrete Beams

A simply supported beam is supported at two points. In most cases, one of the supports is a pin and the other is a roller (Figure 3-1). Simply supported beams are statically determinate. Singly reinforced rectangular beams usually contain single layer of reinforcement in the tension zone of the beam (Figure 3-3). The reinforcement is placed a little higher than the extreme

tension fiber of the beam. The other controlling parameters of the beam are the width b , the height h , and the effective depth d (distance from top of the beam to the centroid of the reinforcement), and the area of steel, A_s as shown in Figure 3-3. A protective concrete cover is essential around the reinforcement bars in order to ensure the composite action between steel and concrete and to protect the reinforcing steel from being exposed to weathering action. The minimum requirement for cover is specified in the ACI Code (2008) as 1.5 in for rectangular beams. When calculating the flexural strength of the reinforced concrete section, the tensile strength of the concrete is not considered (Section 10.2.5 ACI 2008). So, the effective depth d of the section is more important than the overall depth h . When the effective depth d is not sufficient, reinforcement is added in compression zone of the beam which is referred to as a doubly reinforced beam. In continuous beams, tension reinforcement is provided in both the top and the bottom of the section since positive bending occurs at or near mid span and negative bending occurs over the supports.

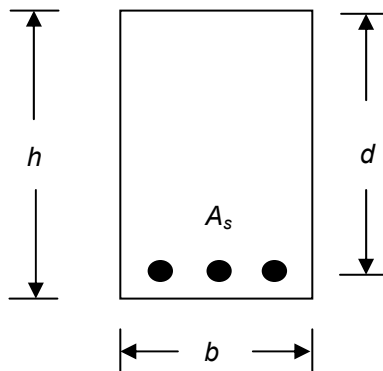


Figure 3-3. Singly Reinforced Concrete Beam Section

3.4 Singly Reinforced Concrete Beam Analysis

The design of a singly reinforced concrete beam must conform to the specifications of the ACI Code (2008) or guidelines that are not necessarily stated in the ACI Code (2008) but are practiced in construction of reinforced concrete structures.

3.4.1 Computation of Beam Capacity

According to ACI Code (2008) 11.1.1, the applied factored shear force V_u in any section of a beam should be:

$$V_u \leq \phi V_n \quad (3-1)$$

$$V_n = V_c + V_s \quad (3-2)$$

where V_n is the nominal shear strength of the beam, V_c is the shear strength provided by the concrete, V_s is the shear strength provided by the web reinforcement, and ϕ is the strength reduction factor for shear ($\phi = 0.75$).

The nominal strength is the sum of the contribution of concrete and the web reinforcement if provided. So Equation (3-1) becomes:

$$V_u \leq \phi V_c + \frac{\phi A_v f_{yt} d}{s} \quad (3-3)$$

where A_v is the total cross-sectional area of web steel, f_{yt} is the yield strength of shear reinforcement, and s is the spacing between vertical stirrups used as web reinforcement.

The nominal shear strength of concrete V_c , including the contributions of aggregate interlock, dowel action of main reinforcing bars, and the uncracked concrete is given by ACI Code (2008) Equation 11-5:

$$V_c = \left[1.9\lambda\sqrt{f'_c} + 2500\frac{\rho V_u d}{M_u} \right] bd \leq 3.5\lambda\sqrt{f'_c}bd \quad (3-4)$$

where M_u is the ultimate moment and λ is a modification factor ($\lambda = 1.0$ for normal weight concrete). In Equation (3-4), $V_u d/M_u$ should not be taken more than 1.0. ρ is the longitudinal reinforcement ratio in Equation (3-4). ρ is calculated as:

$$\rho = \frac{A_s}{bd} \quad (3-5)$$

Equation (3-4) requires the computation of the bending moment and the shear force at every section. An alternative calculation for V_c is permitted by ACI code 11.2.1.

$$V_c = 2\lambda\sqrt{f'_c}bd \quad (3-6)$$

Equation (3-6) provides a conservative result in the regions where the shear-moment ratio is high. However, tests show that for beams constructed with high strength concrete ($f'_c > 6,000$ psi), the concrete contribution to resist shear failure is less than what is predicted by Equations (3-4) and (3-6). For this reason, ACI Code (2008) 11.1.2 limits the value of $\sqrt{f'_c}$ to be used in Equations (3-4) and (3-6) to 100 psi. If a minimum amount of web reinforcement is provided, a value of $\sqrt{f'_c}$ greater than 100 psi can be used in computing V_c . If the factored shear force V_u is not larger than ϕV_c calculated by Equation (3-4) or (3-6), then theoretically no web reinforcement is required.

However, ACI Code (2008) 11.4.6 requires a minimum area of web reinforcement equal to:

$$A_{v,min} = 0.75\sqrt{f'_c} \frac{b_w s}{f_{yt}} \geq 50 \frac{b_w s}{f_{yt}} \quad (3-7)$$

where b_w is the width of the web and $A_{v,min}$ is the total cross-sectional area of the web steel.

Beams which have steel fiber reinforcement, $f'_c < 6,000$ psi, h not greater than 24 in and V_u not greater than $2\phi\sqrt{f'_c}bd$ are not required to have any minimum web reinforcement. However, the usual practice is to use the minimum amount of web reinforcement for shear.

The nominal moment capacity M_n of singly reinforced beam section is defined by ACI code (2008) as:

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \quad (3-8)$$

where f_y is the yield strength of the steel reinforcement and a is the depth of equivalent rectangular stress-block given as:

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad (3-9)$$

According to the provisions of the ACI Code (2008), the nominal flexural strength M_n is reduced by imposing the strength reduction factor ϕ . Equation (3-8) becomes:

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) \quad (3-10)$$

The value of ϕ is dependent upon the section capacity and whether it is a tension or compression controlled section. As per the definition of ACI Code (2008), the tension controlled section is one with a net tensile strain greater than or equal to 0.005; whereas, the compression controlled section is the one with net tensile strain of less than 0.002. The strength reduction factor for a

tension controlled section is $\phi = 0.90$, and for a compression controlled section it is $\phi = 0.65$. The strength reduction factor varies linearly between net tensile strains of 0.002 and 0.005.

3.4.2 Steel Reinforcement Limitations

The ACI Code (2008) has restrictions on the spacing of the reinforcing steel in a beam. The ACI Code (2008) specifies that the distance between two parallel reinforcing steel bars in a row shall be at least the nominal diameter of the bar d_b but not less than 1 in (ACI 7.6.1). The Code (ACI 2008) also prescribes that the minimum cover for beams and columns should be 1.5 in (ACI 7.7.1 c).

According to ACI Code (2008) the steel ratio is restricted to minimum and maximum values. In a very lightly reinforced beam, the flexural strength of the cracked section is not adequate to withstand the moment that produced cracking of the section, and the beam immediately fails without any warning. The minimum steel is provided to ensure resistance against this type of failure. According to ACI Code (2008) Section 10.5, at any section where tensile reinforcement is required by analysis, the minimum reinforcement area $A_{s,min}$ must not be less than:

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} bd \geq 200 \frac{bd}{f_y} \quad (3-11)$$

Minimum reinforcement applies to the section whether it is in positive or negative bending moment.

For under-reinforced behavior ACI Code (2008) Section 10.3.5 establishes a limit for minimum tensile strain ϵ_t at nominal member strength of 0.004 for members subjected to axial loads less than $0.10f'_cA_g$ (where A_g is the gross area of the cross-section).

The reinforcement ratio ρ is:

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \quad (3-12)$$

where β_1 is the parameter for calculating the depth of the equivalent rectangular stress block ($0.65 \leq \beta_1 \leq 0.85$), ϵ_u is the ultimate strain in concrete ($\epsilon_u = 0.003$ to 0.004 for rectangular beams), and ϵ_t is the tensile strain in the steel.

Using $\epsilon_t = 0.004$ the maximum reinforcement ratio ρ_{max} is:

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} \quad (3-13)$$

For tension controlled sections, $\epsilon_t = 0.005$ is allowed by ACI Code (2008). Therefore Equation (3-13) becomes:

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} \quad (3-14)$$

The reinforcement ratio in a beam should be lower than ρ_{max} to ensure under-reinforced behavior of the beam. The value of β_1 is dependent upon f'_c . The relationship between β_1 and f'_c can be expressed as:

$$\begin{aligned} f'_c \leq 4,000 \text{ psi } \beta_1 &= 0.85 \\ f'_c > 4,000 \text{ psi } \beta_1 &= 0.85 - 0.05 \frac{f'_c - 4,000}{1,000} \geq 0.65 \end{aligned} \quad (3-15)$$

3.4.3 Beam Dimensions and Other Restrictions

The ACI Code (2008) sets restrictions on the maximum deflection and minimum thickness of a beam. Exact beam deflections are often difficult to calculate as deflections are a function of time. There are immediate deflections and time dependent deflections in beams. Therefore it is safe to make the beam deeper to avoid excessive deflection. The minimum allowable thickness of the beam depends upon the supports of the beam, whether it is simply supported, continuously supported on one or both sides, or cantilevered. Allowable limits are listed in ACI Table 9-5 (a).

To resist lateral buckling in cases with high axial loads and long members, it is common practice to limit h to two or three times the width of the beam.

3.5 Columns

3.5.1 Column Definition and Variables

The vertical load carrying members in buildings are columns, which transfer the load from the superstructure to the substructure. Columns carry gravity loads from the top to the bottom of the structure. So, columns in the lower stories of a building must be stronger than those in the upper stories. In addition, columns must have enough strength to resist buckling under applied loading. Though columns are considered as compression members, they may have to withstand bending moment transmitted from beams connected to the columns or due to lateral loads applied to the structure. In a multi-story rigid-frame, lateral loads are produced by wind and earthquakes. The horizontal shear developed due to lateral loads in each story produces moments in the columns. Bending moments can also be generated by unbalanced floor loads on both interior and exterior columns and by eccentric loads. In most cases, columns are subjected to both axial compression and bending moment. In general, beam-columns combine beam actions (involves bending and lateral torsional buckling) with column actions (involves compression buckling). So, factors that affect beams and columns also influence the behavior, strength, and design of beam-columns.

There are different kinds of columns; for example, circular and square sections with steel tubing on the outside, circular and square spiral columns with steel reinforcement, and rectangular tied columns with steel reinforcement. In this study, rectangular tied and spiral columns are used in the analyses. All columns are considered as short columns (for which the strength is governed by the strength of the materials and the geometry of the cross section). Tied and spirally reinforced columns have longitudinal reinforcement bars held in place by lateral ties. Depending on the sizes of the longitudinal reinforcement, the sizes of the ties are usually either #3 or #4. If the longitudinal bars are of #10 or smaller, the ACI Code (2008) specifies that at least #3 ties must be used and for bars bigger than #10 no less than #4 ties must be used. A typical section of a column is shown in Figure 3-4, where d' is the distance from the extreme compression fiber in the concrete to the centroid of the compression steel.

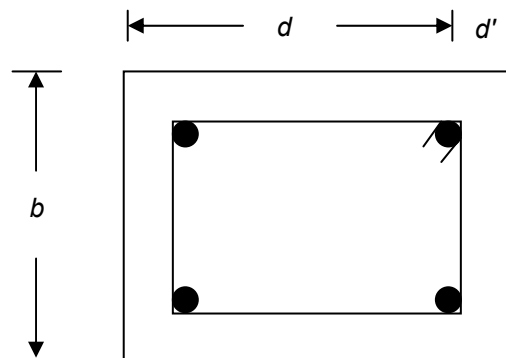


Figure 3-4. Typical Reinforced Concrete Column Section

Rectangular, square, and tied columns are common in construction of reinforced concrete buildings. An adequately tied column not only has to have a strength greater than the applied loads, but it also has to conform to restrictions in the ACI Code (2008).

The ACI Code (2008) restricts the minimum and maximum steel ratio in a column section. The steel ratio in a reinforced concrete column is the ratio between longitudinal reinforcement area and gross concrete area. The gross concrete area A_g is the product of the width b and the

overall depth h . The steel ratio should not be less than 1% and should not be greater than 8% of the column gross section (ACI 10.9.1):

$$0.01A_g \leq A_s \leq 0.08A_g \quad (3-16)$$

The concrete cover is the same as for a beam, but the clear cover in a column is different. In tied or spirally reinforced compression members, the clear distance between longitudinal bars shall not be less than $1.5d_b$ (where d_b is the diameter of the rebar) or 1.5 in , whichever is smaller (ACI 7.6.3).

3.5.2 Column Strength Interaction Diagram

A column strength interaction diagram describes the combination of axial loads and bending moments under which the column will not fail. The vertical axis of the interaction diagram represents the axial load and the horizontal axis represents the bending moment. To obtain a good representation of the interaction diagram, several key points are computed. The strength capacity of any column is compared to the applied axial force and bending moment. If the applied axial force and bending moment fall inside the interaction diagram, the capacity of the column is adequate. Figure 3-5 shows the form of the column strength interaction diagram.

