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Regular and Irregular Type Spacing Error with Tooth Profile Modification Effect on Dynamic Load of Spur Gear System

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REGULAR AND IRREGULAR TYPE SPACING ERROR WITH TOOTH PROFILE MODIFICATION EFFECT ON DYNAMIC LOAD OF SPUR GEAR SYSTEM

by

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ABSTRACT

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The dynamic load of gears with regular type of spacing error such as full-sine and half-sine type has been analyzed in several previous research works. In this study, the irregular type of tooth spacing error is incorporated with the regular type ones to determine their effect on gear dynamic response. Linear and Parabolic tooth profile modification are applied to evaluate their influence on the dynamic loads of gear systems with regular or irregular tooth spacing errors. All dynamic analyses are conducted using the NASA gear dynamics code, DANST program.

The objective of this study is to examine the relationship between gear dynamic load, type and extent of tooth spacing error, and profile modifications. Results obtained from this study can provide proper tooth profile design to minimize the dynamic response of the spur gear systems for a better transmission design.

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CHAPTER 1

1. INTRODUCTION

The gear is a rotating machine part having cut teeth, or cogs, which mesh with another toothed part in order to transmit torque and power. Two or more gears working in tandem are called a transmission and can produce a mechanical advantage through a gear ratio and thus may be considered a simple machine. Geared devices can change the speed, torque, and direction of a power source. The most common situation is for a gear to mesh with another gear, however, a gear can also mesh a non-rotating toothed part, called a rack, thereby producing translation instead of rotation.

The demands on gears for longer lifetime, better power transmission and lower noise emission are increasing. The dynamic tooth load on the rotating gear creates noise and vibration and shortens the life of the gear if the load magnitude is significant. The dynamic tooth load can be affected significantly by the variation of static transmission error in the gear meshing cycle. And the gear tooth profile can influence the static transmission error considerably. Applying profile modification can effectively reduce the dynamic loads of gear systems. Low contact ratio gears (LCRG) creates higher dynamic load during operation. The high contact ratio gears (HCRG) operate with a contact ratio larger than 2. Generally speaking, the higher the contact ratio, the lower the load is applied on a single tooth.

Many researches about gear dynamic load have been done earlier [1-10]. It is found that there is a relationship between dynamic load and transmission error of meshing gear pair [3,4,5]. The total transmission error is the difference between the actual position of the driving gear and the position it would assume if the driven gear were perfectly fixed. Transmission error is mainly caused by the combinations of the deflections of the teeth due to the transmitted load, tooth profile error, tooth spacing error and run-out error from manufacturing processes. One of the important gear errors is spacing which has a significant effect on gear transmission error, gear noise and vibrations. There have been some studies on simple, linearly short span spacing error [9]. In earlier studies, some spacing error is found to have the distribution in the form of a sine wave [3,4], and for industrial settings some spacing error have the distribution in the form of random wave. Therefore this study will select full-sine, half-sine and random wave spacing errors for the investigation. The amplitude or maximum cumulative spacing error will vary from 0.0001 in. to 0.0003 in. with 0.0001-in. increments. Linear and parabolic tooth profile modifications with the amount and length of modification varied systematically will be applied to each case to compare with the unmodified case.

The NASA gear dynamic code DANST (Dynamic Analysis of Spur Gear Transmission) will be used to obtain dynamic load and transmission error data. The results will be plotted for comparison and discussion. Final conclusions will be made based on these analyses.

CHAPTER 2

2. SPUR GEAR SYSTEM

2.1 Generation of the Involute Curve

An involute approximates the path followed by a tetherball as the connecting tether is wound around the center pole. If the center pole has a circular cross-section, then the curve is an involute of a circle. Alternatively, another way to construct the involute of a curve is to replace the taut string by a line segment that is tangent to the curve on one end, while the other traces out the involute.

In Figure 2.1, let line MN roll on the circumference of a circle without slipping. When the line rolls to the position PQ, its original tangent point of A reaches the point K, while tracing out the curve AK during the motion. As the motion continues, the point A will trace out the curve AKC, which is called the involute curve.

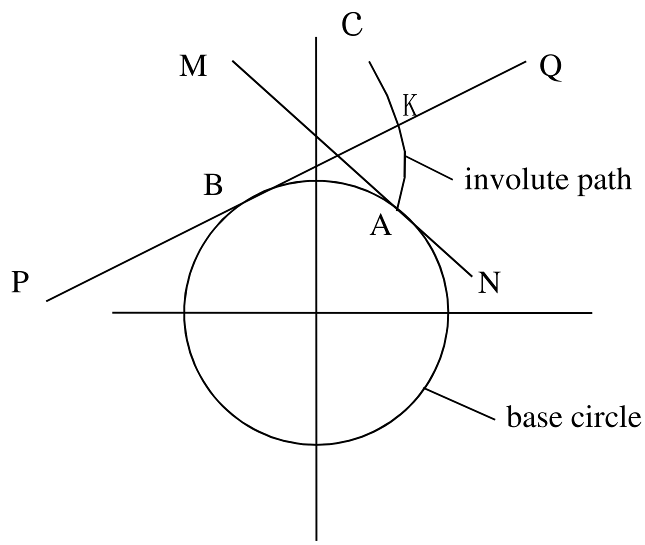

Figure 2.1 Involute curve generation

2.2 Line of Action

The line along which the force between two meshing gear teeth is directed is the line of action. It has the same direction as the force vector. In general, the line of action changes from moment to moment during the period of engagement of a pair of teeth. For involute gears, however, the tooth-to-tooth force is always directed along the same line—that is, the line of action is constant. This implies that for involute gears the path of contact is also a straight line, coincident with the line of action—as is shown in Figure 2.2.

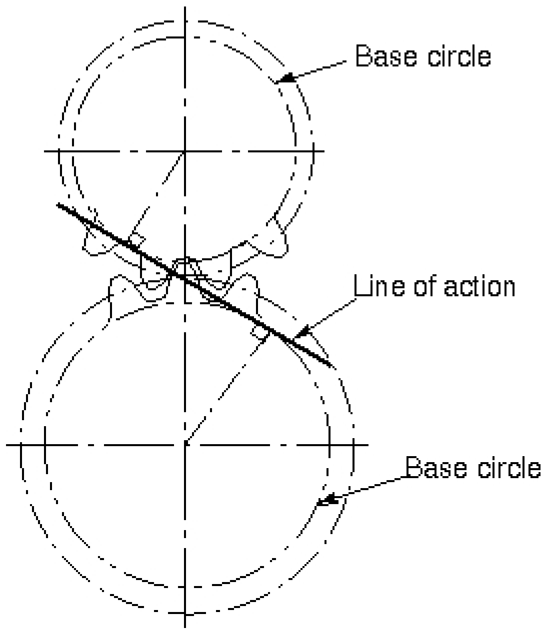

Figure 2.2 Gear line of action

2.3 Tooth Profile Modification

Profile modification can significantly decrease the dynamic load factor of a gear set with any error amplitude. It is an operation in the manufacture of gears. It involves removing part of the face of the gear tooth; it is designed to reduce errors in the regular arrangement of teeth on the gear, which give rise to additional dynamic loads and increase vibrations and noise in the gear train. Errors in manufacture and errors arising from deformation of teeth lead to contact of the teeth outside the pressure line, accompanied by a shock. Profile modification eliminates this and ensures theoretically correct contact on the pressure line.

Tooth profile modification is a deviation of tooth profile from the true involute form which is an effective way to reduce dynamic tooth loads and stresses [11]. Both tooth tip and root can receive profile modification. But extra care must be taken in modifying the roots of the gear teeth because of the complex geometry. In some extreme cases with low-contact-ratio gears, tooth root modification can destroy the effects of tip modification. So usually, tooth tip modification is more preferable.

There are mainly two profile modification variables, modification amount of tip relief (Δ) and modification length (L_n), which can express the extent of the profile modification. Figure 2.3 shows an example of gear tooth with profile modification. Both the modification amount and modification length can be normalized by the reference values. The minimum amount of conventional tip relief and the distance from the tooth tip to the highest point of single tooth contact (HPSTC) are considered as the reference values for the modification amount and modification length separately Figure 2.4. The normalized amount of modification is the ratio of the actual amount of tip modification to the amount of conventional tip relief. It was stated that the amount of conventional tip relief should be equal to twice the maximum spacing error plus the combined tooth deflection evaluated at the HPSTC [11]. Therefore, the normalized conventional profile modification can be expressed with Δ=1 or 100% and L_n=1 or 100%.

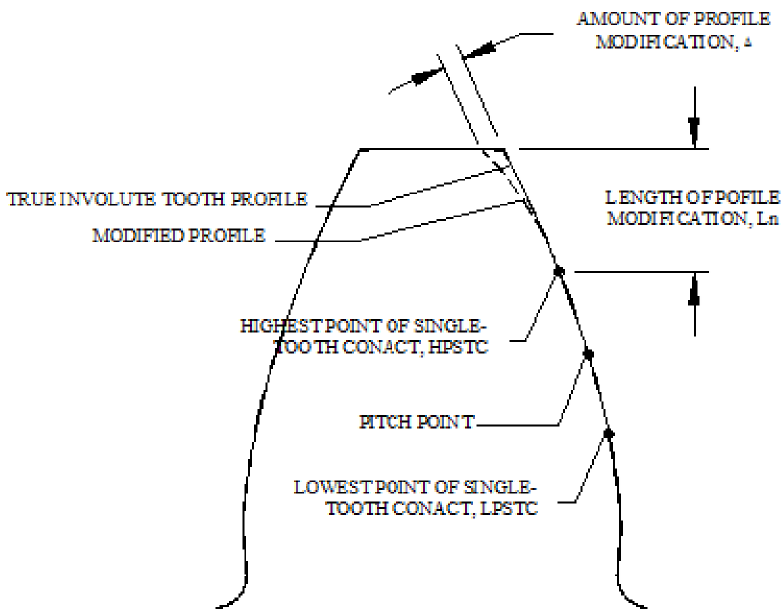

Figure 2.3 Gear tooth with profile modification

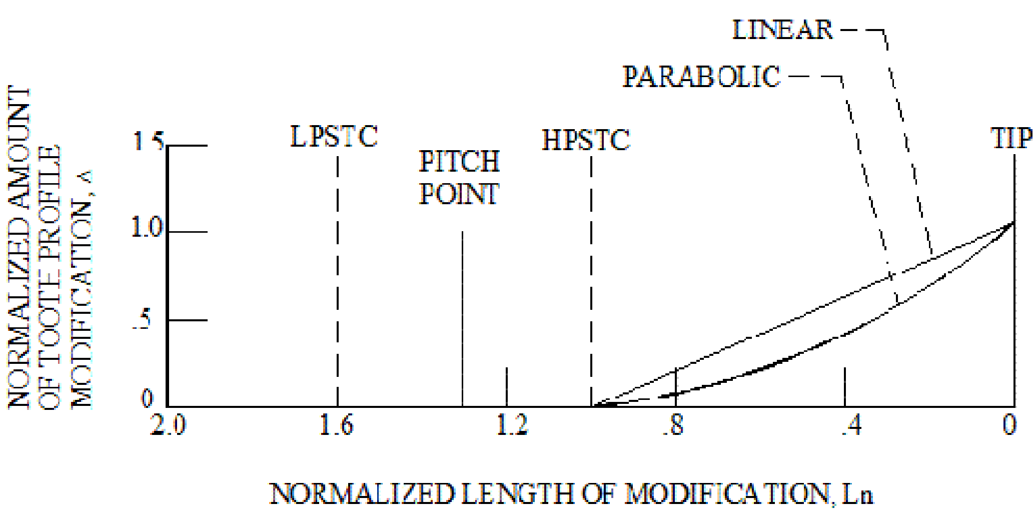

Figure 2.4 Gear tooth with profile modification

There are two types of the profile modification which can be expressed as follows:

$$\Delta_j = \Delta\left[\frac{L_n - L_j}{L_n}\right] \tag{2.1}$$

$$\Delta_j = \Delta\left[\frac{L_n - L_j}{L_n}\right]^2 \tag{2.2}$$

Equation (2.1) is for the linear profile modification; and Equation (2.2) is for the parabolic profile modification.

Where Δ_j is modification amount at point j.
Δ is modification amount at tip.
L_n is normalized length of modification.
L_j is normalized distance between point j and the tip.

2.4 Tooth Spacing Error

Basically, the spacing error have a full-sine and half-sine wave distribution, but this is one way to analysis it in the hypothetical mathematics model, in the real industrial manufacture, the type of spacing error is cannot be controlled. Hence, in this study, an additional analysis is considered where the spacing error is assumed to have a normal random distribution. In the normal distribution, most of spacing errors are very small, only less than 10% of them possibly have large value. This is representation of the actual spacing error.

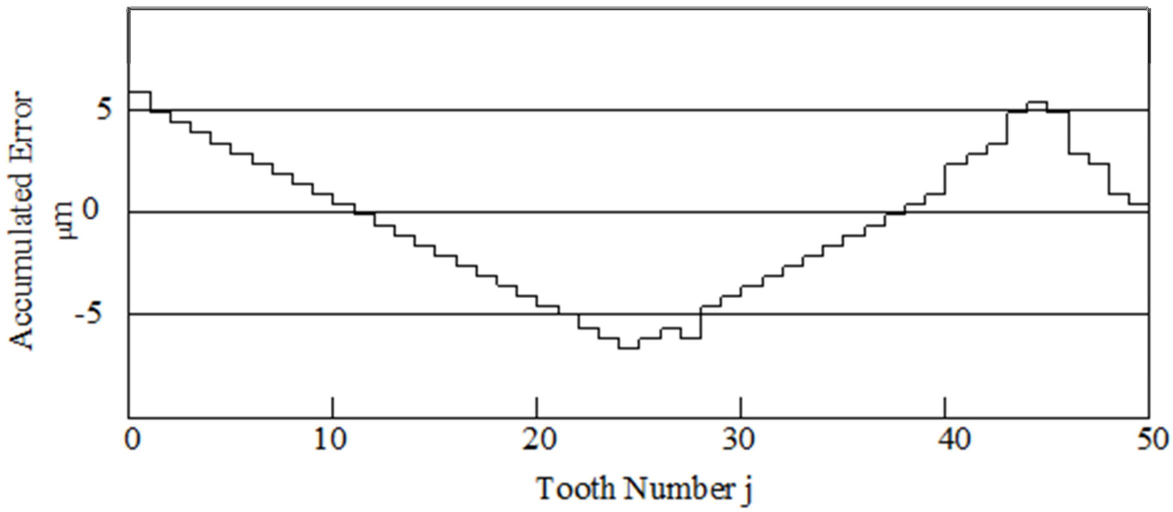

Figure 2.5 Typical Tooth Spacing Errors of a Gear (pitch error graph)

2.5 Static Transmission Error and Load Sharing

The transmission error (TE) is defined as the departure of a meshed gear pair from a constant angular motion. TE may be defined as the instant deviation of the following gear from an ideal nominal value. TE is a result of many contributors and the main items are:

(A) Combined deflection of meshing teeth

(B) Tooth spacing error

(C) Tooth profile error

(D) Run-out error

The total transmission error for a gear pair is the sum of individual errors caused by the above-mentioned sources and is written as:

$$\left(E_T^k\right)_j = \left(\sum_{r=1}^{2} {}_dE_r^k\right)_j + \left(\sum_{r=1}^{2} {}_pE_r^k\right)_j + [P]\left(\sum_{r=1}^{2} {}_sE_r^k\right)_j \tag{2.3}$$

Where,

k: the mating tooth pairs in sequence.
r : driving and driven gears.
P: if k=1 then P=0, otherwise P=1.
E_d: deflection of gear teeth at contact point.
E_p: tooth profile error.
E_s: tooth spacing error.