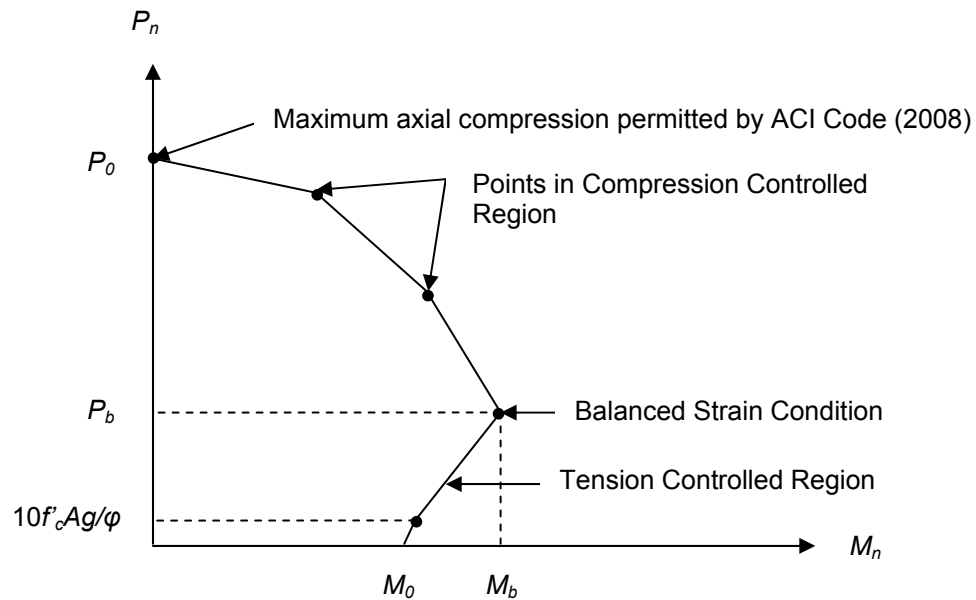


Figure 3-5. Column Strength Interaction Diagram

The points on an interaction diagram are generated by assuming different strains in the longitudinal steel reinforcements. From the strains, the neutral axis depth is calculated. The capacity of the column is then calculated. The axial and bending moment capacities based on the assumed strains can then be connected with a straight line to produce a strength interaction diagram similar to the diagram in Figure 3-5.

The strength interaction diagram has some specific regions (ACI Code 2008), namely the compression-controlled region and the tension-controlled region. These two regions are separated by the point of balanced failure. The maximum axial compression permitted by the ACI Code (2008) limits the maximum axial capacity to $P_{n(max)} = 0.80\phi P$ ($\phi = 0.65$) for tied columns, where P is the nominal axial capacity of the column calculated according to the Equation 10-1 prescribed by ACI Code (2008). The maximum axial capacity $P_{n(max)}$ is referred to as P_0 in Figure 3-5, represents the condition when only axial load is present. To obtain a point in the compression controlled region after the point P_0 , it is assumed that the entire section is in compression and ϕ value is 0.65. The strain in concrete at the extreme fiber of the compression face is taken as 0.003. The strain in steel closest to the tension face is also assumed to be in

compression and is estimated by an iterative procedure. Since the exact strain in the steel is not known, a very small value of strain is assumed initially. The strain is gradually increased until the axial load is equal to P_o . The final strain obtained from the last iteration is then used to compute the axial capacity and moment. Another point in the compression controlled region is obtained assuming that the depth of neutral axis is equal to the effective depth d of the section. The strain in the extreme compression face of concrete is assumed to be 0.003. The steel in the tension zone is assumed to have strain lower than the yield strain. Tension and compression in the steel and the concrete is obtained using the neutral axis location which eventually gives the values of axial load and moment. The balanced strain condition divides the compression controlled region from the tension controlled region. At balanced conditions, it is assumed that the tensile reinforcing steel has reached the yield limit at the same time concrete has reached its ultimate strain. The yield strain of steel, ϵ_y , is equal to f_y/E_s , where E_s is the modulus of elasticity for steel. When the steel strain is at its yield limit, the tensile part of the interaction diagram controls. The upper portion of the tension controlled region of the interaction diagram describes combinations of the axial load and the bending moment with strains closed to balanced condition. In the lower portion of the tension controlled region, the axial load and the bending moment are based on increasing the strain in the tension steel. To obtain a point in this region, the strain in the tension steel can be assumed to be two times f_y/E_s . The neutral axis depth is calculated based on the assumed strain. Once the neutral axis depth is known, the axial load and the bending moment is obtained from the tension and compression provided by the concrete and steel. The increasing strain in tension steel gradually reaches to a point where the axial capacity is $0.10f'_cA_g/\phi$ (ACI code 2008) and the section is fully tension controlled. The last point in the interaction diagram is M_o , which represents the condition when the section is subjected to bending moment only, and no axial load is applied.

CHAPTER 4

BIG BANG-BIG CRUNCH OPTIMIZATION METHOD

4.1 Introduction

Optimization problems can be generally defined as finding values for a set of variables that minimize or maximize an objective function (sometimes referred to as fitness function) which is subjected to a set of conditions that define the range of feasible solutions for the given problem. These conditions are called constraints. In most optimization problems for reinforced concrete structures, a large number of design variables are typically required. As a result, the search space can be large. Evolutionary heuristic algorithms can search large spaces for candidate solutions. These algorithms do not make any assumption about the problem subjected to optimization. Many heuristic algorithms are based on natural phenomena. Common heuristic algorithms based on natural systems are genetic algorithms (GA), simulated annealing (SA), ant colony optimization (ACO), and particle swarm optimization (PSO). In this study, a novel optimization algorithm, namely Big Bang-Big Crunch (BB-BC), which is based on one of the theories of evolution of the universe, is applied to the design of reinforced concrete frames. BB-BC is a population based heuristic search algorithm which can handle both continuous and discrete variables. The BB-BC algorithm has two phases: Big Bang and Big Crunch phase. In the Big Bang phase, energy dissipation produces disorder and randomness similar to the Big Bang phenomenon in nature. The Big Bang phase is represented by random distribution of candidate solutions over the search space. In the Big Crunch phase, randomly distributed particles are ordered by a contraction operator which uses the weighted average to calculate a center of mass of the population. Each sequential Big Bang is then created by the random distribution of candidate solutions about the center of mass calculated in the previous step, followed by subsequent Big Crunch to calculate a new center of mass. These cycles are repeated until a convergence criterion is reached. The objective of the optimization is to minimize the total weight (or cost) of the structure subjected to material and performance constraints usually in the form of stress and deflection limits. Each design is evaluated for fitness based on the values of the

design variables. If the design violates any given constraint, it is penalized. The penalized weight or cost represents the actual weight or cost of the structure and the degree to which the constraints are violated. Optimum design of reinforced concrete structures using a BB-BC algorithm is advantageous as it is a numerically simple with few control parameters and provides results that are comparable to other evolutionary algorithms (Camp 2007 and Kaveh and Talatahari 2010).

4.2 Methodology of Big Bang-Big Crunch Algorithm

In the BB-BC algorithm proposed by Erol and Eksin (2006), the initial big-bang is identical to the first step of the other evolutionary methods in that an initial population of candidate solutions is generated randomly over the entire search space. Erol and Eksin (2006) compared this random nature of the Big Bang to the energy dissipation or the transformation from an ordered state (a convergent solution) to a chaotic state (generation of new set of candidate solutions). In the Big Crunch phase following the Big Bang, a contraction operation is applied to randomly distributed candidate solutions. The contraction operator takes the current positions (represented by the values of the design variables) of each candidate solution in the population and its corresponding penalized fitness function value to compute a center of mass. The center of mass is the weighted average of the candidate solution positions where the weight associated with the position of each candidate solution is the inverse of the corresponding penalized fitness function. The averaging is done with respect to the inverse of the penalized fitness function values. The center of mass X_{cm} is computed as:

$$X_{cm} = \frac{\sum_{i=1}^{NC} \frac{X_i}{F_i}}{\sum_{i=1}^{NC} \frac{1}{F_i}} \quad (4-1)$$

where X_i is the position of candidate i in an n -dimensional search space, F_i is the penalized fitness function value of candidate i , and NC is the candidate population size.

New positions $X_{i,new}$ of the candidate solutions for the next iteration of the Big Bang are obtained using:

$$X_{i,new} = X_{cm} + \sigma \quad (4-2)$$

where σ is the standard deviation of the normal distribution.

In the BB-BC algorithm, the standard deviation σ is related to a subset of search space and obtained from:

$$\sigma = \frac{r \alpha (X_{max} - X_{min})}{n_{cycle}} \quad (4-3)$$

where r is a random number from a standard normal distribution, α is a parameter limiting the size of the search space, X_{max} and X_{min} are the upper and lower limits of the values of the design variables, and n_{cycle} is the number of Big Bang iterations.

For discrete variables, the continuous values $X_{i,new}$ are rounded to the nearest integer value as:

$$X_{i,new} = ROUND \left(X_{cm} + \frac{r \alpha (X_{max} - X_{min})}{n_{cycle}} \right) \quad (4-4)$$

Since normally distributed numbers can exceed ± 1 , it is necessary to limit candidate positions to the prescribed search space boundaries. As a result of this contraction, there is an accumulation of candidate solutions at the search space boundaries (Erol and Eksin 2006). The BB-BC algorithm can be summarized by the pseudo-code in Figure 4-1.

```

Initialize population
Do While (Stopping Criteria Not Satisfied)
    Compute fitness function values,  $F_i$ 
    Determine the center of mass,  $X_{cm}$ 
    Generate new candidate positions,  $X_{i,new}$ 
End Do

```

Figure 4-1. BB-BC Algorithm in Pseudo-code

4.3 Design of Reinforced Concrete Frames Using BB-BC

The BB-BC algorithm allows an open format for constraints statements and it does not require an explicit relationship between the objective function and the constraints. In the optimization process of a reinforced concrete structure the objective is to minimize the cost of the structure. The total cost is the summation of cost for materials and cost for construction. In this research work, another form of the objective function is the amount of CO₂ produced due to the materials used in frames. The estimation of the amount of materials is done in terms of cross-sectional dimensions of the structural members of the frame. So the design variables are mainly the cross-sectional dimensions and the reinforcement. For each candidate frame design, stresses, deflections, and other constraints to ensure proper construction are evaluated to check if the design is feasible. If there is any violation of constraints, a penalty function is applied to the structural weight which increases either the cost or the amount of CO₂. The penalized weight helps focus the search on designs with the smallest structural weight that satisfy the design constraints.

In this study, an optimal frame design is the one where the total cost or CO₂ produced is minimized. The two objective functions are not minimized simultaneously. In this optimization process, the construction of the reinforced concrete frame is split into material and construction components. Unit cost and emissions are estimated for concrete and steel. Unit CO₂ emissions are estimated from the information on the production and placement of concrete and steel in

structures. In addition, cost or CO₂ emissions associated with formwork and scaffolding are considered.

The general form of the optimization problem is given as:

$$\text{Minimize } C = \sum_{i=1,R} p_i u_i (x_1, x_2, \dots, x_n) \quad (4-5)$$

$$\text{Minimize } CO_2 = \sum_{i=1,R} p_i e_i (x_1, x_2, \dots, x_n) \quad (4-6)$$

$$\text{Subject to } c_j(x_1, x_2, \dots, x_n) \leq 0 \quad (4-7)$$

where C is the cost function, CO_2 is the CO₂ emission function, p_i is the unit cost, u_i is the amount of material and construction units, x_i is a design variable, R is the number of material and construction units, e_i is the amount of material and construction units for CO₂ emission, and c_j are the constraint functions. Here the constrained problem is transformed to an unconstrained problem by means of the penalty function.

The BB-BC algorithm for designing reinforced concrete frames follows the general procedure elaborated in Figure 4-1 except for two additional steps implemented by Camp (2007) to improve the computational efficiency and performance. The positions of candidate solutions at the beginning of each Big Bang are normally distributed around a new point between the center of mass, X_{cm} , and the best global solution, X_{best} , as:

$$X_{i,new} = \beta X_{cm} + (1 - \beta) X_{best} + \frac{ra(X_{max} - X_{min})}{n_{cycle}} \quad (4-8)$$

where β is a parameter which controls the influence of the X_{best} on the location of new candidate solutions.

Numerical studies (Camp et al. 2007) indicate that there is significant improvement in the quality of solutions and the computational efficiency of the BB-BC algorithm using Equation (4-8) over the original equation developed by Erol and Eksin (2006). The weighted average of X_{best} and X_{cm} is controlled by β in Equation (4-8), so that the best solution is allowed to influence the direction of search. In addition, another improvement of the BB-BC algorithm is to employ a multiphase search. In a two phase search, the BB-BC algorithm is initially applied to the entire search space. After convergence of Phase I, the Phase II search starts in a reduced search around X_{best} from Phase I. The search space is reduced to 20% of the original search space (Camp 2007).

In order to improve the computational efficiency of BB-BC algorithm, Kaveh et al. (2010) uses the capabilities of PSO. In PSO optimization, the social behavior of bird flocking and fish schooling is modeled as a population of individuals called particles. Their movement is directed by both their own experience and the population's experience. For every iteration, a particle moves towards a direction computed from the local best solution and the global best solution. This concept is used in this research work where the BB-BC algorithm not only utilizes the center of mass but also employs the global best solution to generate the new solution.

A modified version of Equation (4-8) is defined as:

$$X_{i,new} = \alpha_2 X_{cm} + (1 - \alpha_2) [\alpha_3 X_{l,best} + (1 - \alpha_3) X_{g,best}] + \frac{ra(X_{max} - X_{min})}{n_{cycle}} \quad (4-9)$$

where α_2 , α_3 are controlling parameters, $X_{l,best}$ is the local best solution, and $X_{g,best}$ is the global best solution.