There are two kinds of contact zones, the double contact zone and the single contact zone. The whole meshing process begins when the first tooth pair starts contact meshing, and they continue to mesh when the second tooth pair begins to mesh. Then the first tooth pair stops meshing while the second tooth pair continues to mesh. After that, there will be a period which consists of only one meshing tooth pair just before the third tooth pair starts contacting. Thus the meshing process alternates between single contact zone and double contact-zone (Figure 2.6).

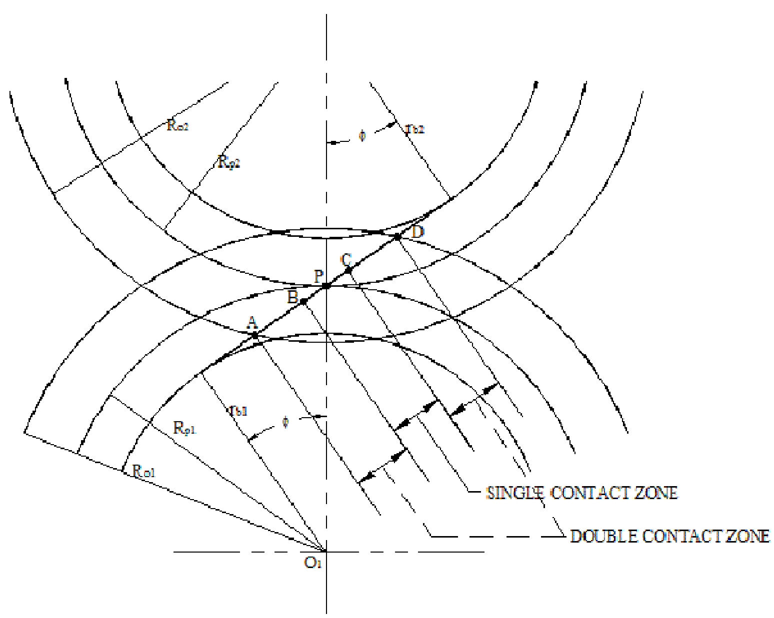

Figure 2.6 - Gear meshing process

During the double contact zones that means there are two tooth pairs in contact, the static transmission error E_t and the shared tooth load W_j for each tooth pair at contact point j may be expressed as follows [3].

$$(E_t^{\alpha})_j = (E_{d1}^{\alpha})_j + (E_{d2}^{\alpha})_j + (E_{p1}^{\alpha})_j + (E_{p2}^{\alpha})_j \tag{2.4}$$

$$(E_t^{\beta})_j = (E_{d1}^{\beta})_j + (E_{d2}^{\beta})_j + (E_{p1}^{\beta})_j + (E_{p2}^{\beta})_j + (E_{s1}^{\beta})_j + (E_{s2}^{\beta})_j \tag{2.5}$$

$$W = W_j^{\alpha} + W_j^{\beta} \tag{2.6}$$

Where E_t: static transmission error at specific contact point. E_t is positive if the driving gear leads the driven gear; otherwise, is negative.

E_d: tooth deflection at specific contact point.

E_p: tooth profile error or modification. (Positive: material is removed from the surface at contact point; Negative: material added to the surface at contact point.)

E_s: tooth spacing error. (Positive: the tooth spacing of the driving gear is less than base pitch or if the tooth spacing of the driving gear is greater than base pitch.)

W : total static transmitted load.

α : constant tooth.

β : entering contact tooth.

Since $$(E_d)_j = (E_{d1})_j + (E_{d2})_j \tag{2.7}$$

$$(E_p)_j = (E_{p1})_j + (E_{p2})_j \tag{2.8}$$

$$(E_s)_j = (E_{s1})_j + (E_{s2})_j \tag{2.9}$$

And during the meshing process in double contact zones, the total transmitting load was shared by two teeth pairs. Thus their static transmission error should be the same, which means:

$$(E_t^{\alpha})_j = (E_t^{\beta})_j \tag{2.10}$$

And $$(E_d)_j = Q_j \cdot W_j \tag{2.11}$$

Therefore, the following equation could be obtained from solving all the equations above.

$$Q_j^{\alpha} \cdot W_j^{\alpha} + (E_p^{\alpha})_j = Q_j^{\beta} \cdot W_j^{\beta} + (E_p^{\beta})_j + (E_s^{\beta})_j \tag{2.12}$$

From Equations (2.3) and (2.9), the following equations could be obtained:

$$W_j^{\alpha} = \frac{W \cdot Q_j^{\beta} - (E_p^{\alpha})_j + (E_p^{\beta})_j + (E_s^{\beta})_j}{Q_j^{\alpha} + Q_j^{\beta}} \tag{2.13}$$

$$W_j^{\beta} = \frac{W \cdot Q_j^{\alpha} + (E_p^{\alpha})_j - (E_p^{\beta})_j - (E_s^{\beta})_j}{Q_j^{\alpha} + Q_j^{\beta}} \tag{2.14}$$

All the above equations are based on the fact that there are two tooth pairs in contact at the same time in the double tooth contact zone. If one tooth pair loses contact, the corresponding doing equations of that tooth pair are eliminated from the foregoing analysis. When the tooth pair is still in contact, the remaining equations will be used to get load and static transmission errors.

2.6 Dynamic Analysis

2.6.1 Equations of motion

Figure 2.7 is the theoretical dynamic model of a spur gear system [2]. The motor connects with a gear system through a shaft. The gear system is driven by the motor (power source) through shaft 1 and transfers energy into the output device through shaft 2.

There have some assumptions will be applied in this part:

1) Damping (due to material in gears and shafting and from lubrication) is expressed as a constant damping coefficient.
2) The differential equations of motion are expressed along the theoretical line of action.
3) The reference point for the tooth deflection is assumed to be located along the tooth centerline at the radius gyration of the gear body.

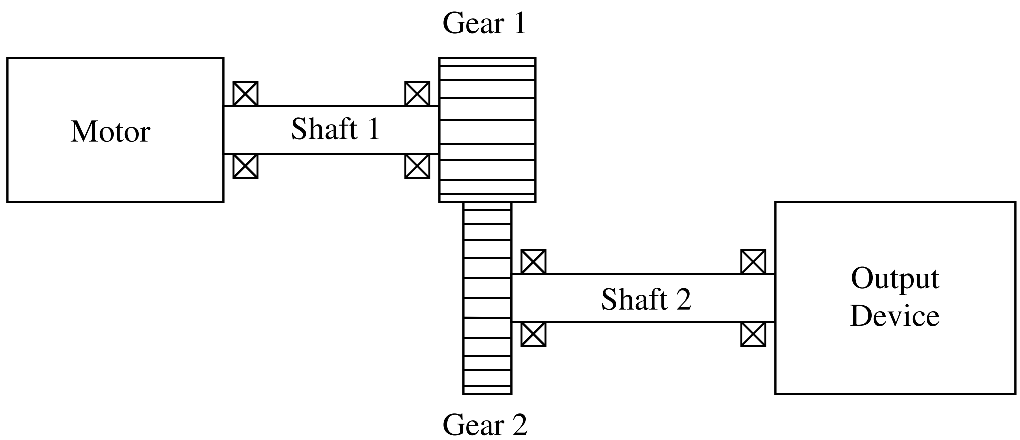

Figure 2.7 Theoretical model of a spur gear system

We can assume the motor, output device and two gears act as masses, and the shafts, gear teeth act as spring of rotational system (Figure 2.8).

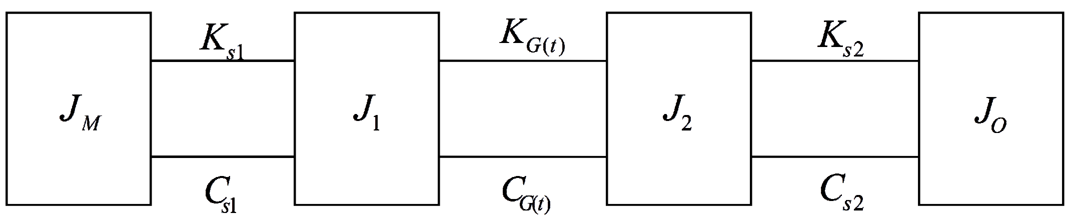

Figure 2.8 Mathematical model of the spur gear system

The motion of the gear system can be expressed by the following differential equations to express the model [2]:

$$J_M\ddot{\theta}_M + C_{s1}(\dot{\theta}_M - \dot{\theta}_1) + K_{s1}(\theta_M - \theta_1) = T_M \tag{2.15}$$

$$J_1\ddot{\theta}_1 + C_{s1}(\dot{\theta}_1 - \dot{\theta}_M) + K_{s1}(\theta_1 - \theta_M) + C_g(t)\left[R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2\right] + K_g(t)\left[R_{b1}(R_{b1}\theta_1 - R_{b2}\theta_2)\right] = T_{f1}(t) \tag{2.16}$$

$$J_2\ddot{\theta}_2 + C_{s2}(\dot{\theta}_2 - \dot{\theta}_1) + K_{s2}(\theta_2 - \theta_1) + C_g(t)\left[R_{b2}\dot{\theta}_2 - R_{b1}\dot{\theta}_1\right] + K_g(t)\left[R_{b2}(R_{b2}\theta_2 - R_{b1}\theta_1)\right] = T_{f2}(t) \tag{2.17}$$

$$J_L\ddot{\theta}_L + C_{s2}(\dot{\theta}_L - \dot{\theta}_2) + K_{s2}(\theta_L - \theta_2) = -T_L \tag{2.18}$$

Where: J_M, J_1, J_2, J_L: mass moment of inertia for motor, gear 1, gear 2 and output device.

θ_M, θ_1, θ_2, θ_L: angular rotations of motor, gear 1, gear 2 and output device.

C_{s1}, C_{s2}, $C_g(t)$: damping coefficients of the shafts and the gears.

K_{s1}, K_{s2}, $K_g(t)$: stiffnesses of the shafts and gears.

T_M, T_L, $T_{f1}(t)$, $T_{f2}(t)$: motor and load torques and frictional torques on the gear.

R_{b1}, R_{b2}: base circle radii of the gears.

t: time.

$\dot{\theta}$: angular velocity.

$\ddot{\theta}$: angular acceleration.

The damping coefficients and the stiffness of the gear tooth mesh have to be determined first. The equations of damping in shaft and gear mesh are obtained by Kasuba and Evans [12].

$$C_{s1} = 2\xi_1 \sqrt{\frac{K_{s1}}{\frac{1}{J_M} + \frac{1}{J_1}}} \tag{2.19}$$

$$C_{s2} = 2\xi_2 \sqrt{\frac{K_{s2}}{\frac{1}{J_O} + \frac{1}{J_2}}} \tag{2.20}$$

$$C_g(t) = 2\xi_g \sqrt{\frac{K_g(t)}{\frac{R_{b1}^2}{J_1} + \frac{R_{b2}^2}{J_2}}} \tag{2.21}$$

Where: ξ_1, ξ_2 : damping ratios of the first and second shaft which have values between: 0.005-0.0075 [14, 15].

ξ_g : damping ratio of the gear mesh and which lies between: 0.03-0.17 [12].

Because the gear meshing process has been explained earlier, the gear meshing stiffness can be obtained as follows:

$$K_g(t) = \frac{W(t)}{Q(t)} \tag{2.22}$$

Where: $W(t)$: transmitted input load.

$Q(t)$: deformation of the tooth (total transmission error) in the direction of $W(t)$.

The shaft stiffness K_S can be expressed by the following equation:

$$K_S = \frac{JG}{l} \tag{2.23}$$

Where: G : the shear modulus of the shaft.

l : the length of the shaft.

J : the polar moment of inertia of the shaft

$$J = \frac{\pi\rho(D_o^2 - D_i^2)}{32}$$

D_o and D_i: the outside and inside diameter of the shaft.

ρ : the mass density.

2.6.2 Dynamic tooth load and dynamic load factor

The dynamic tooth load at contact point j is the product of the relative gear tooth displacements $(R_{b1}\theta_1 - R_{b2}\theta_2)$ and the corresponding meshing stiffness plus the product of the velocities with the damping coefficient. During the gear meshing process, those following conditions may happen [2].

1) $(R_{b1}\theta_1 - R_{b2}\theta_2) > 0$.

This is the normal case during meshing process [16].

The dynamic tooth load (W_d) at the point j is:

$$(W_d)_j = (K_g)_j - (R_{b1}\theta_1 - R_{b2}\theta_2)_j + \left(C_g\right)_j(R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2)_j \tag{2.24}$$

2) $(R_{b1}\theta_1 - R_{b2}\theta_2) \leq 0$.

In this case, gear will separate and the contact will be lost which does not usually happen.

$(W_d)_j = 0$

3) $(R_{b1}\theta_1 - R_{b2}\theta_2) < 0$ and $\left|(R_{b1}\theta_1 - R_{b2}\theta_2)\right| > \delta$.

In this condition, Gear 1 will collide with gear 2 at the back side. The dynamic tooth load at the point j could be obtained by:

$$(W_d)_j = (K_g)_j(R_{b2}\theta_2 - R_{b1}\theta_1)_j + (C_g)_j(R_{b2}\dot{\theta}_2 - R_{b1}\dot{\theta}_1)_j \qquad (2.25)$$

After the gear dynamic load has been calculated, the dynamic load factor can be determined. The dynamic load factor is the ratio of the dynamic tooth load divided by the static applied load. There have a relationship between the dynamic load factor and the tooth spacing error in this study.

2.7 Iterative Procedures

In order to solve equations (2.15)-(2.18), DANST is used in this study to simulate the dynamic loading in the gear meshing process. Figure 2.9 displays a flowchart of the generalized computational procedure for the solution of the solution of the governing differential equations. The equations were linearized by dividing the mesh period into small intervals. A constant input torque T_M was assumed. The output torque T_L was considered to fluctuate as a result of time-varying stiffness, friction, and damping in the mesh.

To start the solution iteration process, initial values of the angular displacements are obtained by preloading the input shaft the nominal torque carried by the system. Initial values of the angular speed are taken from the nominal system operating speed.

The iterative procedure is as follows: the calculated values of the angular displacement and speed after one mesh period are compared with the assumed initial values. Unless the differences between them are smaller than a preset tolerance, the procedure is repeated using the average of the initial and calculated values as new initial conditions.

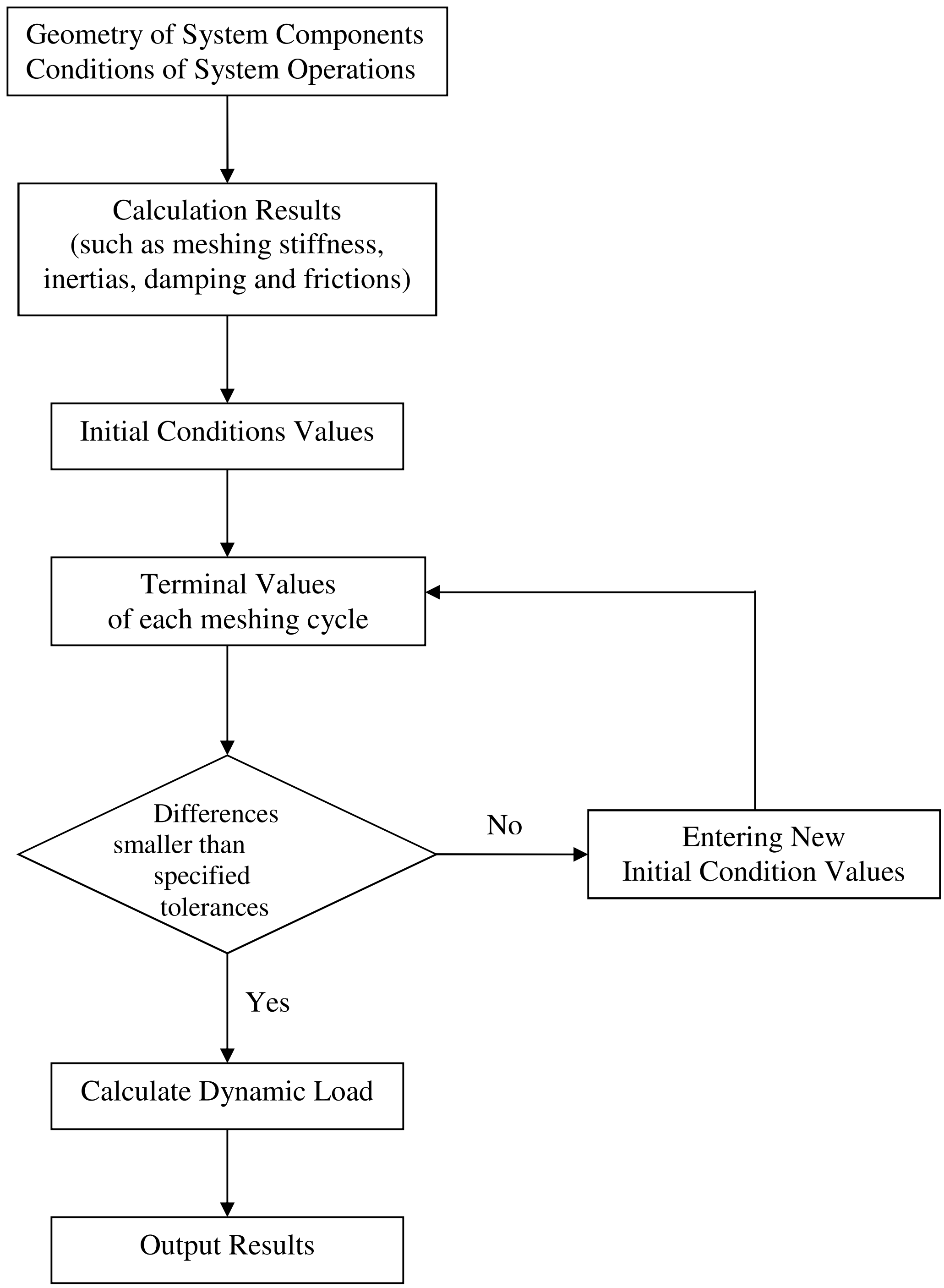

Figure 2.9 Whole procedure of calculating dynamic load

CHAPTER 3

3. APPLICATIONS AND RESULTS

3.1 Tooth Spacing Error and Profile Modification

In a gear system, some tooth spacing errors are found to have the form of a sine function [2,3,13]. Therefore, full-sine and half-sine spacing error will be used to investigate their effects on the dynamic tooth load in this study, see Figure 3.1. Another type of spacing error, random distribution spacing error is also used in the study. Random distribution means for that all spacing errors in a gear, most are very small and only less than 10% of them have big error values. This type of spacing error is close to some of the actual gear error (Figure 3.2). The magnitudes of the spacing error used in this study are varied between 0.0001 and 0.0003in, with 0.0001in as an increment.

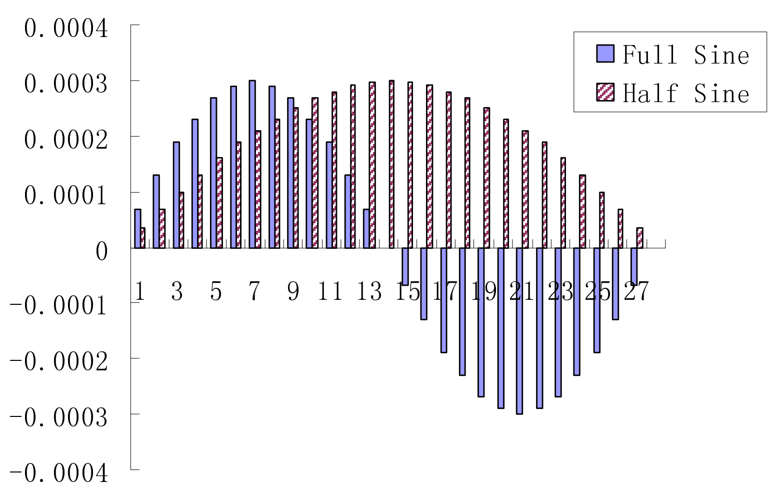

Figure 3.1 Full and Half-sine form cumulative spacing error distribution for a 28-teeth gear. The maximum error value is 0.0003in [3].

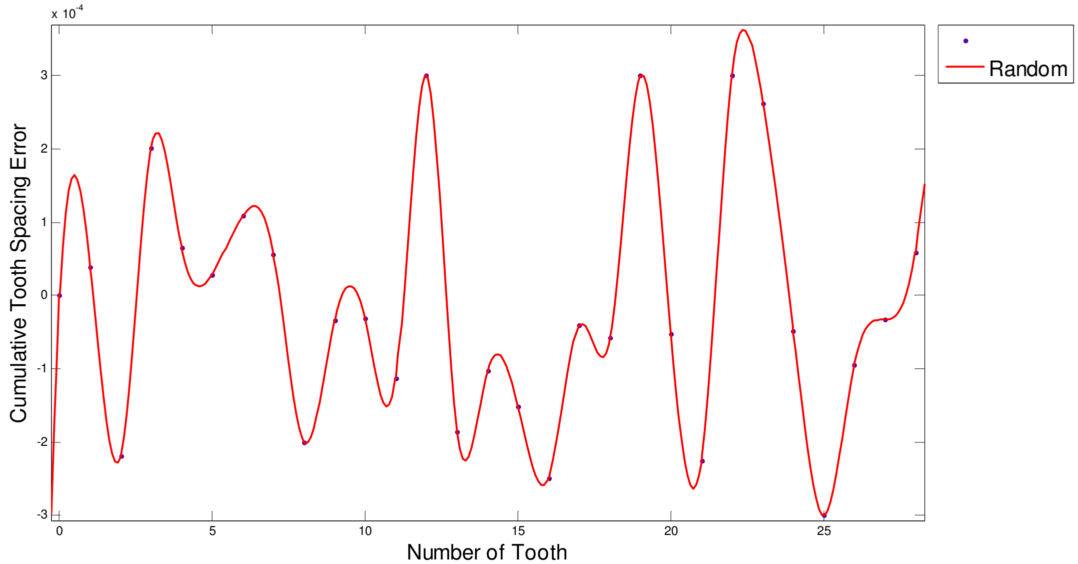

Figure 3.2 Random form cumulative spacing error distribution for a 28-teeth gear. The maximum error value is 0.0003in.

In this study, two kinds of tooth profiles modifications are used were in this study in order to obtain the most effective one. There are two parameters related to the profile modification: the amount of modification (Δ) and the length of modification (Ln). Both of these two parameter start at the highest point of single tooth contact (HPSTC) See (Figure 3.3). Both profile modification parameters are varied systematically. Let one parameter changed and the other one be the unit.

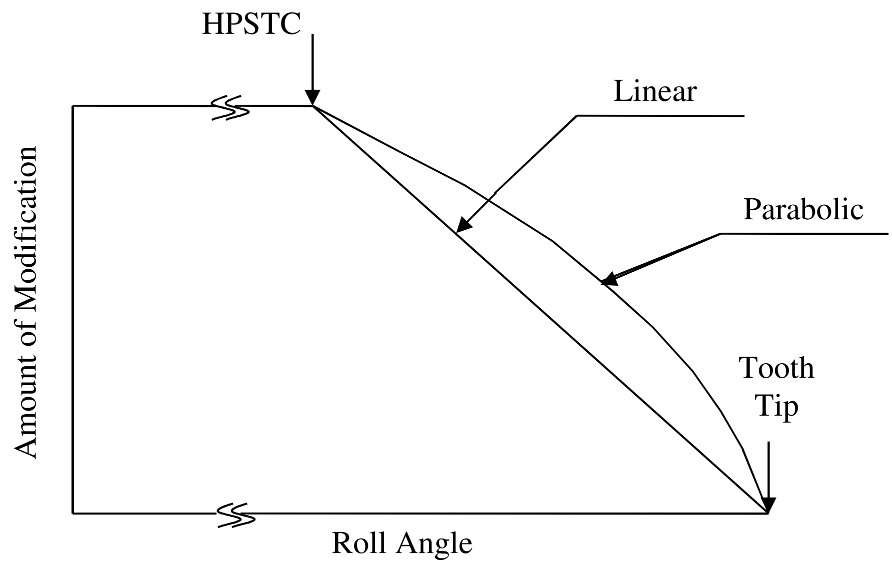

Figure 3.3 Two types of gear tooth profile modification: linear and parabolic.

A gear set with a driving gear of 28 teeth and a driven gear of 42 teeth is used for the analysis in this study. The corresponding parameters of this gear set are shown in the following table (Table 3.1).

Table 3.1 Gears parameters

Driving gear teeth	28
Driven gear teeth	42
Diametral pitch (teeth/in)	8.0
Face width (inch)	0.5
Design load (lb)	1500
Theoretical contact ratio	1.618
Pressure angle (degree)	20

3.2 Influence of Tooth Profile Modification

From Figure 3.4 to Figure 3.6, three types of tooth spacing error is used to compare the dynamic load factor. The length and amount parameters are of full magnitude (Δ=100%, Ln=100%). The cumulative spacing error is increased from 0 to 0.0003 in, In these figures, the linear modification shown to be better than the parabolic and non-modification types, and the parabolic type is better than non-modification type. With both linear and parabolic being good choices to reduce the dynamic load for all types of spacing error, however, when spacing error is greater than 0.0002 in (around 0.0003 in), the non-modified one is better than the parabolic profile modification for all cases with full-sine spacing error.

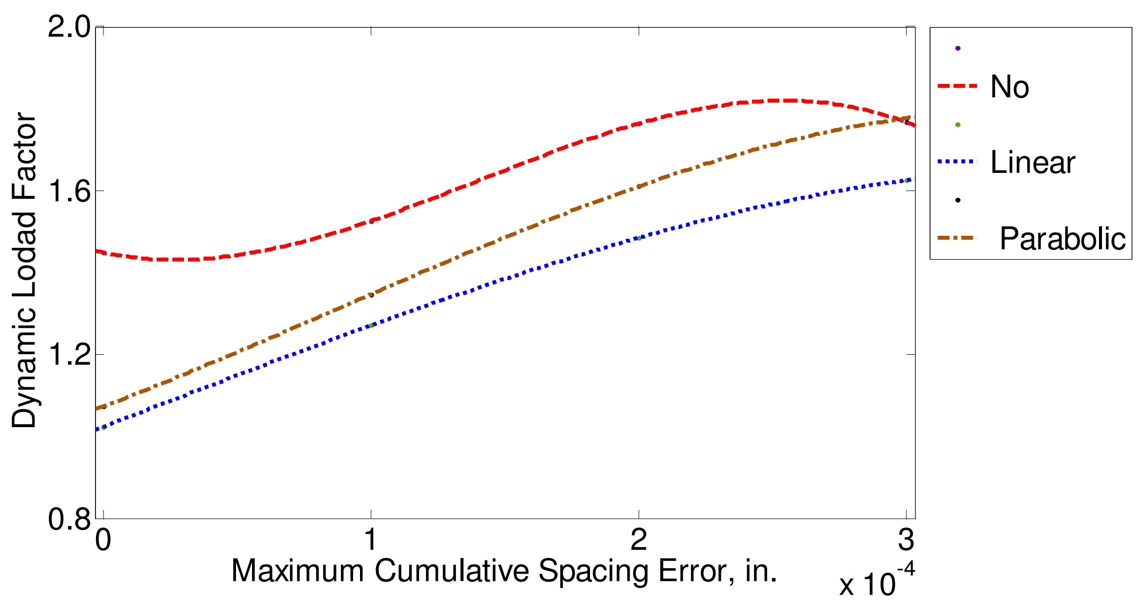

Figure 3.4 Random type of spacing error with no-modification, linear and parabolic modifications (Δ=100%, Ln=100%).

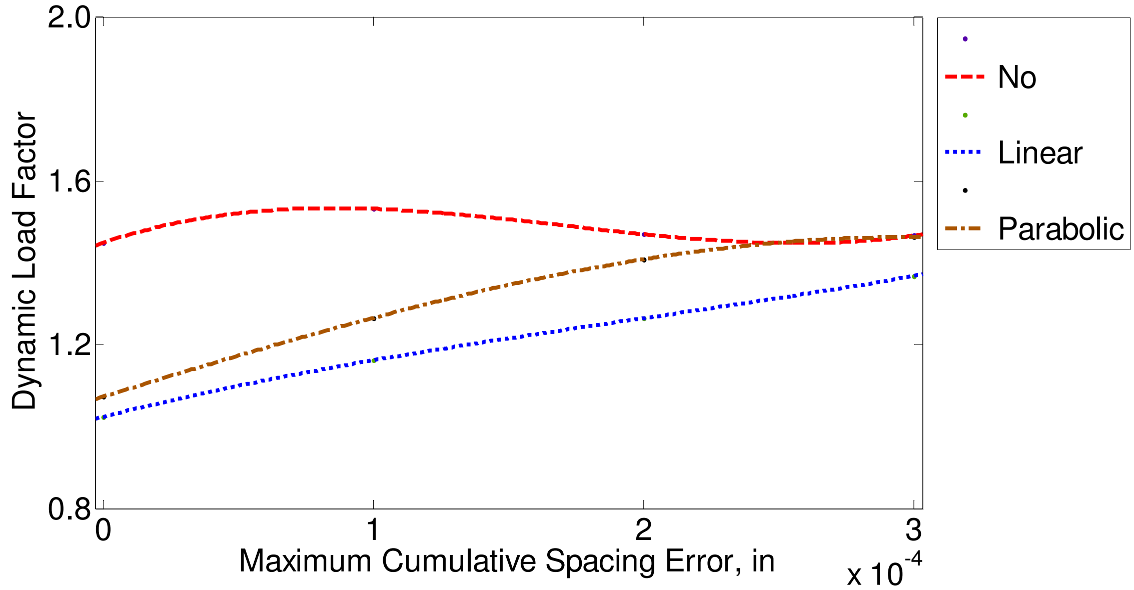

Figure 3.5 Full-sine type of spacing error with no-modification, linear and parabolic profile modifications (Δ=100%, Ln=100%).

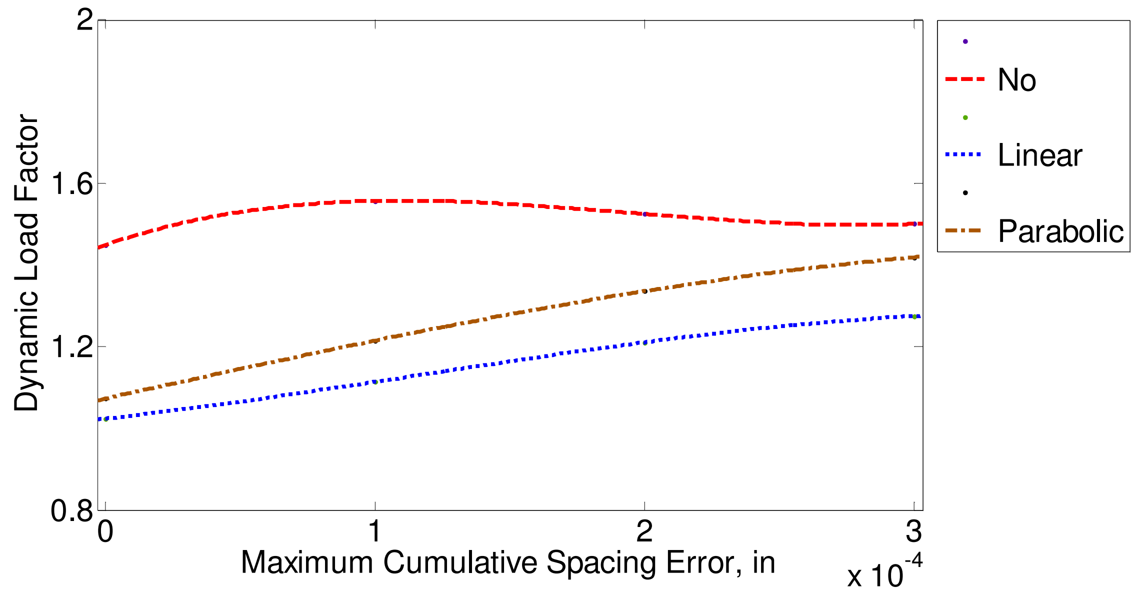

Figure 3.6 Half-sine type of spacing error with no-modification, linear and parabolic profile modifications (Δ=100%, Ln=100%).