$X_{i,new}$ is a weighted average between local and global best solutions. In Equation (4-9) α_2 and α_3 are adjustable parameters that control the influence of the local and the global best solutions on the new positions of candidate solutions. A study is done on a one-bay one-story frame to determine the value of α_2 and α_3 . The possible values range from 0 to 1 with an

increment of 0.1. A total of 121 possible combinations of values for the parameters α_2 and α_3 are tested, where each pair of values gave a different average magnitude of the cost function. From this study, the values of α_2 and α_3 are taken as 0.3 and 0.5 respectively. The constant value of α_1 is taken as 1.0 (Camp 2007). In the previous research work by Kaveh et al. (2010), the values of the parameters are 1.0, 0.40 and 0.80 for α , α_2 and α_3 . Table 4-1 lists the results of the one-bay one-story study. Figure 4-2 shows the change of the parameter values with respect to cost.

Table 4-1. Values of α_2 and α_3 for Average Cost of One-Bay One-Story Frame

α_2 α_3	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0	2,546.6	2,541.2	2,535.1	2,531.2	2,528.3	2,525.3	2,525.5	2,523.9
0.1	2,534.3	2,538.1	2,531.3	2,533.1	2,534.8	2,525.1	2,516.6	2,516.9
0.2	2,536.7	2,542.1	2,525.6	2,531.0	2,523.6	2,527.2	2,521.7	2,521.4
0.3	2,538.3	2,532.5	2,532.3	2,522.9	2,530.5	2,531.8	2,523.9	2,524.3
0.4	2,523.0	2,526.0	2,533.8	2,531.7	2,525.5	2,524.5	2,532.9	2,531.9
0.5	2,541.0	2,531.1	2,524.8	2,537.0	2,536.0	2,537.8	2,535.6	2,533.1
0.6	2,548.8	2,542.1	2,544.5	2,539.1	2,557.8	2,546.1	2,542.3	2,545.9
0.7	2,552.8	2,552.1	2,550.8	2,547.5	2,568.4	2,565.2	2,546.8	2,545.5
0.8	2,570.5	2,572.9	2,559.4	2,560.5	2,566.9	2,555.5	2,564.7	2,574.0
0.9	2,585.6	2,572.6	2,567.4	2,576.7	2,567.2	2,564.2	2,560.3	2,568.5
1	2,572.9	2,575.8	2,557.3	2,583.9	2,574.2	2,572.7	2,569.4	2,577.3

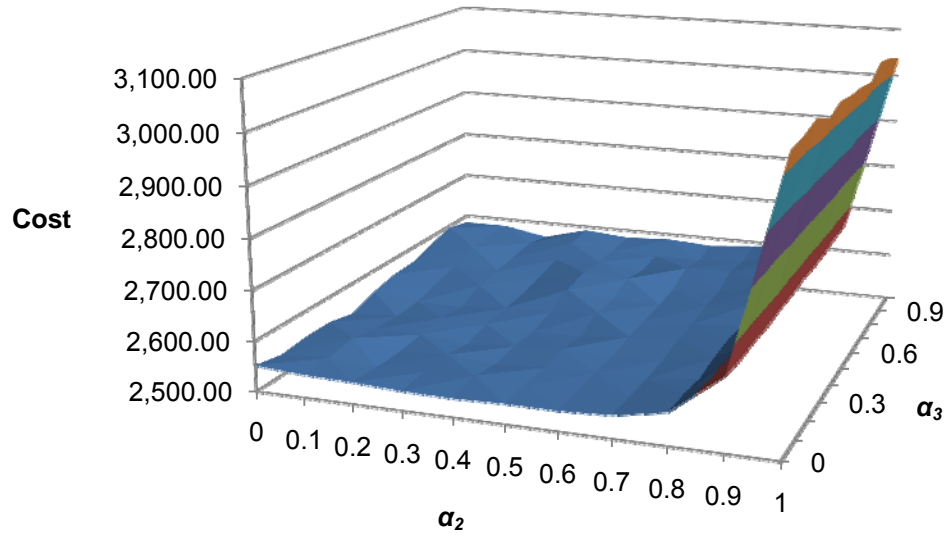


Figure 4-2. Values of α_2 and α_3 for One-Bay One-Story Frame

The BB-BC frame design process begins by randomly selecting values for the depth, width, and steel reinforcement for each beam and column. The values for the design variables are chosen within the prescribed limits. The fitness function of each candidate design is then computed using Equation (4-5) for cost and (4-6) for the amount of CO₂. Each design is also analyzed to determine if all the constraints are satisfied. The constraints are divided into beam constraints and column constraints.

A reinforced beam must have a structural capacity to resist the factored applied loading and also meet the various specifications given in ACI Code (2008). If the shear or moment capacity is less than what is required the beam cost is penalized. In addition, ACI Code (2008) has restrictions and limitations on the cross-sectional geometry and quantity of steel reinforcement. The basic form of the constraints c_i are:

$$c_i = \begin{cases} 0 & \text{if } m_i < 0 \\ m_i & \text{if } m_i > 0 \end{cases} \quad (4-10)$$

where m_i is the normalized degree of violation of constrain c_i .

The penalty over normalized moment capacity is:

$$m_1 = \frac{|M_u| - \phi M_n}{\phi M_n} \quad (4-11)$$

Since each beam section is being considered as singly reinforced, the capacity provided by the positive and negative reinforcement is checked separately.

According to ACI Code (2008) the minimum amount of reinforcement that should be provided in a beam is:

$$\rho_{min} = 3 \frac{\sqrt{f'_c}}{f_y} \text{ and } \rho_{min} \geq \frac{200}{f_y} \quad (4-12)$$

If ρ is less than ρ_{min} , the penalty for violating the minimum reinforcement ratio is:

$$m_2 = \rho_{min} - \rho \quad (4-13)$$

To ensure that the beam will fail in tension, an upper limit on the reinforcement ration is defined as:

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \quad (4-14)$$

where ϵ_t value is dependent on section flexural property (see Chapter 3). If ρ is greater than ρ_{max} then the maximum reinforcement penalty is:

$$m_3 = \rho_{max} - \rho \quad (4-15)$$

The ACI Code (2008) specifies a minimum thickness to resist deflection for non-prestressed beams for various support conditions. If the thickness of the beam is less than the minimum thickness h_{min} prescribed by the ACI Code (2008) the beam thickness penalty is:

$$m_4 = \frac{h_{min} - h}{h_{min}} \quad (4-16)$$

The ACI Code (2008) limits the minimum spacing between parallel bars in a layer. The minimum clear spacing t_d between parallel bars should be 1.5 times d_b but not less than 1 in. So the minimum width of the beam b_{min} becomes:

$$b_{min} = 2x_c + 2x_t + d_b n_b + t_d (n_b - 1) \quad (4-17)$$

where x_c is clear cover for reinforcement, x_t is the diameter of stirrups, n_b is the number of bars, and t_d is the total width excluding bar diameter, clear cover, and diameter of stirrups. If the width is less than the minimum width, the penalty value is:

$$m_5 = \frac{b_{min} - b}{b_{min}} \quad (4-18)$$

There is also restriction on the maximum depth of a beam, h_{max} , which is based on the width of the beam. If the depth h is 2.5 times greater than the width, the beam is considered as deep beam. As the beams considered here are not deep beams, the penalty for maximum thickness h_{max} is:

$$m_6 = \frac{h - h_{max}}{h_{max}} \quad (4-19)$$

For a given geometry, if the rectangular stress-block depth a is greater than the effective depth d the penalty is:

$$m_7 = \frac{a - d}{d} \quad (4-20)$$

A reinforced concrete column must have sufficient structural capacity to withstand combined effects of axial force and bending moment while meeting specifications defined in ACI Code (2008). A load-moment interaction diagram as shown in Figure 3-5 is constructed for each column in a frame. If the combination of the axial load and the bending moment on a column fall inside the load-moment interaction diagram, the capacity of the designed column is adequate. If not, then the column does not have adequate strength and is penalized. The penalty for violating the load-moment diagram constraint is:

$$c_8 = \begin{cases} 0 & \text{if Load-Moment Interaction Satisfied} \\ m_8 & \text{if Load-Moment Interaction Not Satisfied} \end{cases} \quad (4-21)$$

The quantity m_8 is a measure of the degree of violation from the load-moment interaction diagram. The capacity of a column is checked by the residue theorem technique (Camp et al. 2003) which determines whether the column lies within the load-moment diagram. The numerical value for load moment interaction penalty is:

$$m_8 = \frac{r_1}{r_0} - 1 \quad (4-22)$$

where r_1 is the radial distance measured from the load-moment combination for a particular column to the origin of the interaction diagram and r_0 is the radial distance from the intersection of the vector r_1 and the load-moment curve to the origin.

The ACI Code (2008) limits the longitudinal reinforcement in compression members such that it shall not be less than 1% or more than 8% of the gross area A_g or the total area of the section. The maximum or minimum reinforcement penalty is:

$$m_{\theta} = \begin{cases} 0.01 - \rho_g & \text{if } \rho_g \leq 0.01 \\ \rho_g - 0.08 & \text{if } \rho_g > 0.08 \end{cases} \quad \rho_g = \frac{A_s}{A_g} \quad (4-21)$$

The ACI Code (2008) has specifications for tied reinforced compression members. According to the specifications, the clear distance between longitudinal bars shall not be less than 1.5 times d_b or 1.5 in. The longitudinal bar spacing penalty is:

$$m_{10} = \frac{d_{min} - d_c}{d_{min}} \quad (4-22)$$

where d_{min} is the allowable clear spacing between the bars according to the ACI Code (2008) and d_c is the spacing between the longitudinal reinforcement.

A penalty function is used to enforce the constraints c_j on the objective function. The total objective function penalty θ_i for a candidate design i is a function of the summation of capacity, reinforcement configuration, and geometric constraints defined as:

$$\theta_i = \left(1 + \sum_{k=1}^{10} c_k \right)^{\eta} \quad (4-23)$$

where η is a positive penalty exponent (typically > 1).

The penalized objective function F_i is a product of the objective function of candidate design i and its total penalty:

$$F_i = \theta_i f_i \quad (4-24)$$

The penalty function imposes a numerical penalty on the value of the objective function that tends to reflect the degree to which the constraints are violated by a candidate set of design variables. As the value of the penalty function exponent η increases, the penalty for a given candidate design increases. In Phase I of the BB-BC algorithm, if $\eta > 2$, the search tends to be more exploitive and less explorative; generating solutions that while feasible, are too costly to be considered good designs. However, during Phase II, in a reduced search space, a larger penalty should be applied to control the unfavorable tendency of convergence to light, but slightly infeasible designs. For all the frame design examples, $\eta = 2$ in Phase I and $\eta = 4$ in Phase II.

An initial population is generated randomly within the entire search space using a uniform random number distribution. The penalized objective function for each candidate solution, defined by a set of design variables, is calculated as defined in Equation (4-24). The next set of candidate solutions is normally distributed about the center of mass X_{cm} and the best global solution $X_{g\ best}$ according to Equation (4-9). New candidate populations are generated iteratively going through sequences of Big Bangs and Big Crunches until the global best solution $X_{g\ best}$ has not changed for a number of consecutive iterations; with this condition reached, the BB-BC algorithm is considered to have converged to a solution. At this point, Phase I of the BB-BC search is complete and Phase II search is initiated in the region surrounding $X_{g\ best}$. In Phase II, a local search space is defined around $X_{g\ best}$ from Phase I and the immediate neighborhood of each design. In Phase II, a new set of candidate solutions $X_{i,new}$ are randomly generated within the local search space with $X_{g\ best}$ from Phase I being retained or reset. Convergence for each phase is determined when the value of $X_{g\ best}$ has not improved for a specified number of analyses. Selection of $X_{g\ best}$ is limited to solutions that are feasible, in other words, designs that have no penalty applied to their objective function values.

The unique characteristics of the hybrid BB-BC are its simplified numerical structure and its dependence on a relatively small number of controlling parameters. Without considering some common parameters like population size and convergence criteria, BB-BC has three controlling parameters. When a multi-phase search is applied, another parameter referred to as search reduction parameter is needed. In contrast, a simple GA requires a minimum of three parameters and may need more in complex algorithms. PSO requires values for at least five parameters. A simple ACO requires four parameters with an additional search space reduction parameter for performing multi-phase search.

CHAPTER 5

DESIGN EXAMPLES

5.1 Introduction

In this chapter, three design examples will be presented. The designs obtained using BB-BC will be compared to the other researchers' designs.

5.2 Design Examples

Among the three design examples considered for this research, the smallest geometry is a two-bay four-story frame. The other two examples are two different two-bay six-story frames. The two-bay six-story frame is considered twice with different bay distance, design constraints, and applied loadings.

5.3 Two-Bay Six-Story Frame Design Using Genetic Algorithm

The two-bay six-story frame shown in Figure 5-1 was originally designed by Rajeev and Krishnamoorthy (1998). The design conformed to the Indian Standard Code of Practice (IS 1978) for reinforced concrete and did not account for the shear capacity of the beam sections. The structural members were divided into three column groups and four beam groups (see Figure 5-2). Each column group consisted of all co-linear columns extending from ground to roof; therefore, there are 3 column groups. The beams were divided into two main groups, floor beams and roof beams. Each of the two beam groups had two subdivisions based on the bay length. The reason for grouping the beams was to reduce the number of design variables in the structure. In addition, the grouping also reduced the combinatorial complexity of the optimization problem.

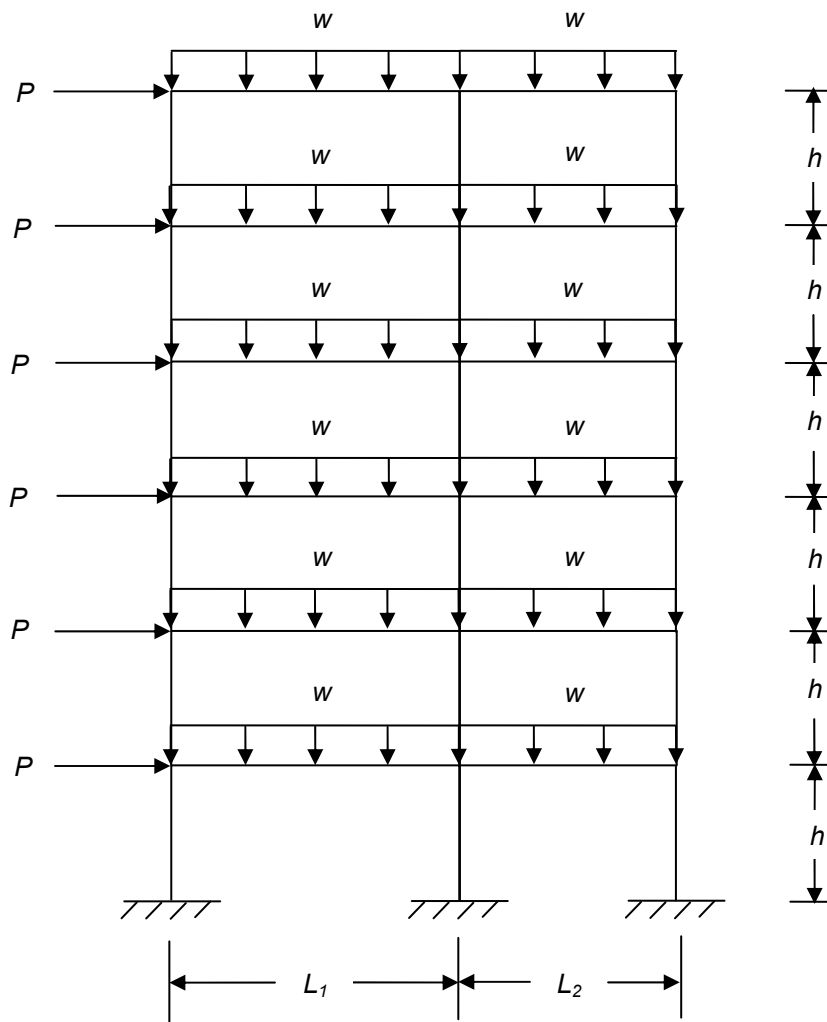


Figure 5-1. Geometry and Loading for Two-Bay Six-story Frame

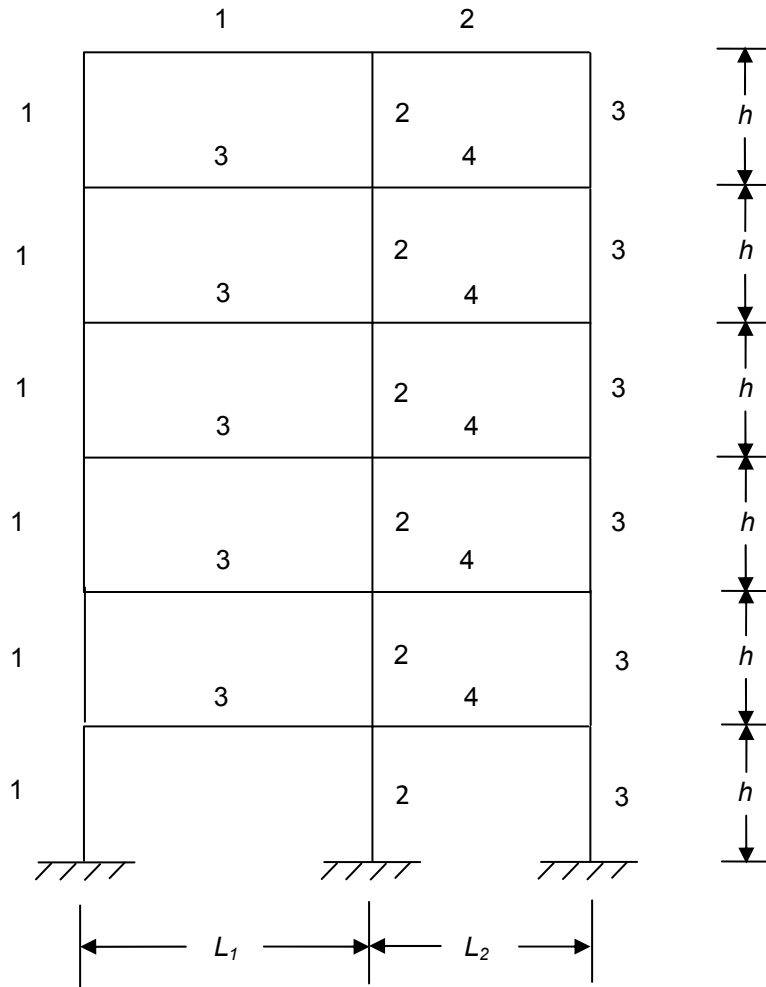


Figure 5-2. Beam and Column Groups for Two-Bay Six-Story Frame

Each story in the structure had a height of 4 m (13.12 ft), for a total height of 24 m (78.72 ft). The width of the left bay was 6 m (19.69 ft) while the right bay had a width of 4 m (13.12 ft) for a total bay width of 10 m (32.81 ft). A factored uniformly distributed vertical load of $w = 30\text{ kN/m}$ ($2,056\text{ lb/ft}$) was applied on every beam in the structure and lateral loads of $P = 10\text{ kN}$ ($2,248\text{ lb}$) were applied at each story. The unit weight of concrete was approximately 2323 kg/m^3 (145 lb/ft^3) and the unit weight of steel was 7849 kg/m^3 (490 lb/ft^3). The 28-days strength of concrete was $f'_c = 20\text{ MPa}$ ($3,000\text{ psi}$) and yield strength of steel was $f_y = 414\text{ MPa}$ ($60,000\text{ psi}$).

Camp et al. (2003) proposed the design of the two-bay six-story frame previously done by Rajeev and Krishnamoorthy (1998). The design of the frame by Camp et al. (2003) was done using the ACI Code (1999). The geometry, design variables, loading, and optimization algorithm were the same as Rajeev and Krishnamoorthy (1998). The major differences between these two designs were the design code, the cost objective function, and the choice of beam column sections. Table 5-1 lists the search space for the two-bay six-story frame design.

Table 5-1. Search Space Parameters for Two-Bay Six-Story Frame.

Column	b		h		A_s	
	(m)	(in)	(m)	(in)	Number of bars	Bar size
Min	0.15	6	0.18	7	4	3
Max	0.56	22	0.56	22	12	11
Increment	0.03	1	0.03	1	2	1
Beam	b		h		A_s	
	(m)	(in)	(m)	(in)	Number of bars	Bar size
Min	0.20	8	0.30	12	1	3
Max	0.46	18	0.84	33	4	11
Increment	0.03	1	0.03	1	1	1

In the design proposed by Camp et al. (2003), the design variables for beams were width, depth, compression, and tension reinforcement. Feasible beam cross-sections in compliance with ACI Code (1999), were generated and sorted based on the amount of reinforcement provided in the section. The sorting was in ascending order (small to large). The Beam section with the smallest amount of reinforcement was placed first in the database and the section with the largest amount of reinforcement was placed last. The smallest area of steel that can be placed in a beam is one #3 bar and the maximum is four #11 bars for 36 possible bar combinations. Table 5-2 lists the steel patterns from smallest to largest. Then each section was checked for minimum spacing between parallel bars in a row, maximum and minimum reinforcement ratio, placement of bars within and between the rows, maximum depth, and minimum width of beams. The sections within the limits prescribed by ACI Code (1999) for the criteria mentioned were considered feasible beam sections. The possible number of beam sections based on fulfilling the ACI Code (1999)

and placement constraints were determined to be 50,526. The feasible beam sections and their corresponding steel reinforcing are shown in Table 5-3.

Table 5-2. Ordered Steel Area from Smallest to Largest

Index #	Bar Combination	Area of Steel	
		(mm^2)	(in^2)
1	3	71.29	0.1105
2	4	126.64	0.1963
3	3,3	142.58	0.2210
4	5	197.94	0.3068
...
33	9,9,9,9	2580.64	4
34	11,11,11	3022.25	4.6845
35	10,10,10,10	3269.15	5.0672
36	11,11,11,11	4029.67	6.2460

Table 5-3. Ordered Feasible Beam Table

Index #	b		h		Steel Bar Combination	
	(m)	(in)	(m)	(in)	$A_{tension}$	$A_{compression}$
1	0.20	8	0.46	18	7	7
2	0.20	8	0.46	18	7	9
3	0.20	8	0.46	18	7	10
4	0.20	8	0.46	18	7	11
...
50,523	0.53	21	0.84	33	36	33
50,524	0.53	21	0.84	33	36	34
50,525	0.53	21	0.84	33	36	35
50,526	0.53	21	0.84	33	36	36

For column sections, the numbers of possible bars were limited to an even number to avoid possible eccentricity in the column. Figure 5-3 shows five reinforcing topologies represent combinations of four, six, eight, ten or twelve bars. Column design variables were height and width of the cross-section, size of longitudinal reinforcing bars, and topology. The possible bar

sizes were the same as beams, ranging from #3 to #11 bar. To simplify the design, the reinforcing bars in the column had the same size. All the feasible column designs based on placement of the bars and the reinforcing limits were determined in a manner similar to that used for beams.

Feasible columns were sorted by the moment capacity of the section at balanced condition. Table 5-4 lists the ordered feasible column sections and their corresponding longitudinal steel.

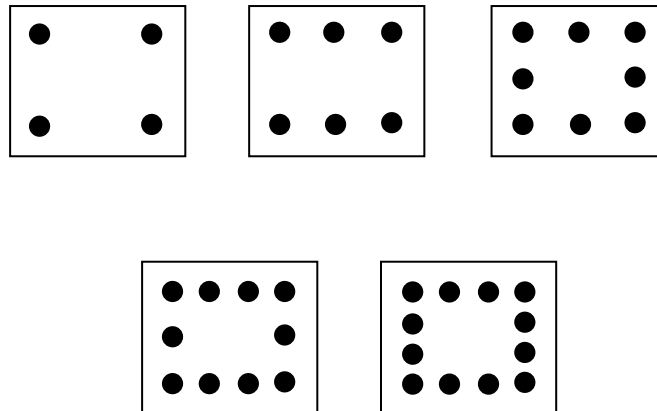


Figure 5-3. Possible Topologies for Column Design for Two-Bay Six-Story Frame

Table 5-4. Ordered Feasible Column Table

Index #	<i>b</i>		<i>h</i>		Steel Bar Combination	
	(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)	Number of Bars	Bar #
1	0.18	7	0.18	7	6	3
2	0.18	7	0.18	7	8	3
3	0.18	7	0.18	7	4	4
4	0.18	7	0.18	7	4	5
...
3,108	0.51	20	0.56	22	12	11
3,109	0.56	22	0.56	22	10	11
3,110	0.51	20	0.56	22	12	11
3,111	0.56	22	0.56	22	12	11

The objective for this two-bay four-story frame design was to minimize structural cost. Camp et al. (2003) implemented a GA to find the near optimal solution of this cost optimization problem. The mathematical form of the objective function is:

$$F = \sum_{i=1}^{n_b+n_c} \{C_c b_i h_i + C_s A_{s_i} + 2C_f (b_i + h_i)\} l_i \quad (5-1)$$

where C_c is the unit cost of concrete, C_s is the unit cost of steel reinforcement, A_{s_i} is the area of the reinforcing bar, C_f is the unit cost of formwork, b_i is the width of beam or column section, h_i is the height of beam or column section, l_i is the length of the member, n_b is the number of beams, and n_c is the number of columns.

The cost of concrete, steel, and formwork was given as $\$735/m^3$ ($\$20.81/ft^3$), $\$7.1/kg$ ($\$1,578/ft^3$), and $\$54/m^2$ ($\$5.02/ft^2$) respectively (Rajeev and Krishnamoorthy 1998). The cost function used by Rajeev and Krishnamoorthy (1998) was slightly different from the cost function used by Camp et al. (2003). The cost function used by Rajeev and Krishnamoorthy (1998) includes two rows of reinforcement in both compression and tension zones as per their requirements. The outer row of reinforcement is continuous and has the same number of equal sized bars throughout the co-linear spans of the frame. The cost function also incorporates the bar cut-off lengths for the inner row of the reinforcement in both the tension and compression bar groups. The design presented by Camp et al. (2003) utilizes one row of continuous reinforcement in both the positive and negative moment zones and excludes bar cut-off.