For the group of Figure 3.7 to Figure 3.9, the normalized length of profile modification is reduced to 70%, but the amount of modification remains at Δ=100% of the normalized value. As shown in the figures the gears with no spacing error but have certain profile modifications achieved reduced dynamic load better than the non-modified one. For the random wave type spacing error, when the spacing error is greater than 0.0015 in, the non-modified one has better reduction in the dynamic load. For the full-sine type of spacing error, when the error magnitude is greater than 0.0002in, the non-modified gears are better than the modified ones. However, the shorter length profile modification actually increases dynamic load factor in this condition.

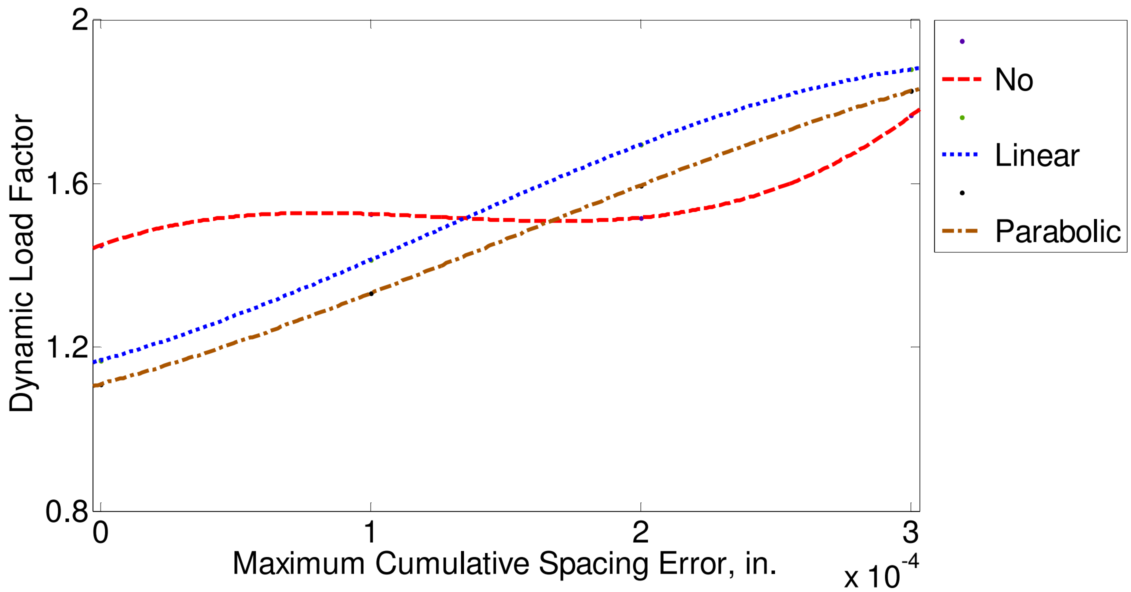

Figure 3.7 Random type of spacing error with no-modification, linear and parabolic modifications (Δ=100%, Ln=70%).

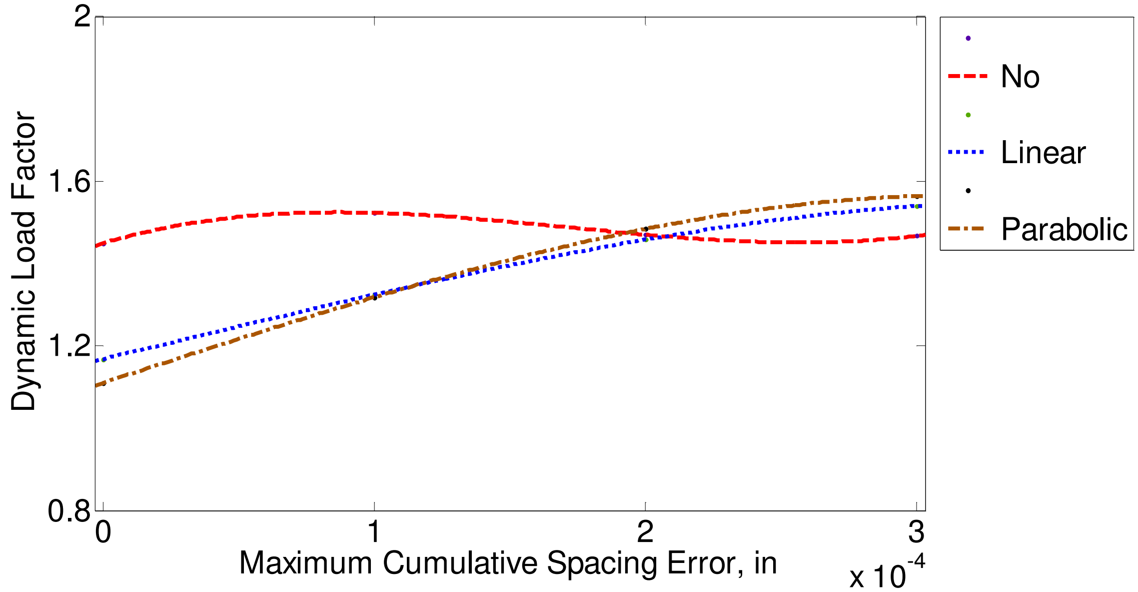

Figure 3.8 Full-sine type of spacing error with no-modification, linear and parabolic profile modifications (Δ=100%, Ln=70%).

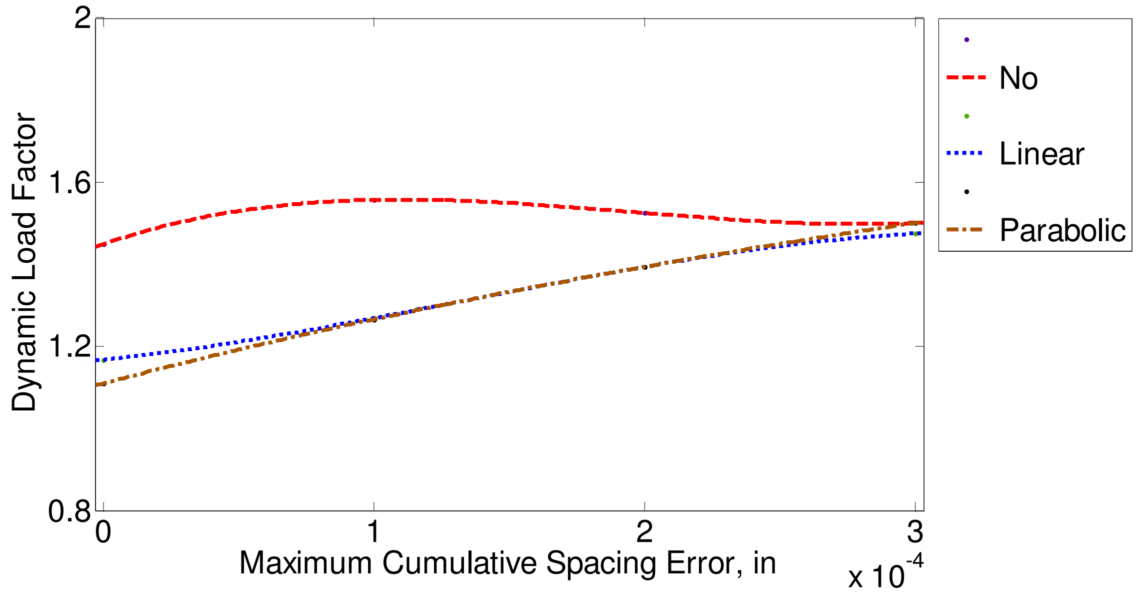

Figure 3.9 Half-sine type of spacing error with no-modification, linear and parabolic profile modifications (Δ=100%, Ln=70%).

For Figure 3.10 to Figure 3.12, the modification amount is Δ=50% with a modification length Ln=100%, the spacing error is varied from 0 to 0.0003 in. The linear type is the best one in these three figures. It is better than the non-modified one and parabolic type. Comparing these figures with Figure 3.7 though Figure 3.9, it is obvious that the lower amount of profile modification could reduce the dynamic load, and a reduced modification amount produces very significant dynamic load reduction for gears with no spacing error.

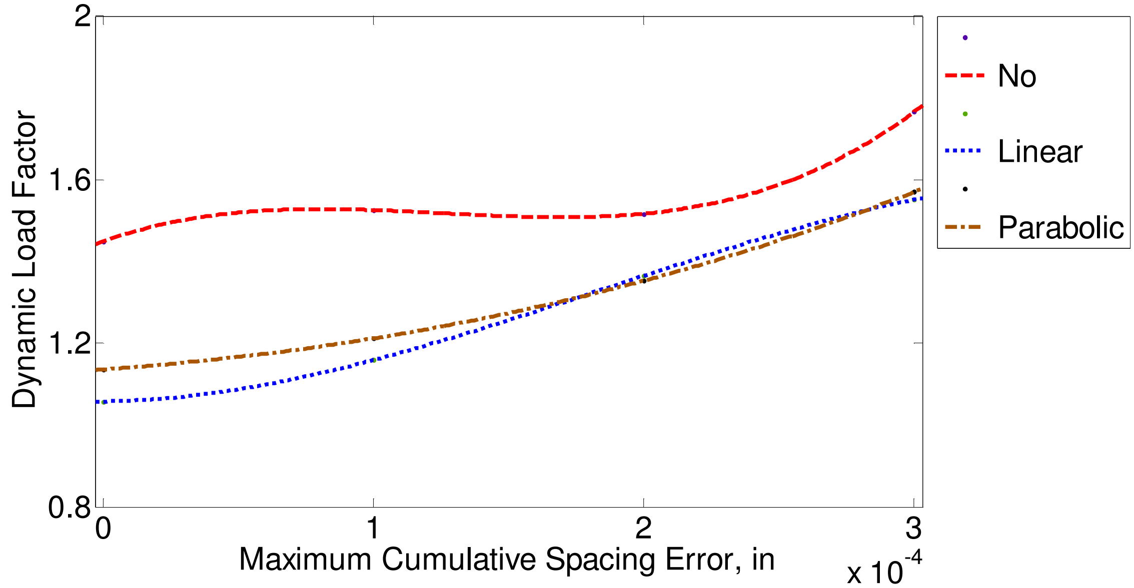

Figure 3.10 Random type of spacing error with no-modification, linear and parabolic modifications (Δ=50%, Ln=100%).

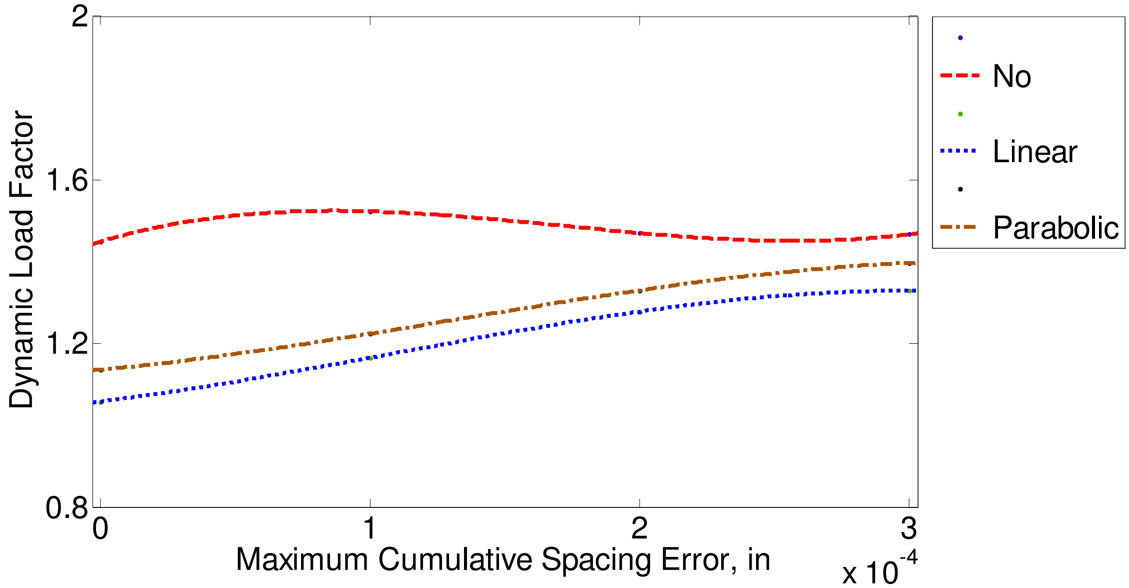

Figure 3.11 Full-sine type of spacing error with no-modification, linear and parabolic profile modifications (Δ=50%, Ln=100%).

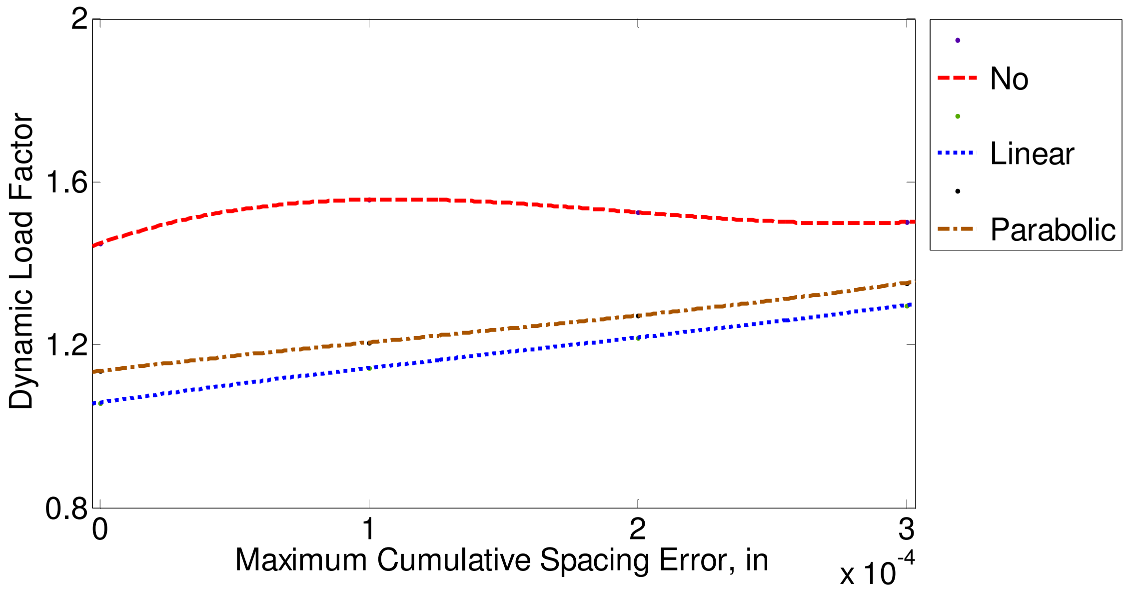

Figure 3.12 Half-sine type of spacing error with no-modification, linear and parabolic profile modifications (Δ=50%, Ln=100%).

3.3 Influence of Type and Length of Tooth Profile Modification

For Figures 3.13, Figure 3.14 and Figure 3.15, the amount of profile modification is kept at 100%, and the length of profile modification is reduced to 85%. The figures compare the combination of the three different types of spacing error and two profile modification types. The dynamic load factor of the linear type is always the lower one. However, when the spacing error is small, there is no significant difference between the dynamic load factors of all types of profile modification, When the value of spacing error is close to 0.0003in, the parabolic modification is the better choice.