The design proposed by Rajeev and Krishnamoorthy (1998) was only partially acceptable according to the ACI Code (1999). Table 5-5 shows the comparison between the designs, without considering shear reinforcement, done by Camp et al. (2003) and Rajeev and Krishnamoorthy (1998). The best design developed by Camp et al. (2003) had a cost of \$ 24,959, whereas the Rajeev and Krishnamoorthy design had a cost of \$26,052, a cost reduction of 4.2%. Though the cost reduction was small, the design generated by Camp et al. (2003) conformed to the

specifications of ACI Code (1999), while the beams in element groups 1, 2 and 3 and the columns in element groups 6 and 7 of the Rajeev and Krishnamoorthy (1998) design did not conform to the ACI Code (1999). The factored nominal moment capacities of these sections are between 3.4% and 125% less than the required moment. In order for this design to be acceptable according to ACI specifications, 19 beam and column elements must be redesigned, which would increase the cost significantly. The cost of the beam designs by Camp et al. (2003) was higher than the Rajeev and Krishnamoorthy (1998) design and the cost of the columns was lower. The beams designs prescribed by Camp et al. (2003) had required the moment capacity. All the column groups in Camp et al. (2003) design had rectangular cross-section with the larger dimension placed in the direction of lateral loads.

The computational time for 300 generations with a population size of 300 was about 13 hour. A large population was required due to the size of the geometrically feasible design space consisted of approximately 1.96×10^{29} possible solutions.

Table 5-5. Design Results for Two-Bay Six-Story Frame

Rajeev & Krishnamoorthy (1998)		Beam Group Number				Column Group Number		
		1	2	3	4	5	6	7
<i>b</i>	(<i>m</i>)	0.20	0.20	0.20	0.20	0.25	0.25	0.25
	(<i>in</i>)	7.90	7.90	7.90	7.90	9.90	9.90	9.90
<i>h</i>	(<i>m</i>)	0.35	23.09	0.35	0.29	0.25	0.25	0.29
	(<i>in</i>)	13.8	9.9	13.8	11.8	9.9	9.9	11.8
$A_{s\ bottom} (in^2)$		2 #4	2 #4	2 #5	1 #11	6 #7	6 #7	6 #7
$A_{s\ top} (in^2)$		2 #5	2 #5	1 #9	1 #9			
Cost		\$26,052						
Camp et al.(2003)		Beam Group Number				Column Group Number		
		1	2	3	4	5	6	7
<i>b</i>	(<i>m</i>)	0.28	0.33	0.23	0.20	0.18	0.18	0.18
	(<i>in</i>)	11	13	9	8	7	7	7
<i>h</i>	(<i>m</i>)	0.56	0.48	0.56	0.48	0.20	0.46	0.28
	(<i>in</i>)	22	19	22	19	8	18	11
$A_{s\ bottom} (in^2)$		2 #6	1 #5	4 #4	1 #6	4 #5	4 #7	4 #4
$A_{s\ top} (in^2)$		2 #8	2 #7	1 #11	2 #5			
Cost		\$24,959						

5.4 Two-Bay Six-Story Frame Design Example by BB-BC Algorithm

A new design of the two-bay six-story frame previously done by Camp et al. (2003) and Rajeev and Krishnamoorthy (1998) is proposed in the current research work. The geometry, loading, and topology are same as the previous designs described in Section 5.3. However, the set of feasible beam and column sections are not generated for this design. For each beam there are four design variables: width, height, tension reinforcement, and compression reinforcement. For each column there are four design variables: width, thickness, rebar sizes, and reinforcement topology. The total number of design variables is 28. Like the previous design, the moment and the shear capacity in each beam and the moment capacity in each column are checked. The main difference between these two designs is the optimization algorithm. The new design is done using the BB-BC algorithm instead of a GA. Table 5-6 lists the results of the BB-BC design.

Table 5-6. Design Results for Two-Bay Six-Story Frame Using BB-BC

Camp et al.(2003)		Beam Group Number				Column Group Number		
		1	2	3	4	5	6	7
<i>b</i>	(<i>m</i>)	0.28	0.33	0.23	0.20	0.18	0.18	0.18
	(<i>in</i>)	11	13	9	8	7	7	7
<i>h</i>	(<i>m</i>)	0.56	0.48	0.56	0.48	0.20	0.46	0.28
	(<i>in</i>)	22	19	22	19	8	18	11
$A_{s\ bottom} (in^2)$		2 #6	1 #5	4 #4	1 #6	4 #5	4 #7	4 #4
$A_{s\ top} (in^2)$		2 #8	2 #7	1 #11	2 #5			
Cost		\$24,959						
BB-BC		Beam Group Number				Column Group Number		
		1	2	3	4	5	6	7
<i>b</i>	(<i>m</i>)	0.36	0.33	0.20	0.23	0.18	0.28	0.15
	(<i>in</i>)	14	13	8	9	7	11	6
<i>h</i>	(<i>m</i>)	0.48	0.43	0.48	0.33	0.28	0.25	0.20
	(<i>in</i>)	19	17	19	13	11	10	8
$A_{s\ bottom} (in^2)$		3 #5	1 #9	2 #6	2 #5	4 #5	8 #5	6 #3
$A_{s\ top} (in^2)$		1 #10	1 #10	2 #9	2 #6			
Cost		\$23,664						

The BB-BC optimization initializes with 300 random candidate solutions. The best solution reported by BB-BC algorithm in Phase I is \$23,911, which is 4.2% less than the best solution \$24,959 given by GA (Camp et al. 2003). In a series of 100 BB-BC design runs, the average cost of the frames is \$27,010.31 with a standard deviation of \$1035.11. The algorithm required 9,927 analyses on average to converge to a solution which is considerably less than the number of analyses needed in case of implementation of GA (Camp et al. 2003) for the same frame.

In Phase II the best solution is \$23,664, which is 5.2% less than design developed using GA (Camp et al. 2003). The average cost in Phase II is \$26,520.55 with a standard deviation of

\$1,069.91. The average number of analyses required for convergence is 12,672. Figure 5-4 shows a typical convergence plot of BB-BC in Phase I and II.

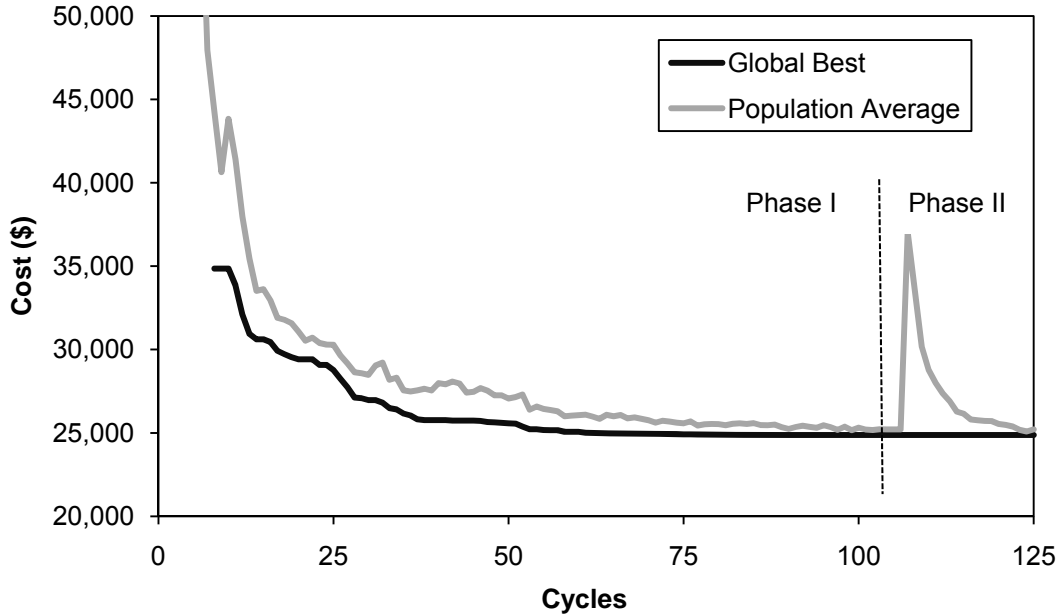


Figure 5-4. Typical Multi-phase Convergence History of Two-Bay Six-Story Frame

5.5 Two-Bay Four-Story Frame Design Example by BB-BC Algorithm

A different two-bay four-story frame was chosen for design using BB-BC algorithm. This frame was originally designed by Paya et al. (2008) using SA in compliance with Spanish Code. Figure 5-5 shows the geometry of the frame where the length L of each bay is 5 m (16.4 ft), and height h of each story is 3 m (9.84 ft). The frame is organized into 4 beam groups and 8 column groups. Each beam group consists of all the beams in a story level, thus beams within a story have same cross sections. Each story is formed by different column groups. There are two different column groups in a story, one group consists of the exterior columns and the other group consists of the interior column. Figure 5-6 shows the beam and column groups. There are more design variables in this example than in the frame presented by Rajeev and Krishnamoorthy (1998) and Camp et al. (2003). The higher number of design variables has increased the complexity of the problem

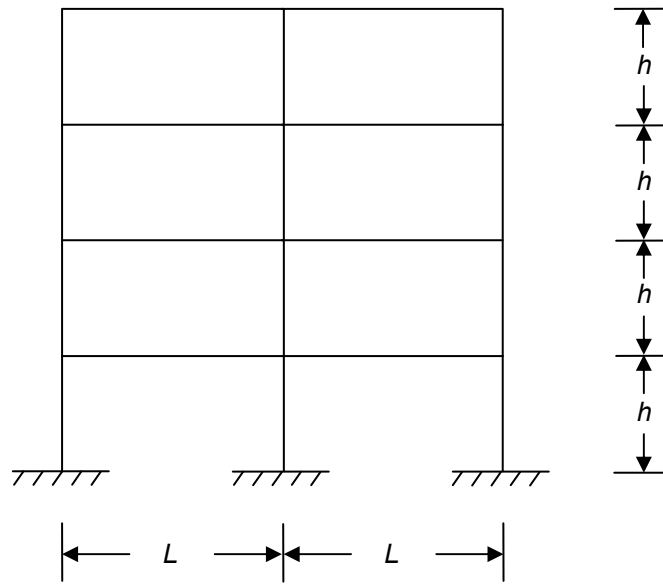


Figure 5-5. Geometry of Two-Bay Four-Story Frame

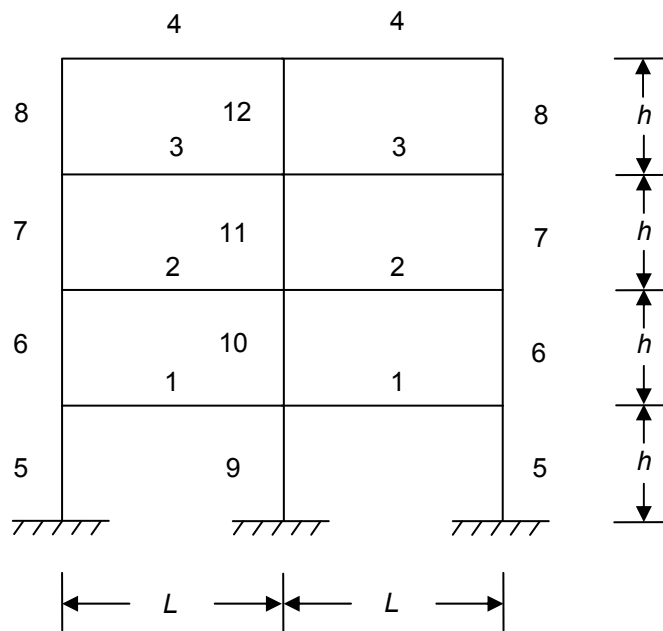


Figure 5-6. Beam and Column Groups for Two-Bay Four-Story Frame

The total height of the structure is 12 m (39.36 ft) and total width is 10 m (32.8 ft). The distance between the planar frames is 5 m (16.4 ft). For the design, uniformly distributed dead and live loads are used on each beam. Dead loads in floor level are different from loads on roof level. Three types of pattern live loading are used to account for the maximum positive and negative moments. In addition, wind loads are applied from both sides of the frame. Different load factors are used to increase dead, live, and wind loads. Strength reduction factors are used to decrease the strength of both steel and concrete. Table 5-7 lists the values of different factors used in the design. Twelve load combinations including the dead, live, and wind load are used for the design of the frame. The combinations, according to Spanish Code, take account of the extreme events that the structure may have to withstand during its lifetime. The loads and combinations are obtained from Paya et al. (2008). The values for uniform loads, wind loads, and combinations are shown in Table 5-8, Table 5-9, and Table 5-10; respectively. Figures 5-7 and 5-8 show dead and pattern loadings and wind loads for two-bay four-story frame. The unit weight of concrete was approximately 2323 kg/m^3 (145 lb/ft^3) while steel had a unit weight of 7849 kg/m^3 (490 lb/ft^3). The yield strength of steel is 500 MPa (72519 psi) for this design, whereas the concrete strength is different in each floor and usually varies in a range from 25 MPa ($3,626 \text{ psi}$) to 50 MPa ($7,252 \text{ psi}$).

Table 5-7. Load and Strength Reduction Factor for Two-Bay Four-Story Frame

Load Type	Load Factors	Values
Dead Load	γ_g	1.5
Live Load	γ_q	1.6
Wind Load	γ_g or $0.9 \gamma_q$	0.9
Material Type	Strength Reduction Factor	Values
Concrete	γ_c	1.5
Steel	γ_s	1.15

Table 5-8. Dead and Live Load Applied on the Frames

Description of the action	Type of Loading	Value	
		(<i>kN/m²</i>)	(<i>lb/ft²</i>)
Dead loads in Floors 1 to 3	Self-weight	3	62.65
	Weight of pavement	1	20.89
Dead loads in roof	Self-weight	3	62.65
	Weight of roof materials	3	62.65
Live load in Floors 1 to 3	Live Load	3	62.65
Live load in roof floor	Live Load	1	20.89

Table 5-9. Wind Load for Two-Bay Four-Story Frame

Story level	Wind Load	
	(<i>kN</i>)	(<i>kip</i>)
roof	5.81	1.31
3	10.74	2.41
2	9.86	2.22
1	8.83	1.99

In Table 5-10, DL indicates dead load, LL1, LL2, and LLTOT indicate the three pattern live loads, W1 is wind load applied on the structure from left to right, and W2 is opposite of W1. The 12 combinations are the summation of loads multiplied by the load factors.

Table 5-10. Load Combinations for Two-Bay Four-Story Frame

Combination number	DL	LL1	LL2	LLTOT	W1	W2
1	γ_g	0	0	0	0	0
2	γ_g	γ_q	0	0	0	0
3	γ_g	0	γ_q	0	0	0
4	γ_g	0	0	γ_q	0	0
5	γ_g	0	0	0	γ_q	0
6	γ_g	0	0	0	0	γ_q
7	γ_g	$0.9\gamma_q$	0	0	$0.9\gamma_q$	0
8	γ_g	0	$0.9\gamma_q$	0	$0.9\gamma_q$	0
9	γ_g	0	0	$0.9\gamma_q$	$0.9\gamma_q$	0
10	γ_g	$0.9\gamma_q$	0	0	0	$0.9\gamma_q$
11	γ_g	0	$0.9\gamma_q$	0	0	$0.9\gamma_q$
12	γ_g	0	0	$0.9\gamma_q$	0	$0.9\gamma_q$

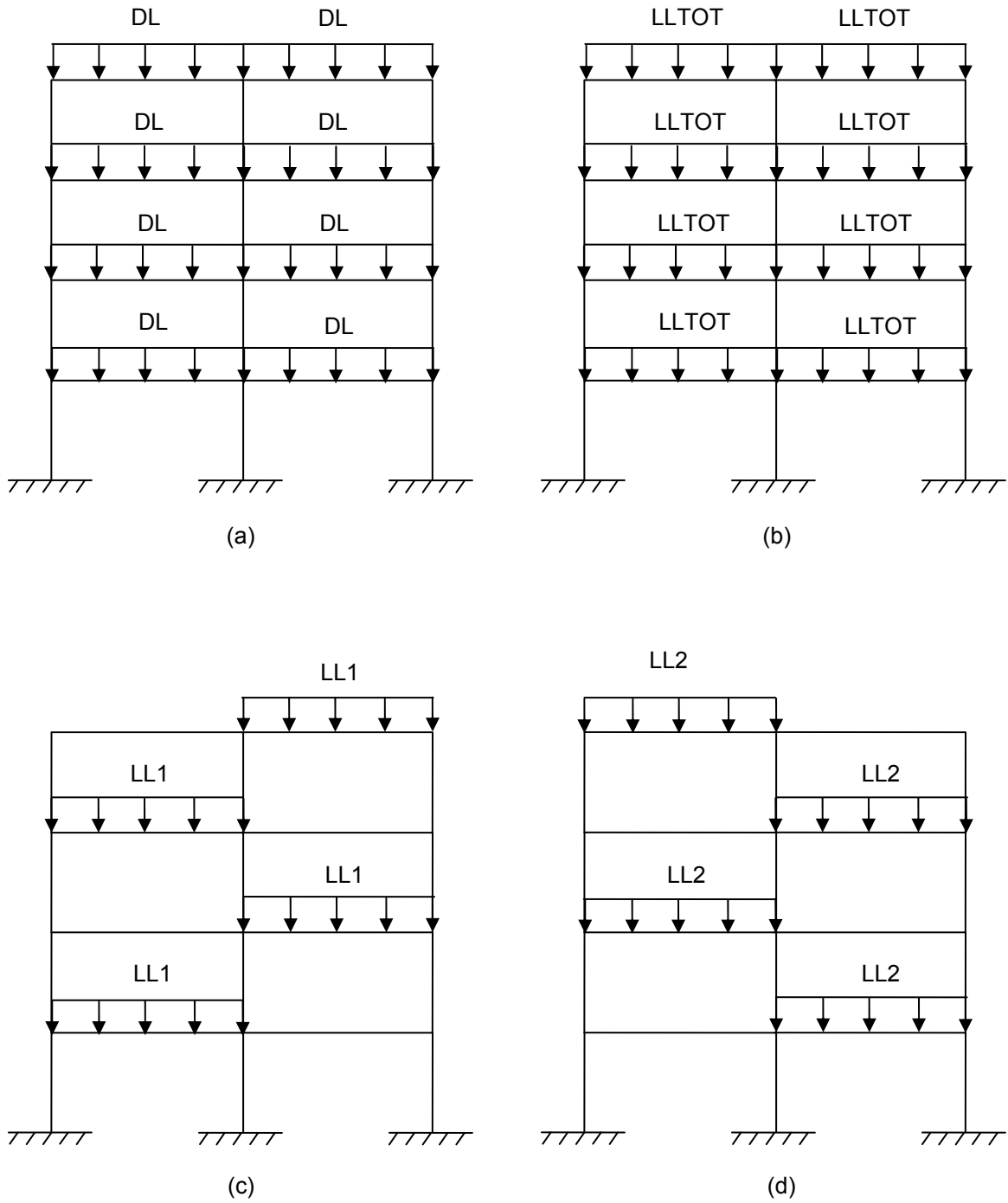
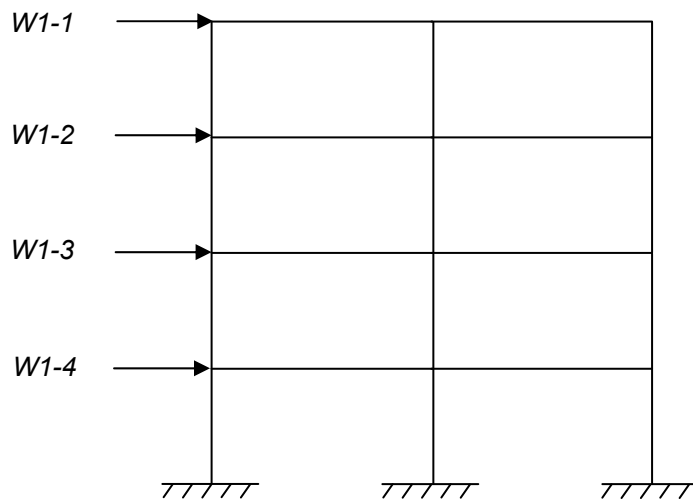
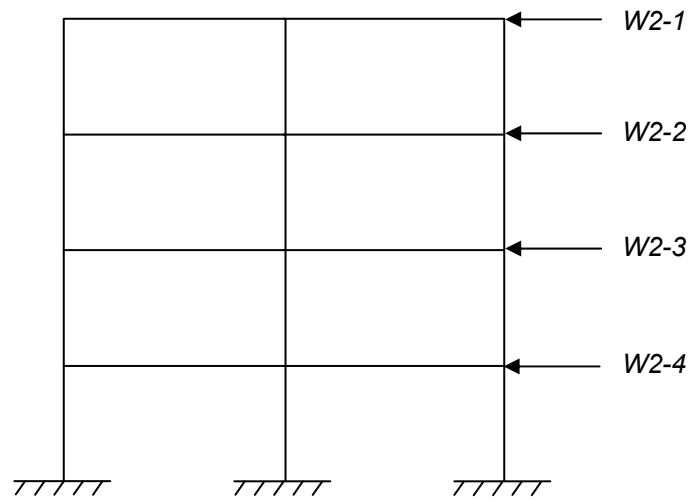


Figure 5-7. Dead and Live Load Patterns for Two-Bay Four-Story Frame



(a)



(b)

Figure 5-8. Wind Load Patterns for Two-Bay Four-Story Frame

The design variables for the beams in two-bay four-story frame are depth, width, top reinforcement, and bottom reinforcement. The beam sections are chosen from a set of values given by Paya et al. (2008). Every section has to fulfill the requirement of the ACI Code (2008). In this work, a set of feasible sections is not generated rather, every section is checked for maximum and minimum steel ratio, minimum width, minimum spacing between parallel bars in a row, shear capacity, and moment capacity. The reinforcement provided in top and bottom of a beam section is chosen from the list of reinforcement patterns (see Table 5-2) used in the previous design by Camp et al. (2003). The cover for beams is 30 *mm* (1.18 *in*) (Paya et al. 2008).

The set of column sections is the same as those used by Paya et al. (2008). The design variables for columns are height, width, longitudinal reinforcing bars, and reinforcement topology. There are six reinforcement topologies of four, six, eight, ten or twelve bars in the column. There are two topologies with six bars using different rebar orientation. Figure 5-9 shows the column topologies. Each column section is checked for maximum and minimum reinforcement ratio, minimum width, minimum spacing between parallel bars, and moment capacity. The possible bar sizes were the same as for beams, ranging from a #3 to a #11 bar. To simplify the design, the bars in the column were the same size. Paya et al. (2008) used European rebars in their design. In this work, U.S. standard rebar equivalent in terms of unit weight (*kg/m*) to European rebar has been used. Table 5-11 lists standard European and U.S. rebar sizes. Table 5-12 lists the extent of search space for the beams and the columns.

In addition to beam and column variables, concrete strength for each story is considered a design variable. The total number of design variables in two-bay four-story frame is 52 with a search space on the order of 2×10^{63} .

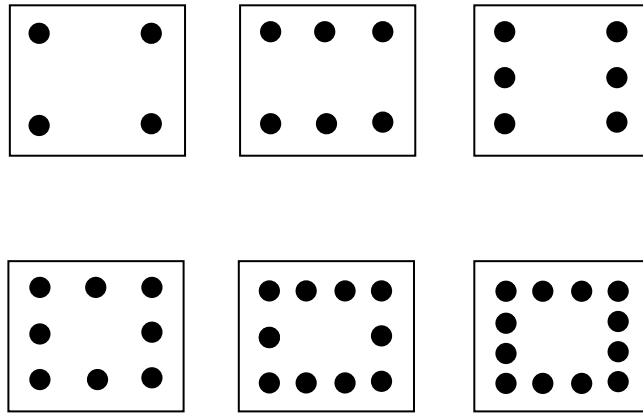


Figure 5-9. Possible Topologies for Column Design

Table 5-11. European and U.S. Standard Rebar

Size (European)	Mass (kg/m)	bar size (U.S.)	Wt (kg/m)
Φ6	0.222	#3	0.561
Φ8	0.395	#4	0.668
Φ10	0.617	#5	1.043
Φ12	0.888	#6	1.502
Φ14	1.210	#7	2.044
Φ16	1.579	#8	2.670
Φ20	2.467	#9	3.400
Φ25	3.855	#10	4.303
Φ28	4.830	#11	5.313
Φ32	6.316	#12	6.424
Φ40	9.868	#14	7.650
Φ50	15.413	#18	13.600

Table 5-12. Search Space Parameters for Two-Bay Six-Story Frame Design by Paya et al. (2008)

Column	b		h		A_s	
	(m)	(in)	(m)	(in)	Number of bars	Bar size
Min	0.25	9.84	0.25	9.84	4	3
Max	1.20	47.24	1.20	47.24	12	11
Increment	0.05	1.97	0.05	1.97	2	1
Beam	b		h		A_s	
	(m)	(in)	(m)	(in)	Number of bars	Bar size
Min	0.15	5.91	0.15	5.91	1	3
Max	1.20	47.24	1.20	47.24	4	11
Increment	0.01	0.39	0.01	0.39	1	1

In this design example, two objective functions are proposed. One is the cost of the structure and the other is amount of CO₂ produced by the materials used for the frames. The cost function is different from the function used in the design done by Camp et al. (2003) as it calculates formworks in beams differently and includes scaffolding cost. In the case of CO₂ objective function, the scaffolding is not included. Both of the objective functions are optimized separately using the BB-BC algorithm. The objective functions for cost and CO₂ for two-bay four-story frame are:

$$\begin{aligned}
 Cost = & \sum_{i=1}^{n_c} \{C_c b_i h_i + C_s A_{s_i} + 2C_f (b_i + h_i)\} l_i \\
 & + \sum_{i=1}^{n_b} \{C_c b_i h_i + C_s A_{s_i} + C_f (b_i + 2h_i)\} l_i + C_{tc} (b_i h_i) l_i
 \end{aligned} \tag{5-2}$$

$$\begin{aligned}
 CO_2 = & \sum_{i=1}^{n_c} \{C_{cc} b_i h_i + C_{sc} A_{s_i} + 2C_{fc} (b_i + h_i)\} l_i \\
 & + \sum_{i=1}^{n_b} \{C_{cc} b_i h_i + C_{sc} A_{s_i} + C_{fc} (b_i + 2h_i)\} l_i
 \end{aligned} \tag{5-3}$$

where n_b is the number of beams, n_c is the number of column, l_i is the length of the member, C_c is the unit cost of concrete, C_s is the unit cost of steel, C_f is the unit cost of formwork, A_{s_i} is the area

of steel reinforcement, C_{tc} is the unit cost of scaffolding, C_{cc} is the unit amount of CO₂ produced for concrete, C_{sc} is the Unit amount of CO₂ produced for steel, C_{fc} is the unit amount of CO₂ produced for formwork. The unit cost and amount of CO₂ emission are obtained from Paya et al. (2008). Table 5-13 lists the unit cost and CO₂ emission.