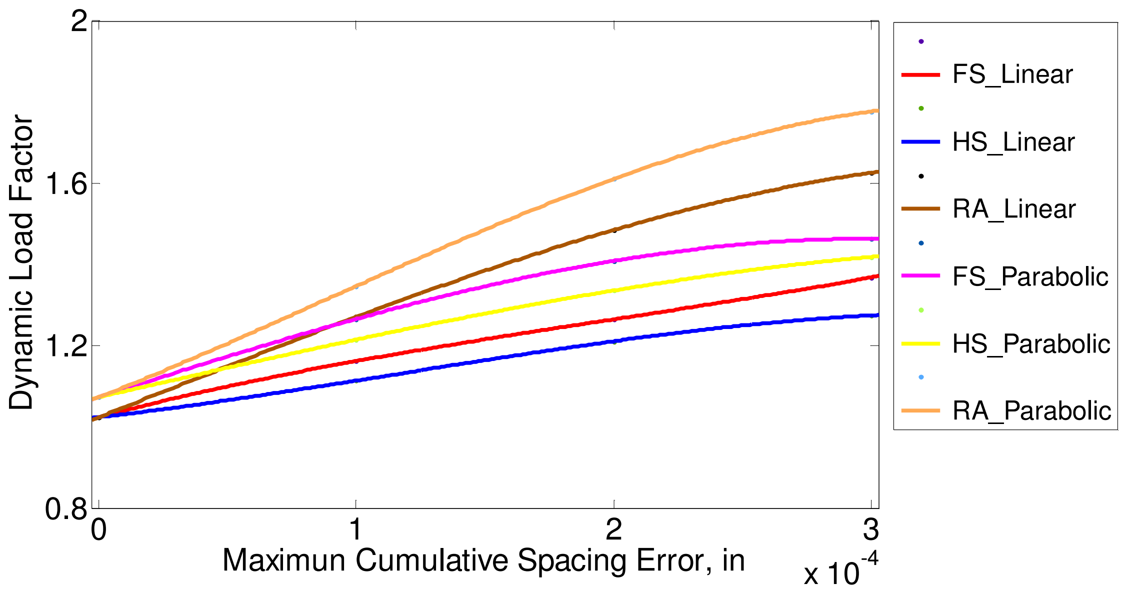

Figure 3.13 Comparison of different type tooth spacing error and profile modifications for (Δ=100%, Ln=100%).

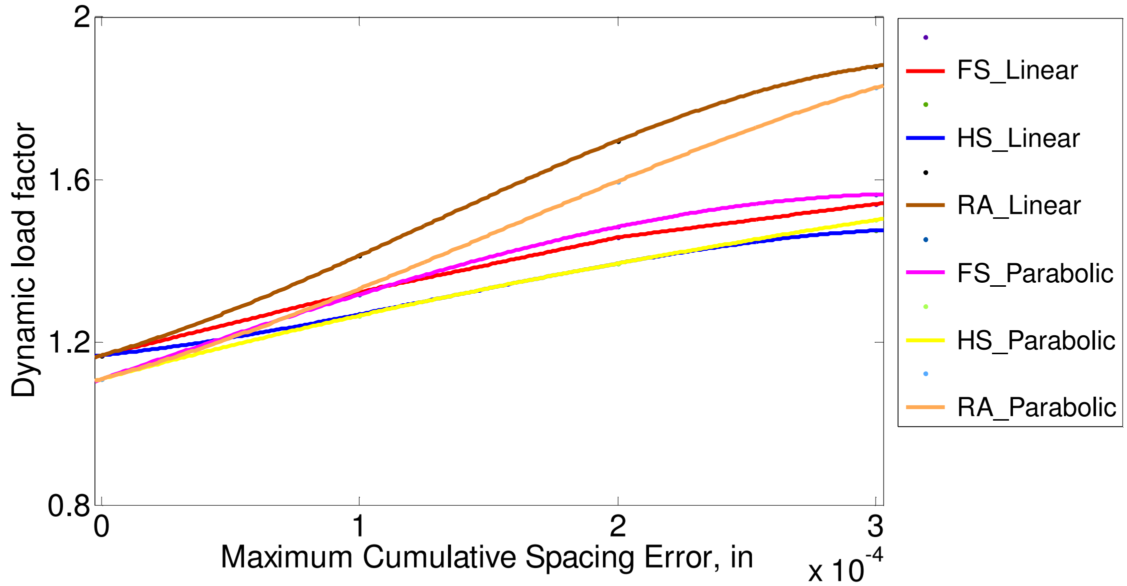

Figure 3.14 Comparison of different type tooth spacing error and profile modifications for (Δ=100%, Ln=65%).

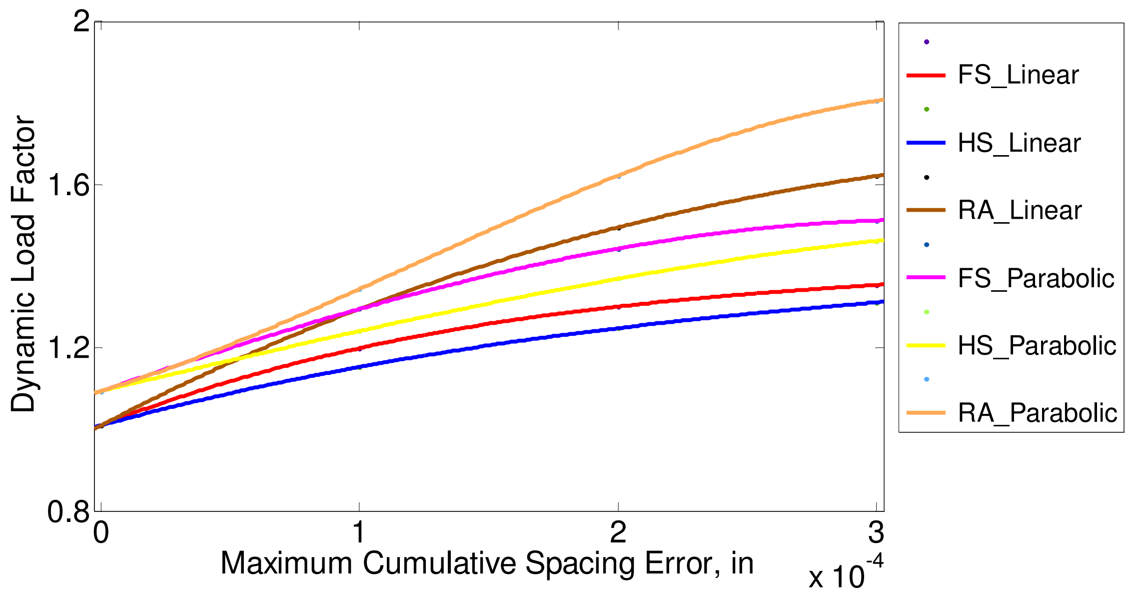

Figure 3.15 Comparison of different type tooth spacing error and profile modifications for (Δ=100%, Ln=85%).

3.4 Influence of Amount and Length of Tooth Profile Modification

The amount and length of tooth profile modification are varied systematically to investigate their dynamic effect on the gear system. First, the normalized modification amount is changed from 0.60 to 1.20 with a 0.20 increment with the normalized modification length kept at a constant value. Then, the normalized modification length is changed from 0.60 to 1.00 with a 0.20 increment with the normalized modification amount kept at a constant value.

3.4.1 Influence of amount of modification

Figure 3.16 to Figure 3.21 show a complicated variation of dynamic load factor. In general, basically the lower modification amount creates lower dynamic load factor when the spacing error increases. Except for Figure 3.18, where for Δ=1.2, the dynamic load factor is lower than the others. Overall, over-modification will create higher dynamic load factor. It can be seen that the gear dynamic load variation over the spacing errors studied is narrower for linear modification than for parabolic one, and the linear type of profile modification can lead to lower dynamic load factor.

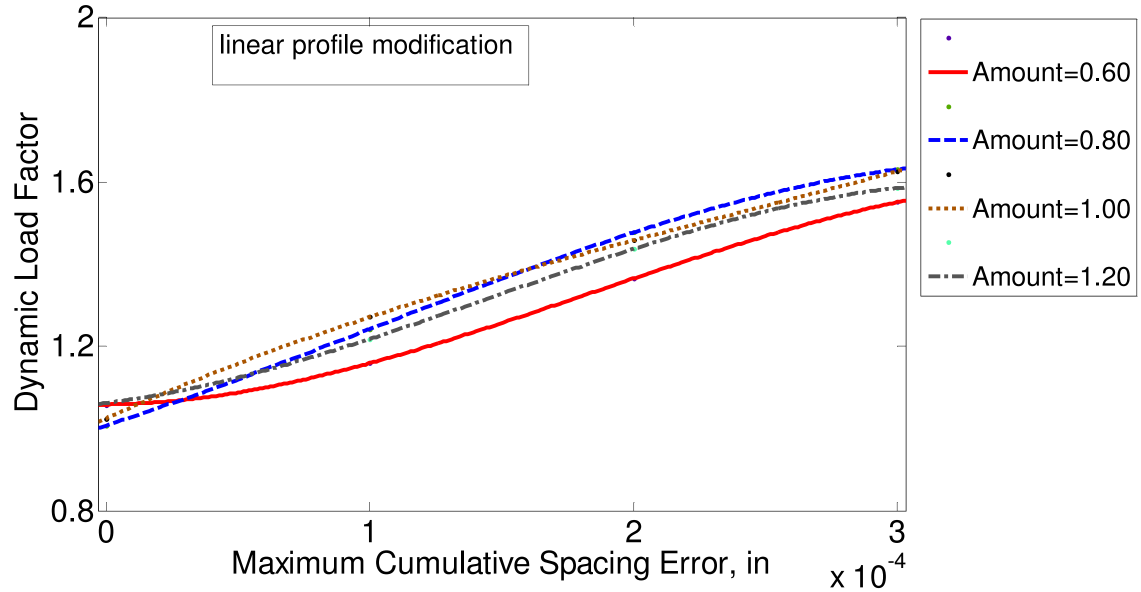

Figure 3.16 Effect of the amount of profile modification on random type of spacing error with linear profile modification (Ln=100%).

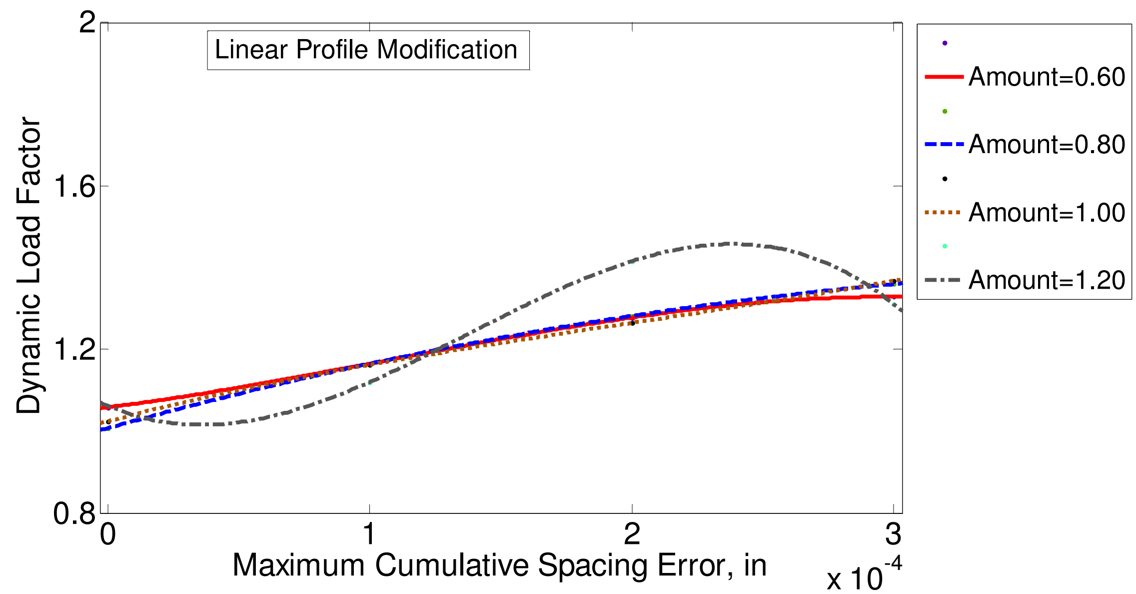

Figure 3.17 Effect of the amount of profile modification on full-sine type of spacing error with linear profile modification (Ln=100%).

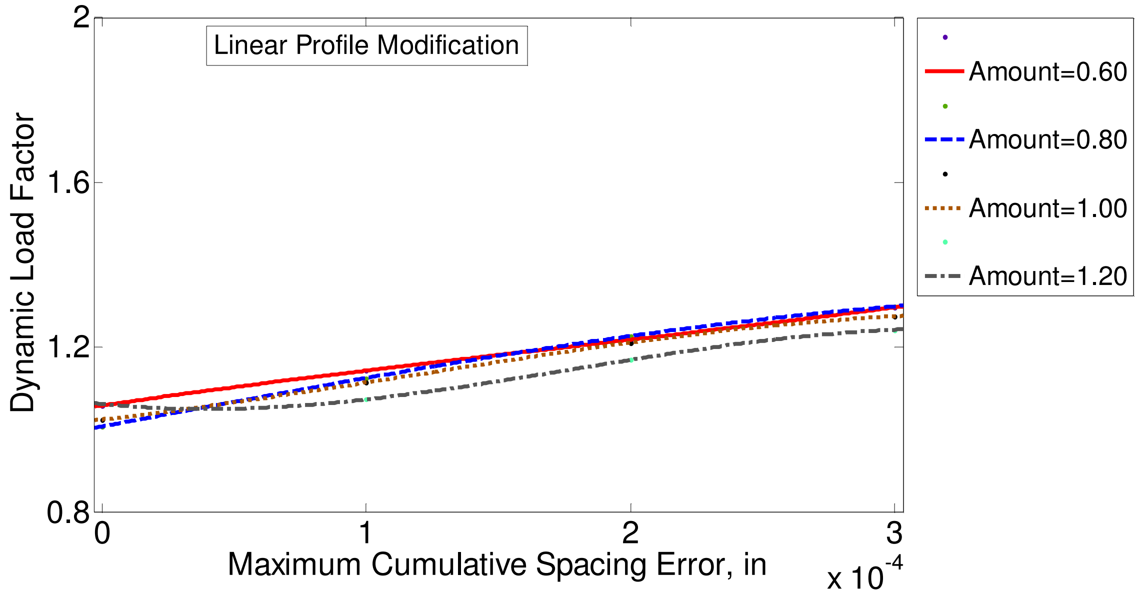

Figure 3.18 Effect of the amount of profile modification on half-sine type of spacing error with linear profile modification (Ln=100%).

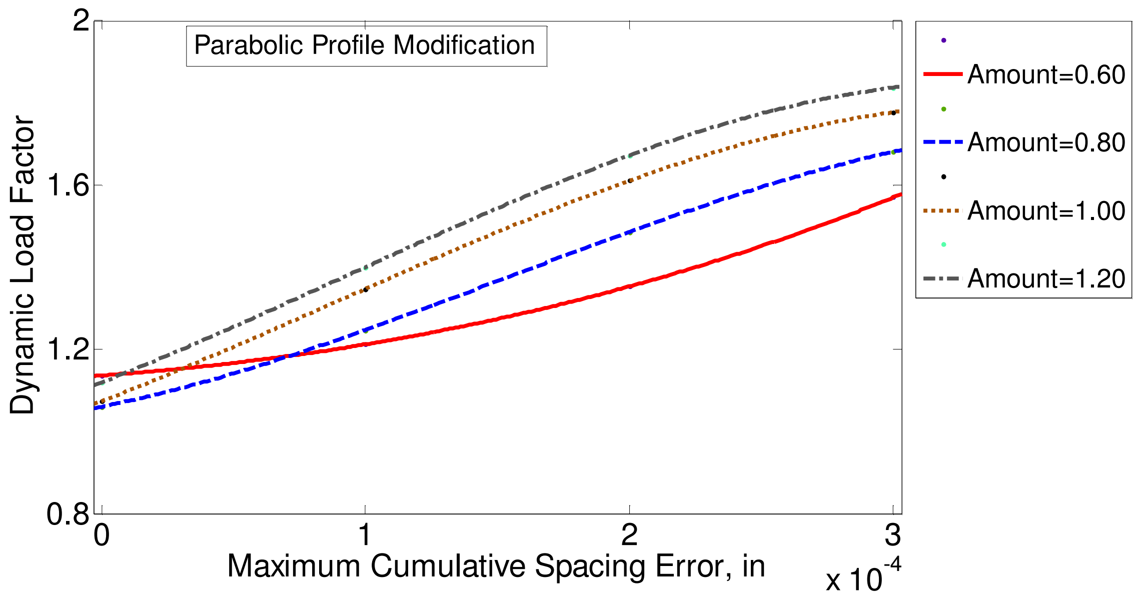

Figure 3.19 Effect of the amount of profile modification on random type of spacing error with parabolic profile modification (Ln=100%).

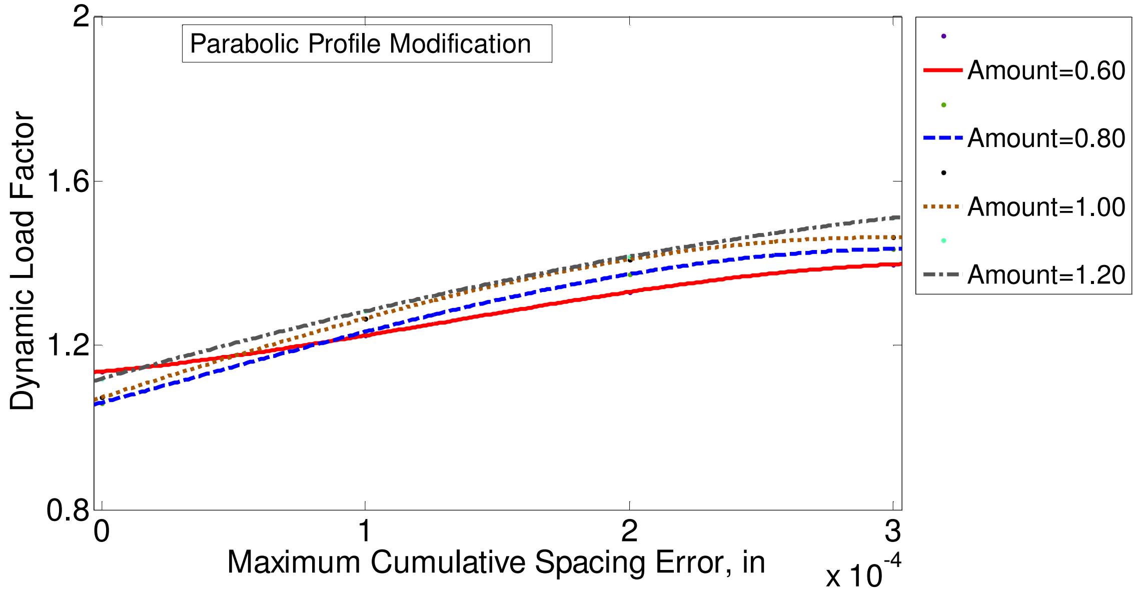

Figure 3.20 Effect the amount of profile modification on full-sine type of spacing error with parabolic profile modification (Ln=100%).

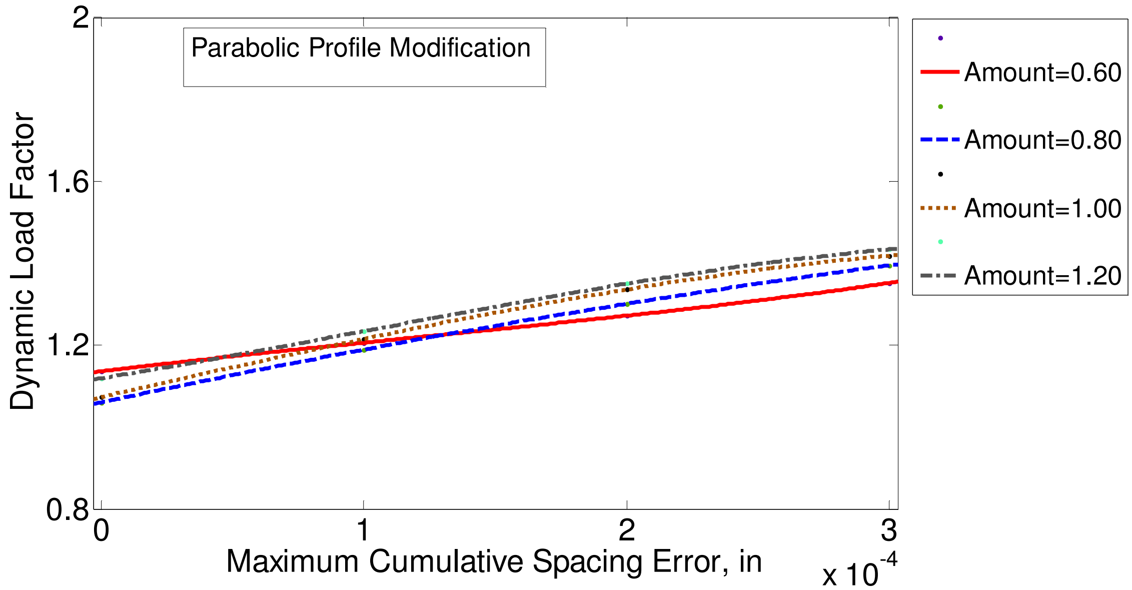

Figure 3.21 Effect of the amount of profile modification on half-sine type of spacing error with parabolic profile modification (Ln=100%).

3.4.2 Influence of length of modification

For Figure 3.22 though Figure 3.27, the amount of linear and parabolic profile modification is kept at 100%, while the length of modification is varied from 0.6 to 1.0, with an increment of 0.2. The dynamic load factor increases in inverse proportion with the length of modification. The normalized modification length of 0.8 and 1.0 are better than the normalized modification length of 0.6. The full modification length is a better choice for the regular type tooth spacing error such as full-sine and Half-sine or even random type. The only exception is for random type of spacing error with parabolic modification, Figure 3.25, where the shorter modification length produces lower dynamic load factor.

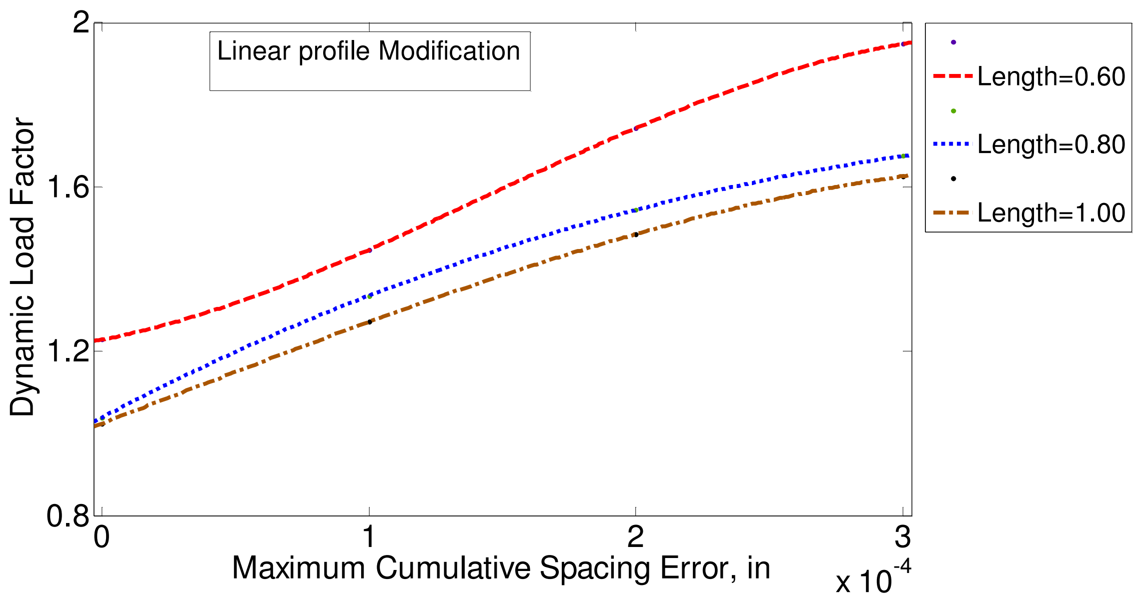

Figure 3.22 Effect of the length of profile modification on random type of spacing error with linear modification (Δ=100%).

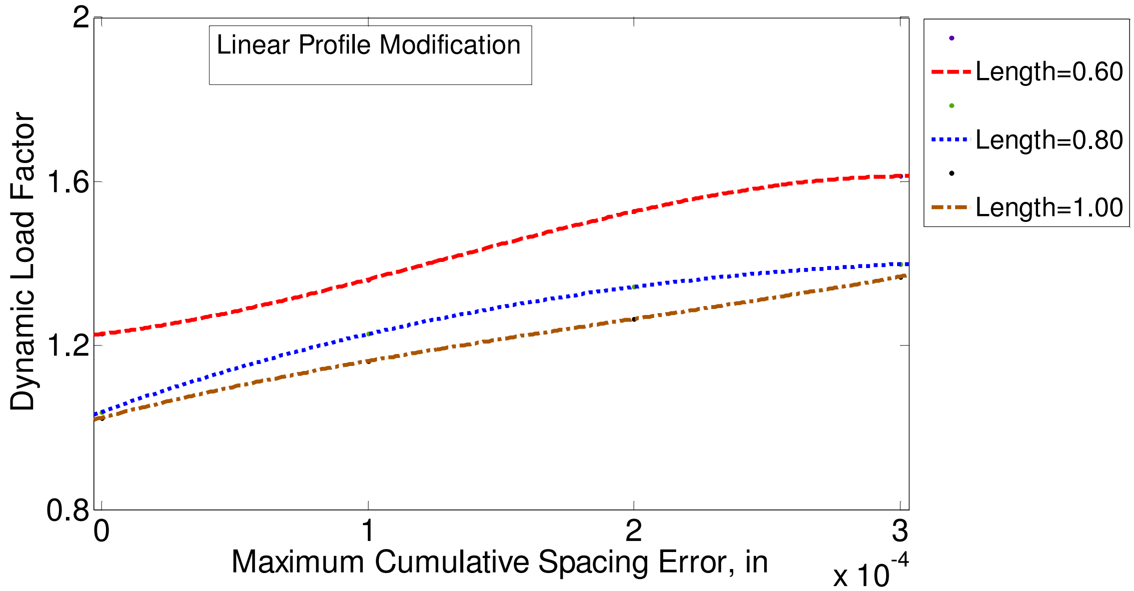

Figure 3.23 Effect of the length of profile modification on full-sine type of spacing error with linear modification (Δ=100%).

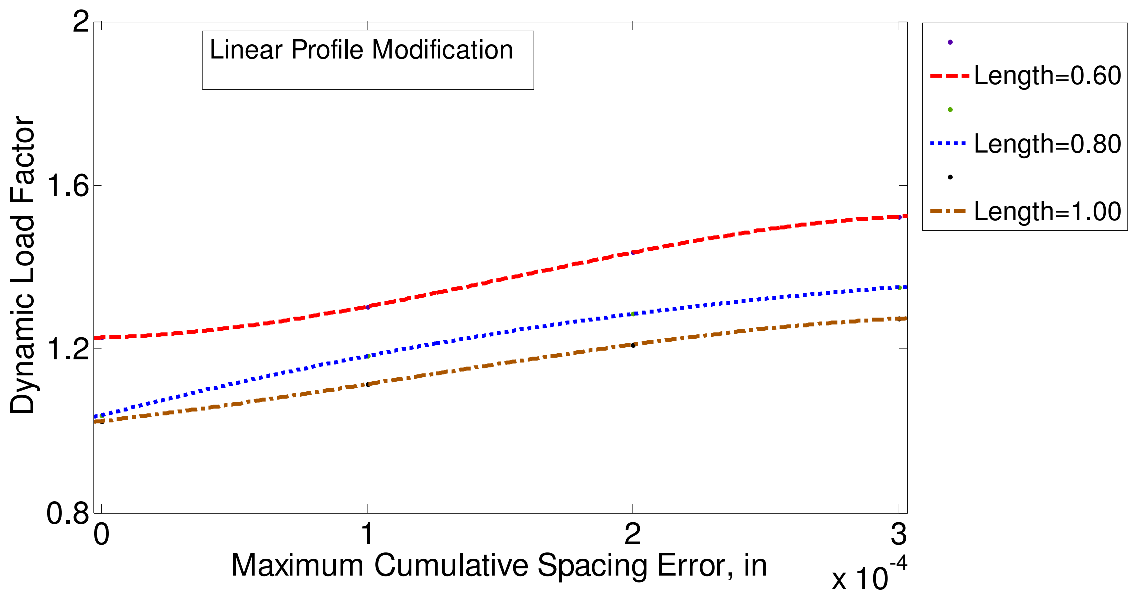

Figure 3.24 Effect of the length of profile modification on half-sine type of spacing error with linear modification (Δ=100%).

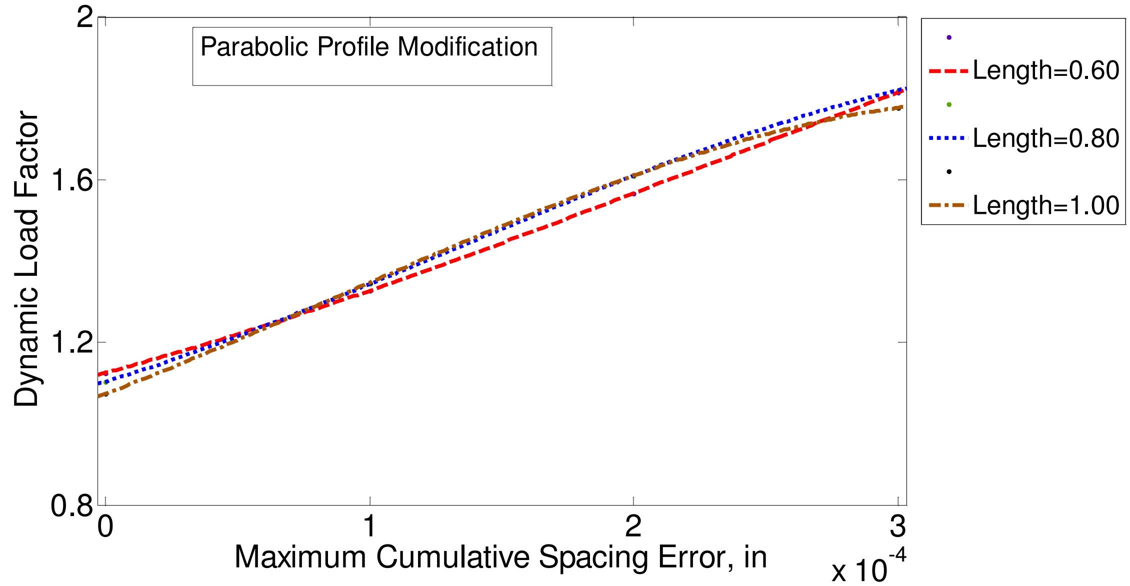

Figure 3.25 Effect of the length of profile modification on random type of spacing error with parabolic modification (Δ=100%).