Table 5-13. Unit Cost and Unit Amount of CO₂ Produced by the Materials

Unit	Description	CO ₂ (kg)	Cost (€)
kg	B-500	3.01	1.30
m ³	Beam (25 MPa)	132.88	78.40
m ³	Beam (30 MPa)	143.48	82.79
m ³	Beam (35 MPa)	143.77	98.47
m ³	Beam (40 MPa)	143.77	105.93
m ³	Beam (45 MPa)	143.77	112.13
m ³	Beam (50 MPa)	143.77	118.60
m ³	Col (25 MPa)	132.88	77.80
m ³	Col (30 MPa)	143.48	82.34
m ³	Col (35 MPa)	143.77	98.03
m ³	Col (40 MPa)	143.77	105.17
m ³	Col (45 MPa)	143.77	111.72
m ³	Col (50 MPa)	143.77	118.26
m ²	Formworks in beams	3.13	25.05
m ²	Formworks in columns	8.90	22.75
m ²	Scaffolding for beams	0	38.89

Note: The value in the parenthesis in the above table is the strength of concrete.

Shear and tie reinforcement are not considered as design variables. The required shear reinforcement and ties in columns are provided according to the code specification. After the cost and CO₂ optimization is completed the shear cost or emission of CO₂ for shear reinforcement in beams and columns are calculated. The additional cost or emission is then added to the main cost or emission values.

There are some adjustable parameters for BB-BC algorithm. The values of the adjustable parameters α_2 and α_3 are 0.3 and 0.5, respectively. The value of α is 1.0. The BB-BC algorithm uses initially 500 candidate solutions and stopping criteria of 5,000 analyses. For cracked section analysis (ACI 10.10.4), the moment of inertia in beams is calculated taking 35% of the gross area

and in column 70% of the gross area. The moment of inertia is reduced on the supposition that the concrete in the section is ineffective in resisting tension.

Paya et al. (2008) implemented SA to optimize cost and CO₂ emission. Table 5-14 lists the design of two-bay four-story frame by Paya et al. (2008).

Table 5-14. Design Result for Cost Objective for Two-Bay Four-Story Frame (Paya et al. 2008)

	Paya et al. (2008)	<i>b</i>		<i>h</i>		$A_{s\ bottom}$ (mm ²)	$A_{s\ top}$ (mm ²)
		(m)	(in)	(m)	(in)		
Beam Group No.	1	0.20	7.87	0.39	15.35	2 Φ 16 1 Φ 16	2 Φ 25 1 Φ 25
	2	0.21	8.27	0.39	15.35	2 Φ 16 1 Φ 25	2 Φ 25 1 Φ 20
	3	0.20	7.87	0.39	15.35	2 Φ 20 1 Φ 25	3 Φ 20
	4	0.21	8.27	0.52	20.47	2 Φ 12 1 Φ 20	2 Φ 16 1 Φ 25
						Column Reinforcement	
Column Group No.	5	0.25	9.84	0.35	13.78	4 Φ 12	
	6	0.25	9.84	0.35	13.78	4 Φ 16	
	7	0.25	9.84	0.35	13.78	4 Φ 16	
	8	0.25	9.84	0.30	11.81	4 Φ 20	
	9	0.25	9.84	0.40	15.74	4 Φ 12	
	10	0.25	9.84	0.35	13.78	4 Φ 12	
	11	0.25	9.84	0.30	11.81	4 Φ 12	
	12	0.25	9.84	0.25	9.84	4 Φ 12	
Concrete strength	Story Level					Strength	
						(MPa)	(psi)
	1					45	6,527
	2					45	6,527
	3					40	5,802
	4					40	5,802
Cost	€ 3,670.38						

In Table 5-14, reinforcement design results (Paya et al. 2008) list two values: the first value is for main reinforcement and the second value is for extra reinforcement.

Paya et al. (2008) used European rebars which are lighter than standard U.S. rebars. In addition, Paya et al. (2008) considered shear reinforcement and column ties as design variables. The proposed shear design by Paya et al. (2008) is divided into three zones (left, right, and middle) of a beam span. For shear and tie reinforcement, European bar Φ 6, Φ 8 and Φ 10 were

used which are lighter than the required ACI shear reinforcement. The shear and tie design conformed to Spanish Code. Paya et al. (2008) used extra top and bottom reinforcement in positive and negative moment zones of beams. The extra reinforcement was also considered as design variables. Bar cut-off is done for extra top and bottom reinforcement. In beam-column intersection zone of the interior span the extra top reinforcement is extended to $0.4L$ whereas in the exterior intersection zone this length is $0.2L$ and for bottom extra reinforcement the length is $0.8L$.

Since the European code and rebars are not directly comparable to the ACI code, an equivalent two-bay four-story frame is proposed that is designed by Paya et al. (2008). In the equivalent frame, all the design variables are the same as those presented by Paya et al. (2008); however, the cost of the frame is computed using equivalent U.S. standard rebars without considering the cost of shear reinforcement or the effects of rebar cut-off. Table 5-15 lists the cost of the design for the equivalent two-bay four-story frame. Table 5-16 lists the BB-BC design of the equivalent two-bay four-story.

Table 5-15. Design Result for Cost Objective for Equivalent Two-Bay Four-Story Frame

	BB-BC	<i>b</i>		<i>h</i>		$A_{s\ bottom} (in^2)$	$A_{s\ top}(in^2)$
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.20	7.87	0.39	15.35	3 #5	3 #8
	2	0.21	8.27	0.39	15.35	1 #8, 2 #5	2 #8, 1 #7
	3	0.20	7.87	0.39	15.35	1 #8, 2 #7	3 #7
	4	0.21	8.27	0.52	20.47	1 #7, 2 #4	1 #8, 2 #5
						Column Reinforcement	
Column Group	5	0.25	9.84	0.35	13.78	4 #4	
	6	0.25	9.84	0.35	13.78	4 #5	
	7	0.25	9.84	0.35	13.78	4 #5	
	8	0.25	9.84	0.30	11.81	4 #7	
	9	0.25	9.84	0.40	15.74	4 #4	
	10	0.25	9.84	0.35	13.78	4 #4	
	11	0.25	9.84	0.30	11.81	4 #4	
	12	0.25	9.84	0.25	9.84	4 #4	
Concrete Strength	Story Level					Strength	
						(<i>MPa</i>)	(<i>psi</i>)
	1					45	6,527
	2					45	6,527
	3					40	5,802
	4					40	5,802
Cost	€ 3,706.22						

Table 5-16. Design Result for Cost Objective for Two-Bay Four-Story Frame Using BB-BC

	BB-BC	<i>b</i>		<i>h</i>		$A_{s\ bottom}$ (in^2)	$A_{s\ top}(in^2)$
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.20	7.87	0.48	18.89	1 #8	2 #8
	2	0.20	7.87	0.46	18.11	1 #8	2 #8
	3	0.20	7.87	0.49	19.29	1 #8	1 #11
	4	0.22	8.66	0.555	21.84	1 #8	1 #10
						Column Reinforcement	
Column Group No.	5	0.25	9.84	0.35	13.78	6 #3	
	6	0.25	9.84	0.35	13.78	6 #5	
	7	0.25	9.84	0.25	9.84	6 #5	
	8	0.25	9.84	0.25	9.84	4 #7	
	9	0.50	19.68	0.25	9.84	6 #5	
	10	0.25	9.84	0.25	9.84	10 #5	
	11	0.25	9.84	0.25	9.84	6 #4	
	12	0.25	9.84	0.25	9.84	6 #3	
Concrete Strength	Story Level					Strength	
						(<i>MPa</i>)	(<i>psi</i>)
	1					30	4,351
	2					30	4,351
	3					30	4,351
	4					25	3,626
Cost	€ 3,494.88						

The best solution developed by BB-BC for cost €3,494.88 in Phase II. In Phase II, the average cost is €3,783.59, the standard deviation is €125.80, and the average numbers of analyses required for convergence is 53,425. The best cost in Phase I is €3,560.85 with an average cost and standard deviation of €3,809.68 and €142.83 respectively. The average number of analyses required in Phase I is 46,015. The best cost solution given by BB-BC is 5.7% less than the cost of the frame design equivalent to the design proposed by Paya et al. (2008).

The CO₂ optimization was done in the same manner as cost. Instead of unit cost, the unit amount of CO₂ emission is used. Though shear and tie reinforcement were not considered as design variables, it was provided as per the requirement of the ACI Code (2008). Standard U.S. rebars were used with no bar cut-off. Table 5-17 and Table 5-18 list the design result for CO₂ emissions for two-bay four-story frame with and without considering shear and tie reinforcement.

Table 5-17. Design Result (With Shear) for CO₂ Emissions for Two-Bay Four-Story Frame

	BB-BC	<i>b</i>		<i>h</i>		$A_{s\ bottom}$ (<i>in</i> ²)	$A_{s\ top}$ (<i>in</i> ²)
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.23	9.05	0.57	22.44	2 #5	4 #5
	2	0.23	9.05	0.57	22.44	2 #5	2 #7
	3	0.24	9.45	0.58	22.83	4 #4	2 #7
	4	0.24	9.45	0.59	23.22	4 #4	4 #5
						Column Reinforcement	
Column Group No.	5	0.25	9.84	0.40	15.74	4 #4	
	6	0.25	9.84	0.40	15.74	6 #4	
	7	0.25	9.84	0.25	9.84	4 #5	
	8	0.25	9.84	0.25	9.84	6 #5	
	9	0.35	13.78	0.40	15.74	6 #4	
	10	0.30	11.81	0.30	11.81	4 #6	
	11	0.25	9.84	0.25	9.84	12 #3	
	12	0.25	9.84	0.25	9.84	8 #3	
Concrete Strength	Story Level					Strength	
						(<i>MPa</i>)	(<i>psi</i>)
	1					25	3,626
	2					25	3,626
	3					25	3,626
	4					25	3,626
CO ₂ (<i>kg</i>)	4,694.77						

The best solution reported for CO₂ emissions by BB-BC is 4,694.77 kg in Phase II. The average CO₂ emissions in Phase II is 4,987.32 kg with a standard deviation of 132.13 kg and average number of analyses performed is 51,070. In Phase I, the best cost obtained is 4,751.42 kg with a standard deviation of 135.49 kg and the average number of analysis performed is 44,575. The average CO₂ emissions in phase I is 5,013.08 kg.

Table 5-18. Design Result (Without Shear) for CO₂ Emissions for Two-Bay Four-Story Frame

	BB-BC	<i>b</i>		<i>h</i>		<i>A_{s bottom}</i> (in ²)	<i>A_{s top}</i> (in ²)
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.19	7.47	0.46	18.11	2 #5	3 #6
	2	0.20	7.87	0.50	19.68	2 #5	1 #10
	3	0.21	8.27	0.51	20.07	2 #5	2 #7
	4	0.22	8.66	0.52	20.47	1 #8	2 #7
						Column Reinforcement	
Column Group No.	5	0.25	9.84	0.70	27.55	8 #3	
	6	0.25	9.84	0.45	17.71	4 #3	
	7	0.25	9.84	0.45	17.71	6 #3	
	8	0.25	9.84	0.45	17.71	4 #6	
	9	0.30	11.081	0.45	17.71	6 #6	
	10	0.25	9.84	0.30	11.81	6 #3	
	11	0.25	9.84	0.30	11.81	6 #4	
	12	0.25	9.84	0.25	9.84	6 #3	
Concrete Strength	Story Level					Strength	
						(<i>MPa</i>)	(<i>psi</i>)
	1					45	6,527
	2					45	6,527
	3					35	5,076
	4					25	3,626
CO ₂ (<i>kg</i>)	3,736.20						

The best solution reported by BB-BC for CO₂ emissions excluding the shear reinforcement is 3,736.20 in both Phase I and II. In Phase I, the average emissions is 4,057.84 *kg*, the standard deviation is 172.90 *kg*, and average number of analyses to converge are 40,125. In Phase II, the average emission is 4,013.15 *kg* with a standard deviation of 145.10 *kg*. The average number of analyses performed in Phase II are 47,940.

5.6 New Two-Bay Six-Story Frame Design Example by BB-BC Algorithm

The design of a two-bay six-story frame originally designed by Paya et al. (2008) is also proposed in this study. The story height and bay width of the frame is same as the previously mentioned two-bay four-story frame. The total height of the frame is 18 *m* (59.04 *ft*) and total width is 10 *m* (32.8 *ft*). There are six beam groups and twelve column groups. Figure 5-10 shows the geometry of the frame and Figure 5-11 shows the beam and column groups.