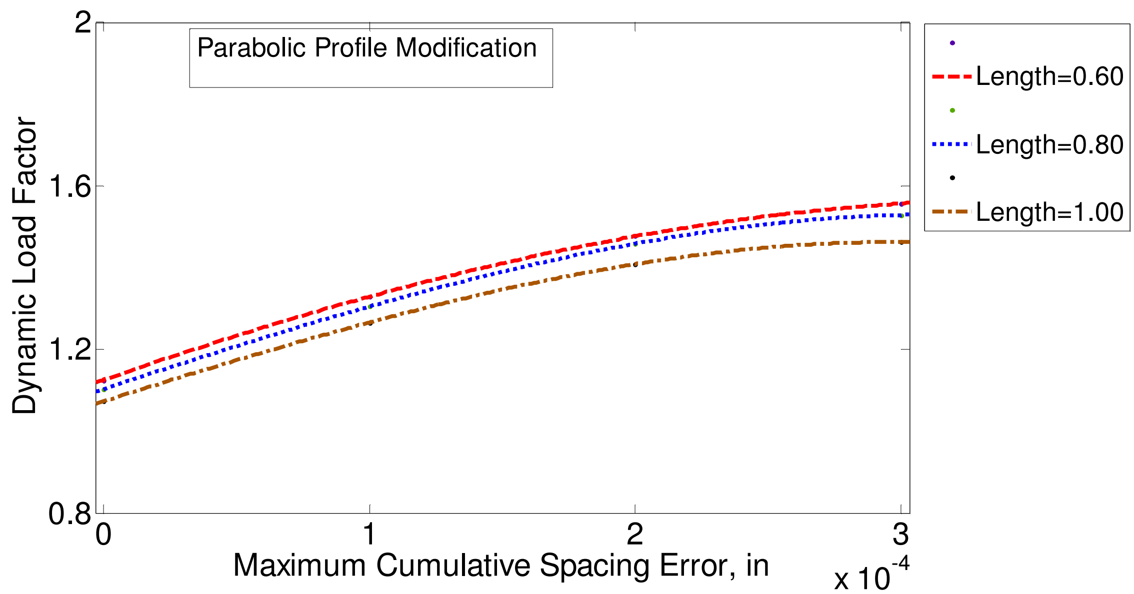

Figure 3.26 Effect of the length of profile modification on full-sine type of spacing error with parabolic modification (Δ=100%).

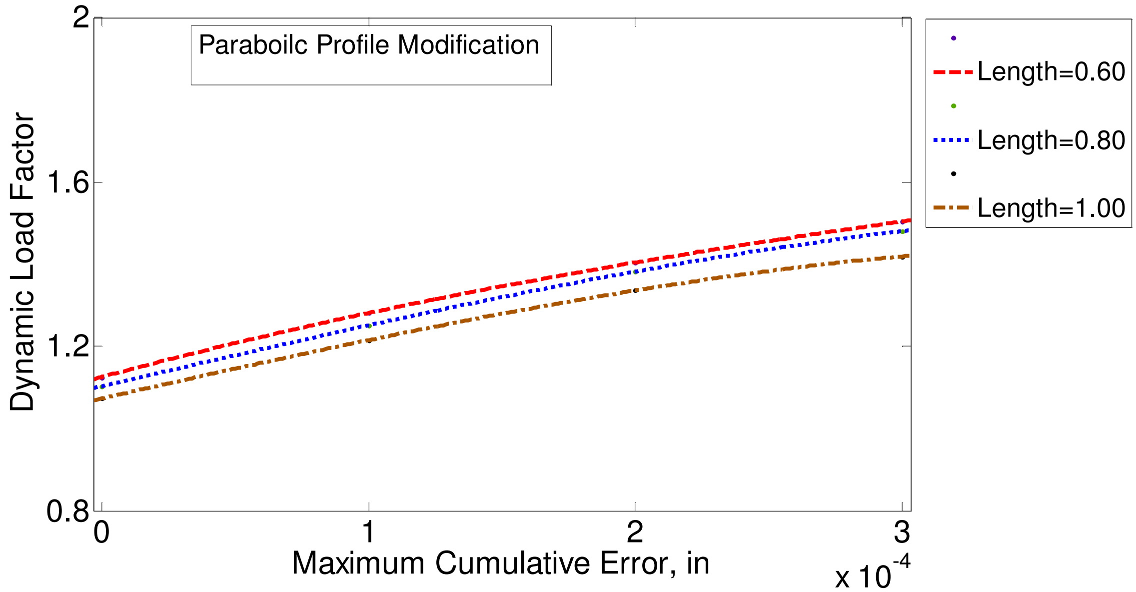

Figure 3.27 Effect of the length of profile modification on half-sine type of spacing error with parabolic modification (Δ=100%).

3.5 Dynamic Load Factor Speed Survey Analysis

The operating speed affects the dynamic response of a gear transmission system. Speed survey would display the variation of dynamic load factor in an extensive range of gear rotating speed. In this part of investigation the operating speed is increased from 2000 rpm to 10000 rpm with a 1000 rpm increment.

3.5.1 Influence of tooth profile modification in speed survey

Figure 3.28 though Figure 3.33 displays a comparison of the linear and parabolic profile modifications with different types of spacing error. In general, when the operating speed reaches around 7000 rpm, for all types of profile modification, the dynamic load factor becomes lower when compared with other speeds. When the operating speed is over 7000 rpm, the non-modified gears are the best choice to reduce dynamic load. The natural frequency of the gear transmission system is about 6000 rpm.

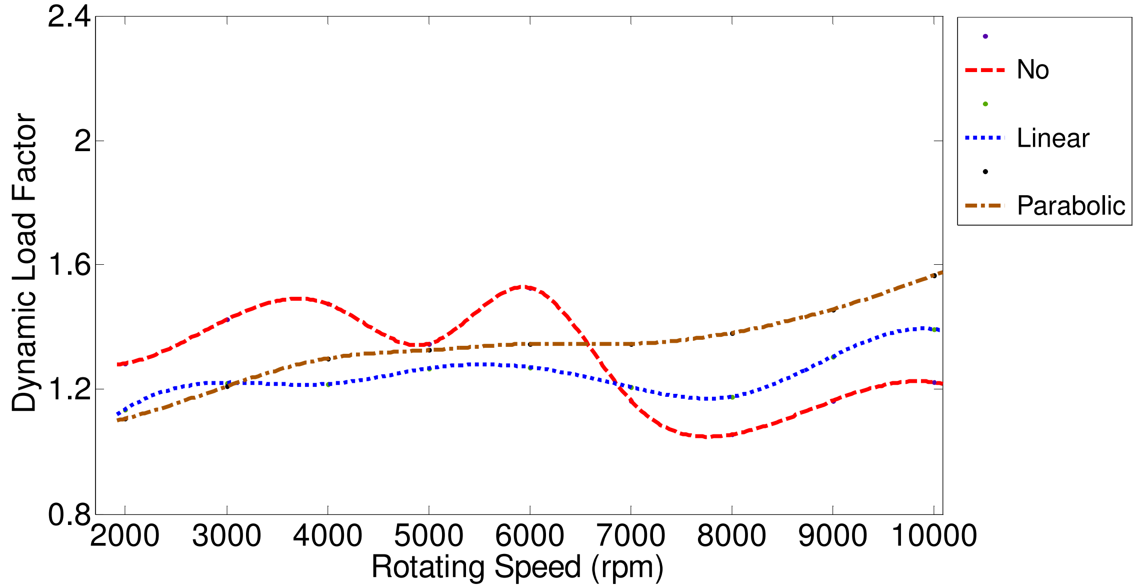

Figure 3.28 Full amount and length of tooth profile modification with maximum cumulative random tooth spacing error of 0.0001 in.

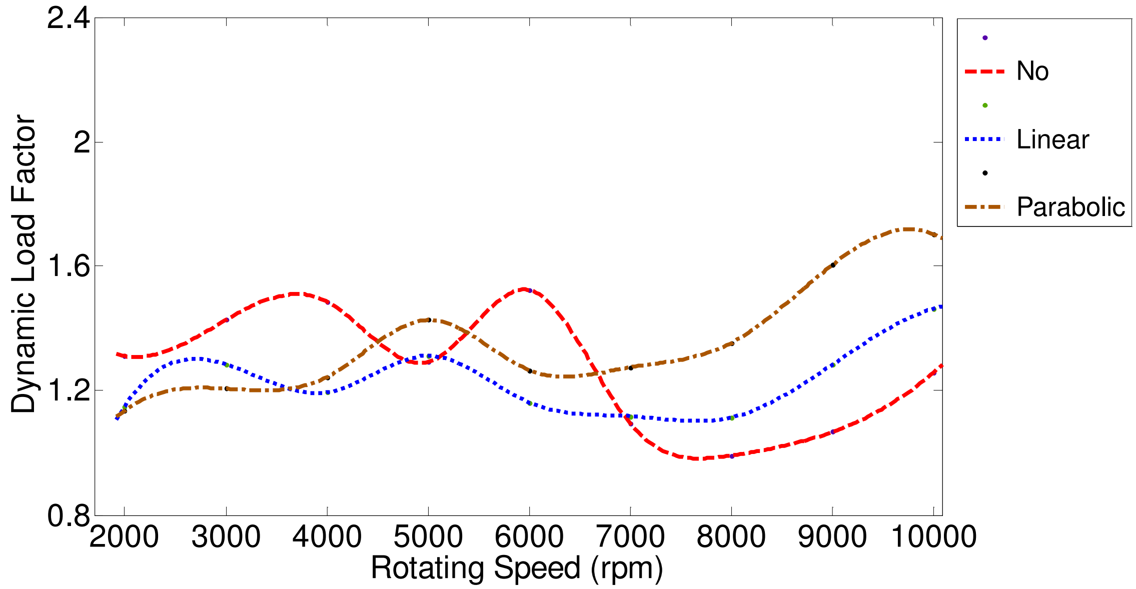

Figure 3.29 Full amount and length of tooth profile modification with maximum cumulative full-sine tooth spacing error of 0.0001 in.

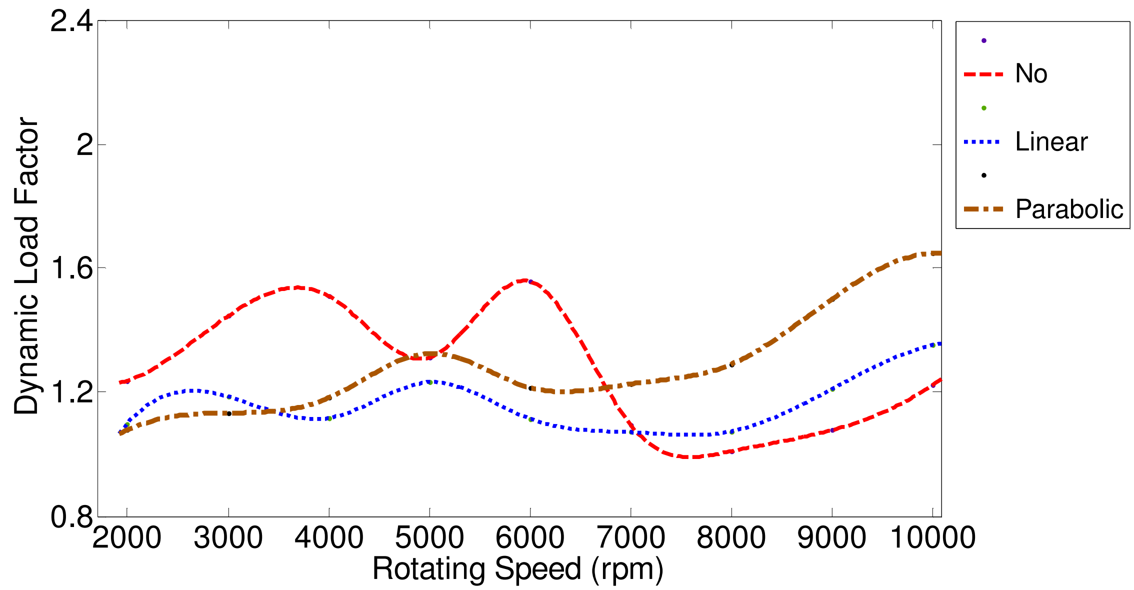

Figure 3.30 Full amount and length of tooth profile modification with maximum cumulative half-sine tooth spacing error of 0.0001 in.

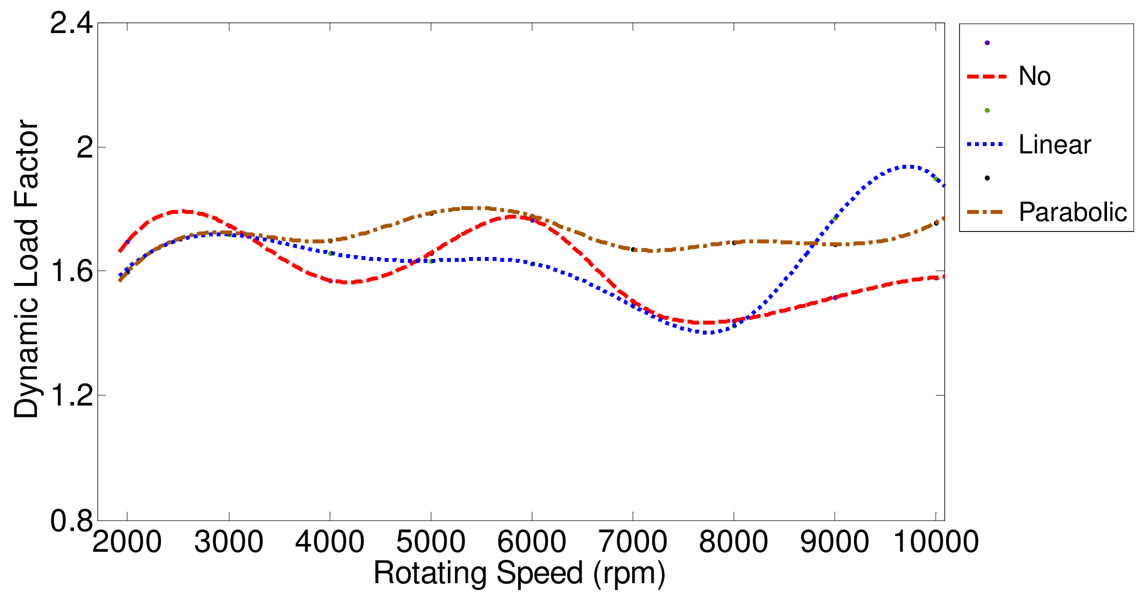

Figure 3.31 Full amount and length of tooth profile modification with maximum cumulative random tooth spacing error of 0.0003 in.

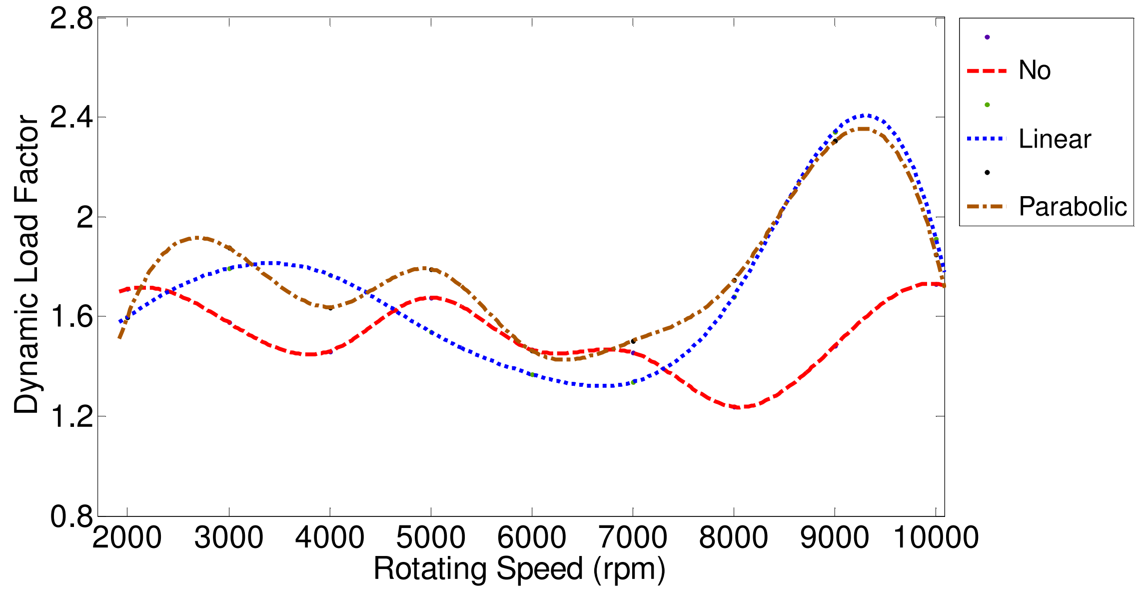

Figure 3.32 Full amount and length of tooth profile modification with maximum cumulative full-sine tooth spacing error of 0.0003 in.