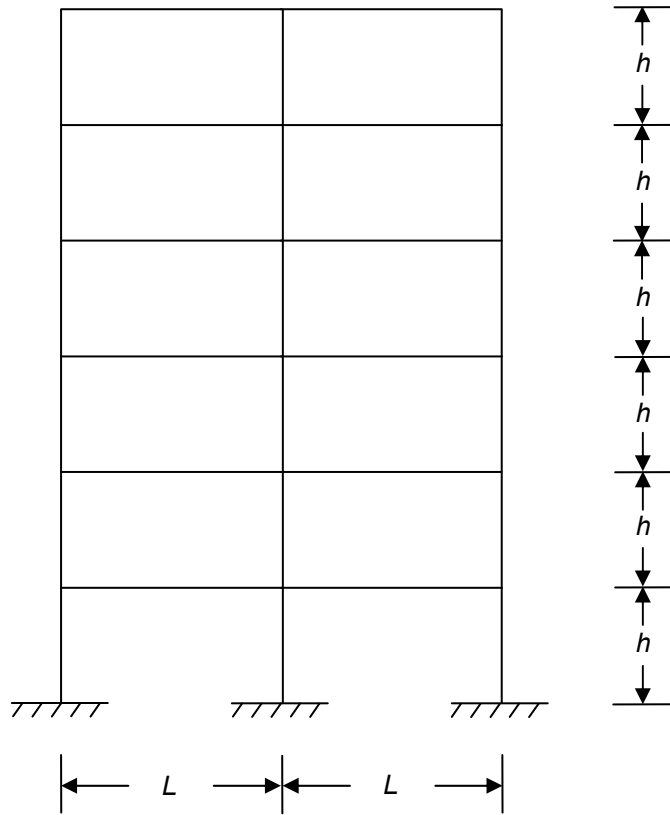


Figure 5-10. Geometry for Two-Bay Six-Story Frame

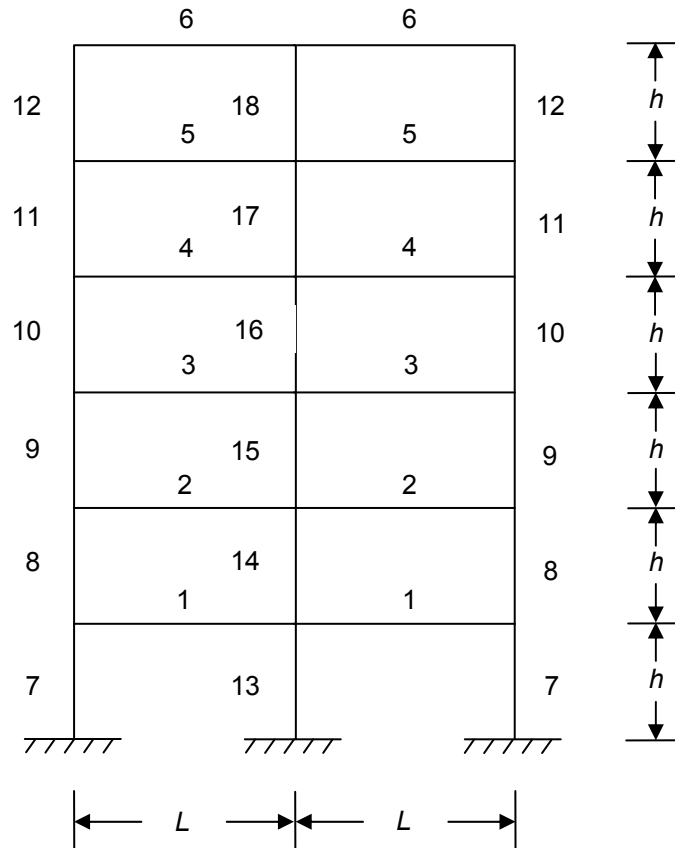


Figure 5-11. Beam and Column Groups for Two-Bay Six-Story Frame

The dead load and pattern live load magnitude is also same as the two-bay six-story frame (see Table 5-8) the load factors and combinations are also same as the previous example (see Table 5-10 and (Figure 5-7, 5-8)). Only the wind loads are different. The wind load value for each story is same as used by Paya et al. (2008). Table 5-19 lists the magnitude of the wind load in each story.

Table 5-19. Wind Load for Two-Bay Six-Story Frame

Story level	Wind Load (kN)	Wind Load (kip)
roof	6.62	1.49
5	12.36	2.78
4	11.62	2.61
3	10.74	2.41
2	9.86	2.22
1	8.83	1.99

The extent of search space for the beams and the columns is same as before (see Table 5-12). The design variables for both the beams and the columns are also the same as for the two-bay four-story frame. There are 78 design variables for this two-bay six-story frame.

The objective functions for both cost and CO₂ emissions are the same as described in Equations (5-2) and (5-3) respectively. The number of initial candidate solutions, total number of run and number of cycles in each run are kept the same as for the two-bay four-story frame.

In two-bay six-story frame design, standard U.S. rebar with no bar cut-off was used for both the cost and CO₂ objective functions. Designs were done both with and without shear and tie reinforcement. The CO₂ emissions are computed without considering shear and tie reinforcement. Tables 5-20 and 5-21 list the design result for cost of two-bay six-story frame with and without shear and tie reinforcement. Table 5-22 lists the design result for CO₂ emissions for two-bay six-story frame without shear and tie reinforcement.

Table 5-20. Design Result (with Shear) for Cost Objective for Two-Bay Six-Story Frame

	BB-BC	<i>b</i>		<i>h</i>		$A_{s\ bottom} (in^2)$	$A_{s\ top} (in^2)$
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.18	7.08	0.45	17.71	2 #8	2 #9
	2	0.21	8.27	0.52	20.45	3 #5	2 #9
	3	0.21	8.27	0.49	19.29	4 #4	3 #7
	4	0.22	8.27	0.55	21.65	2 #6	1 #11
	5	0.20	7.87	0.45	17.71	1 #10	2 #8
	6	0.19	7.48	0.47	18.50	1 #9	2 #8
						Column Reinforcement	
Column Group No.	7	0.25	9.84	0.35	13.78	10 #6	
	8	0.25	9.84	0.30	11.81	6 #6	
	9	0.25	9.84	0.30	11.81	6 #6	
	10	0.25	9.84	0.30	11.81	10 #4	
	11	0.25	9.84	0.30	11.81	8 #5	
	12	0.25	9.84	0.30	11.81	6 #7	
	13	0.25	9.84	0.85	33.46	6 #4	
	14	0.25	9.84	0.45	17.71	6 #7	
	15	0.25	9.84	0.35	13.78	6 #7	
	16	0.25	9.84	0.35	13.78	4 #5	
	17	0.25	9.84	0.25	9.84	10 #4	
	18	0.25	9.84	0.25	9.84	6 #5	
Concrete Strength		Story Level				Strength	
						(<i>MPa</i>)	(<i>psi</i>)
		1				40	6,527
		2				35	5,076
		3				30	4,351
		4				30	4,351
		5				30	4,351
		6				30	4,351
Cost	€7,138.79						

The best solution obtained for cost in Phase I and II from BB-BC design for the two-bay six-story frame is €7,138.79. The average cost in Phase I is €7,568.12 and in phase II is €7,545.63 with standard deviations of €220.36 and €185.43 respectively. The average number of analyses to converge to a solution in Phase I is 47,095 and in Phase II 53,115. The cost is much less when shear reinforcement is not considered.

Table 5-21. Design Result (Without Shear) for Cost Objective for Two-Bay Six-Story Frame

	BB-BC	<i>b</i>		<i>h</i>		$A_{s\ bottom} (in^2)$	$A_{s\ top} (in^2)$
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.19	7.48	0.46	18.11	1 #10	2 #9
	2	0.21	8.27	0.51	20.07	1 #10	3 #7
	3	0.20	7.87	0.49	19.29	3 #5	3 #7
	4	0.22	8.66	0.49	19.29	1 #9	2 #8
	5	0.23	9.05	0.50	19.68	3 #5	1 #11
	6	0.22	8.66	0.55	21.65	4 #4	3 #6
						Column Reinforcement	
Column Group No.	7	0.25	9.84	0.45	17.71	6 #3	
	8	0.25	9.84	0.45	17.71	6 #5	
	9	0.25	9.84	0.40	15.74	10 #3	
	10	0.25	9.84	0.40	15.74	4 #7	
	11	0.25	9.84	0.30	11.81	4 #8	
	12	0.25	9.84	0.25	9.84	6 #6	
	13	0.35	13.78	0.40	15.74	6 #7	
	14	0.30	11.81	0.40	15.74	12 #6	
	15	0.25	9.84	0.30	11.81	6 #9	
	16	0.25	9.84	0.30	11.81	6 #6	
	17	0.25	9.84	0.30	11.81	4 #7	
	18	0.25	9.84	0.25	9.84	8 #6	
Concrete Strength		Story Level				Strength	
						(<i>MPa</i>)	(<i>psi</i>)
		1				40	5,802
		2				30	4,351
		3				30	4,351
		4				25	3,626
		5				25	3,626
		6				25	3,626
Cost	€ 6,113.58						

The best solution obtained for two-bay six-story frame from BB-BC for cost excluding the shear and tie reinforcement in Phase I is €6,113.58 with an average cost of €6,554.53 and standard deviation of €242.57. The average number of analyses required to converge to a solution in Phase I is 54,471. The best solution obtained from Phase II is the same as from Phase I with a different average cost of €6,531.46 and standard deviation of €209.29. The average number of analyses performed in Phase II is 60,665.

Table 5-22. Design Result (Without Shear) for CO₂ Emissions for Two-Bay Six-Story Frame

	BB-BC	<i>b</i>		<i>h</i>		<i>A_{s bottom}</i> (in ²)	<i>A_{s top}</i> (in ²)
		(<i>m</i>)	(<i>in</i>)	(<i>m</i>)	(<i>in</i>)		
Beam Group No.	1	0.20	7.87	0.47	18.50	1 #7	2 #8
	2	0.25	9.84	0.62	24.40	1 #8	3 #6
	3	0.22	8.66	0.55	21.65	1 #7	3 #6
	4	0.23	9.05	0.56	22.04	4 #4	2 #7
	5	0.24	9.45	0.59	23.22	1 #8	1 #10
	6	0.23	9.05	0.57	22.44	4 #4	1 #10
						Column Reinforcement	
Column Group No.	7	0.25	9.84	0.80	31.49	6 #4	
	8	0.25	9.84	0.50	19.68	6 #3	
	9	0.25	9.84	0.40	15.74	6 #3	
	10	0.25	9.84	0.40	15.74	6 #5	
	11	0.25	9.84	0.30	11.81	4 #6	
	12	0.25	9.84	0.25	9.84	4 #7	
	13	0.30	11.81	0.50	19.68	6 #5	
	14	0.30	11.81	0.50	19.68	4 #5	
	15	0.25	9.84	0.40	15.74	6 #6	
	16	0.25	9.84	0.30	11.81	6 #8	
	17	0.25	9.84	0.30	11.81	8 #4	
	18	0.25	9.84	0.25	9.84	6 #5	
Concrete Strength		Story Level				Strength	
						(MPa)	(psi)
		1				30	4,351
		2				30	4,351
		3				30	4,351
		4				25	3,626
		5				25	3,626
		6				25	3,626
CO ₂ (kg)	6,809.44						

The best solution obtained for two-bay six-story frame from BB-BC in Phase II for CO₂ emissions excluding the shear and tie reinforcement is 6,806.93 kg with an average emission of 7,294.92 kg and standard deviation of 331.73 kg. The average number of analyses required to converge to a solution in Phase II is 55,560. The best solution obtained from Phase I is slightly different from Phase II. The best solution is 6,809.44 kg with an average emission of 7,350.16 kg

and standard deviation of 331.73 *kg*. The average number of analyses performed in Phase I is 49,125.

The solution of cost function and CO₂ function change from Phase I to Phase II in two-bay four-story frame design. As the size and complexity of the problem (two-bay six-story) increased, the solutions for both of the objective functions in Phase I and Phase II remained the same.

CHAPTER 6

SUMMARY

6.1 Summary

A hybrid BB-BC algorithm is applied to structural optimization of reinforced concrete frames. The characteristics of reinforced concrete beams and columns members are described and the general structural optimization problem for reinforced concrete frames is formulated. The resulting BB-BC algorithm is applied to the low-cost design of a two-bay six-story frame originally designed by Rajeev and Krishnamoorthy (1998) and refined by Camp et al. (2003). The BB-BC algorithm is also applied to optimization of the cost and the CO₂ emissions associated construction for a two-bay four-story frame and a two-bay six-story frame originally designed by Paya et al. (2008).

The BB-BC designs of reinforced concrete frames showed some improvements over the designs developed by GA and SA. For the two-bay six-story frame originally designed by Rajeev and Krishnamoorthy (1998) and Camp et al. (2003), the BB-BC developed a design that conformed to the ACI Code (1999) and reduced the cost of the structure by 5.2% as compared to the GA design. In order to compare the BB-BC design to the two-bay four-story frame design presented by Paya et al. (2008), which conforms to the Spanish Code and uses metric reinforcement, an equivalent frame using U.S. standard reinforcement that meets the standards set by the ACI Code (2008) is developed. The BB-BC algorithm developed a design that reduced the cost the equivalent ACI frame by 5.7%. In both cases, the BB-BC algorithm was demonstrated improved computationally efficient over both the GA and SA designs.

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