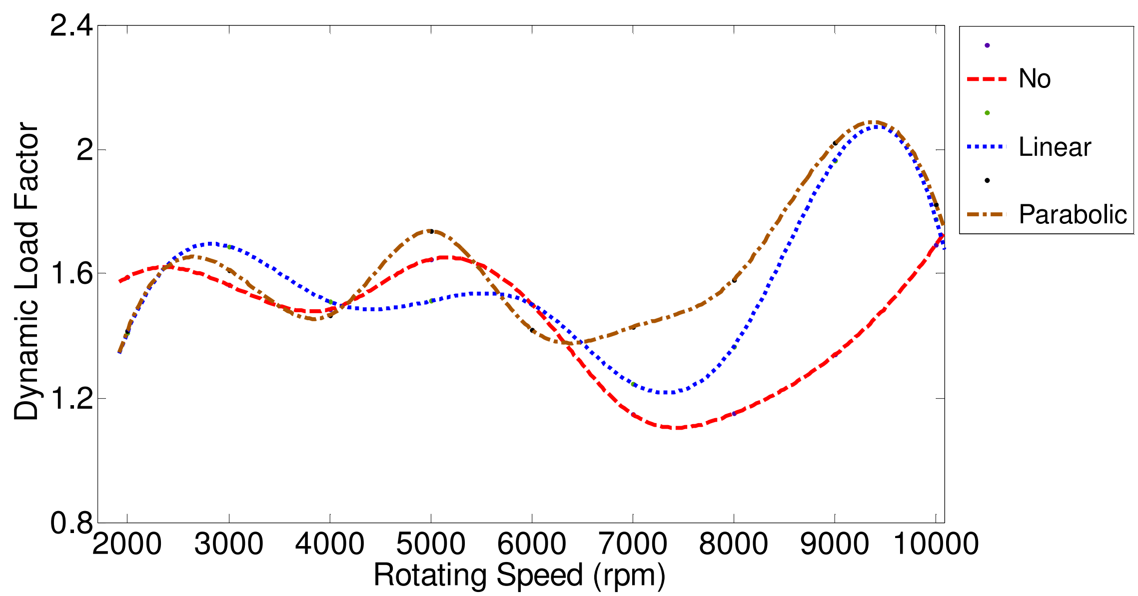

Figure 3.33 Full amount and length of tooth profile modification with maximum cumulative half-sine tooth spacing error of 0.0003 in.

3.5.2 Influence of type of tooth profile modification in speed survey

Figures 3.34 through 3.39 display dynamic load factors of sample gears, with no profile modification, linear profile modification and parabolic modification and with full-sine wave, half-sine wave and random wave forms of spacing errors at 0 in, 0.0001 in, 0.0002 in and 0.0003 in. These figures also depict a general trend of increasing gear dynamic load with higher rotating speed values. However, the sample gears with lower spacing errors have much lower dynamic load than do other sample gears with larger spacing error values, especially for the Full-sine wave of spacing error. Meanwhile, the sample gears with half-sine wave of spacing error have much lower dynamic load factors than that of the sample gears with full-sine wave. Comparing these figures, the sample gears with half-sine wave of spacing error appear to perform better in producing lower dynamic load factors. In this group, for the speed range studied, the sample gears with spacing error at 0.0001 in would create a lower dynamic load around 5000 rpm.

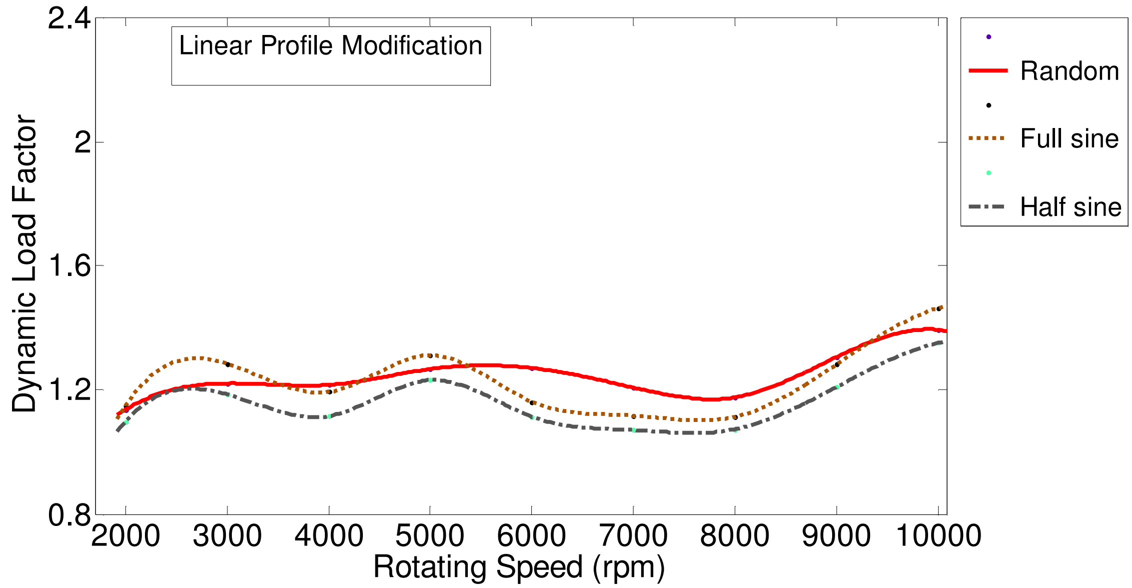

Figure 3.34 Dynamic load factors of sample gears with random, full-sine and half-sine waves of spacing error at 0.0001 in and linear profile modification.

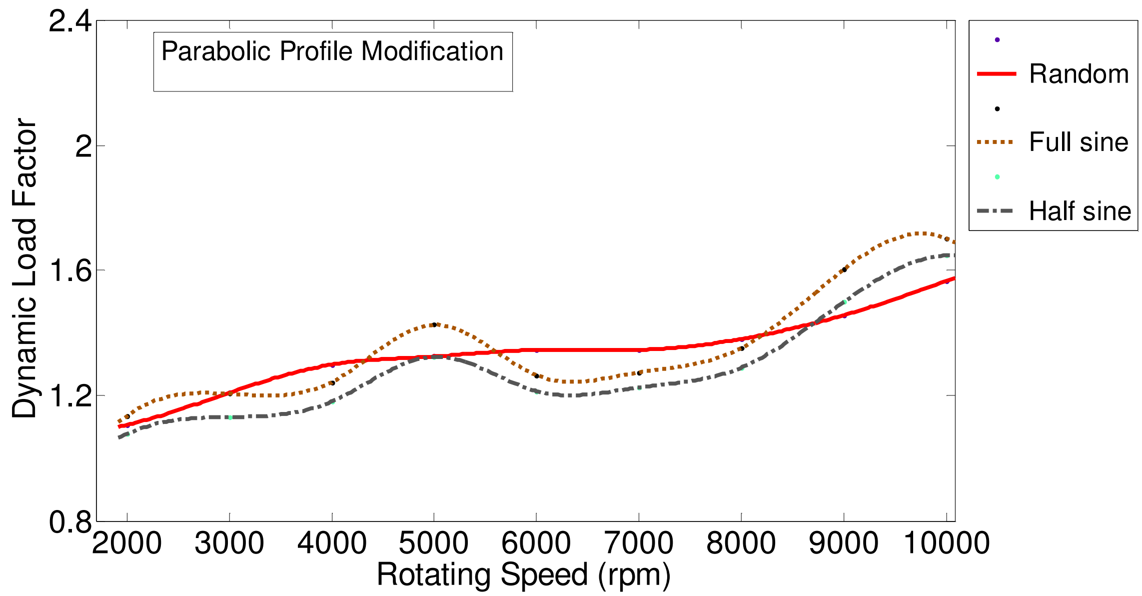

Figure 3.35 Dynamic load factors of sample gears with random, full-sine and half-sine waves of spacing error at 0.0001 in and parabolic profile modification.

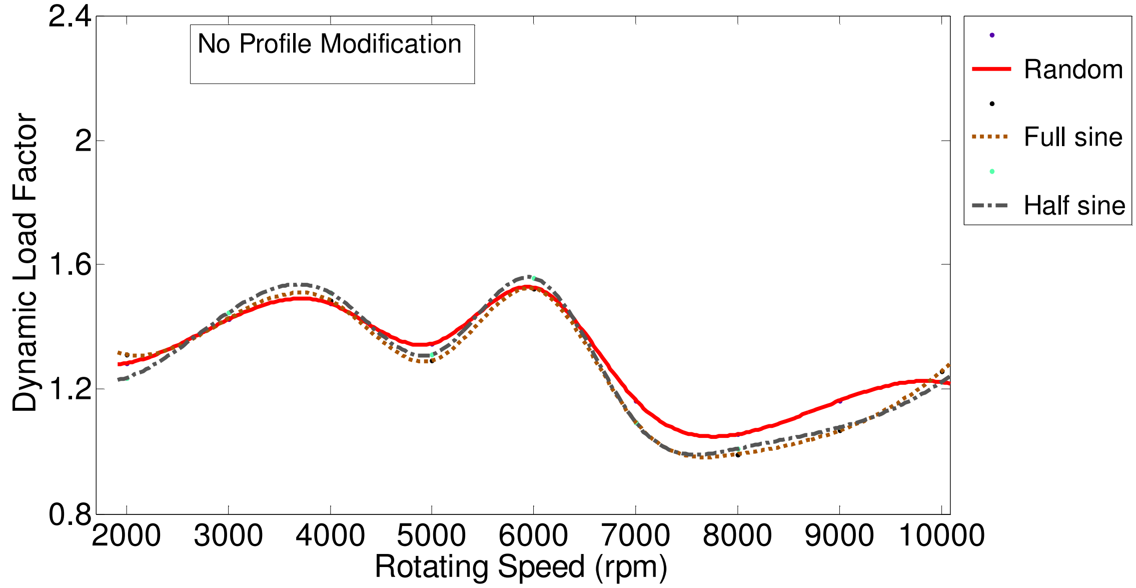

Figure 3.36 Dynamic load factors of sample gears with random, full-sine and half-sine waves of spacing error at 0.0001 in and no profile modification.

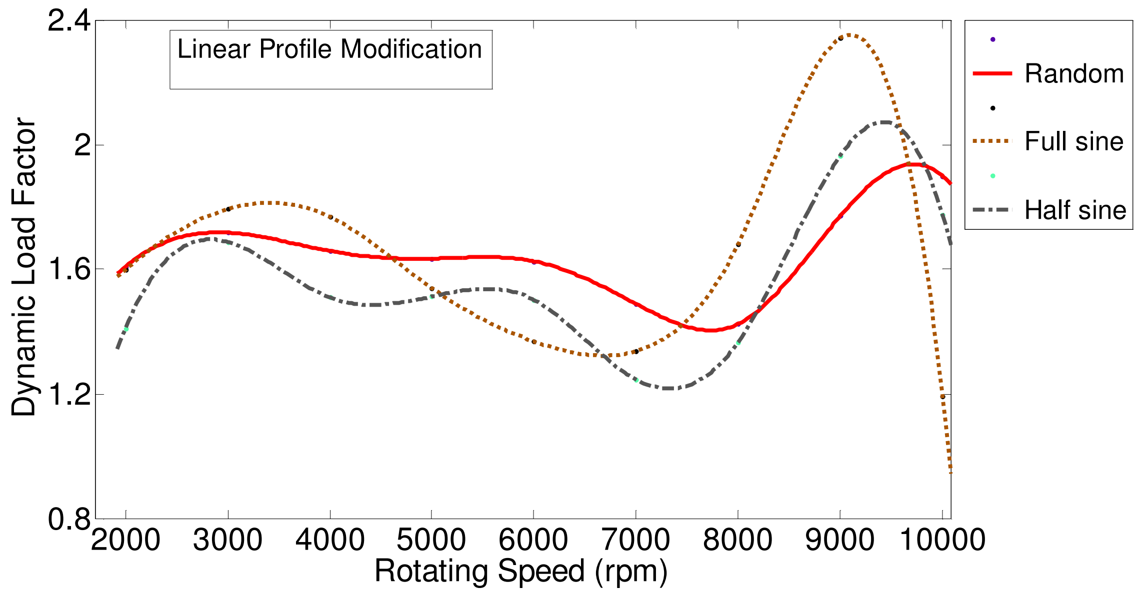

Figure 3.37 Dynamic load factors of sample gears with random, full-sine and half-sine waves of spacing error at 0.0003 in and linear profile modification.

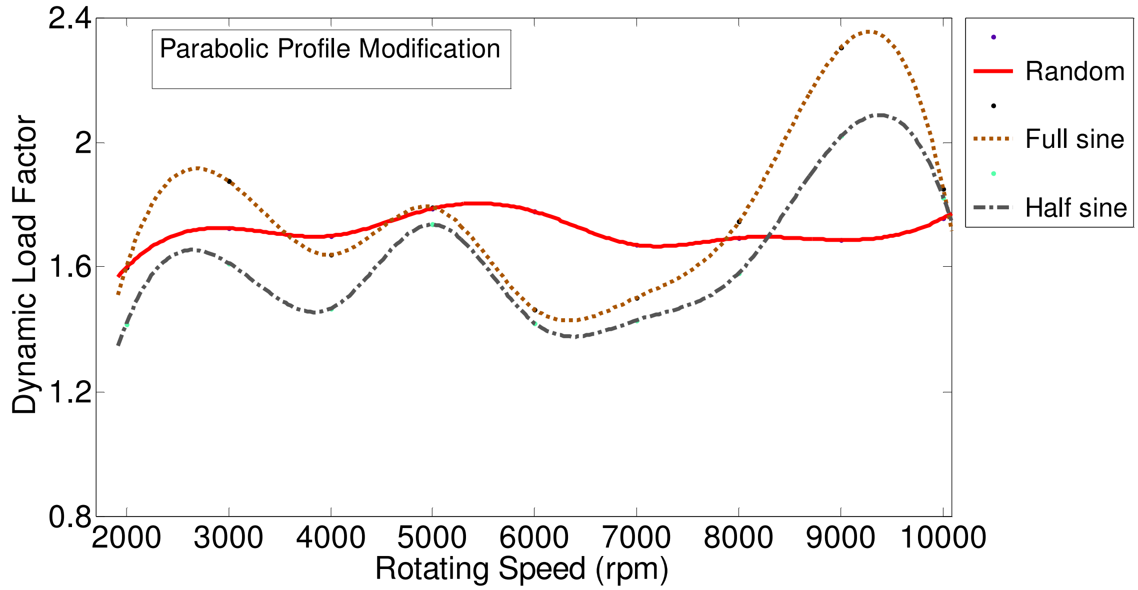

Figure 3.38 Dynamic load factors of sample gears with random, full-sine and half-sine waves of spacing error at 0.0003 in and parabolic profile modification.

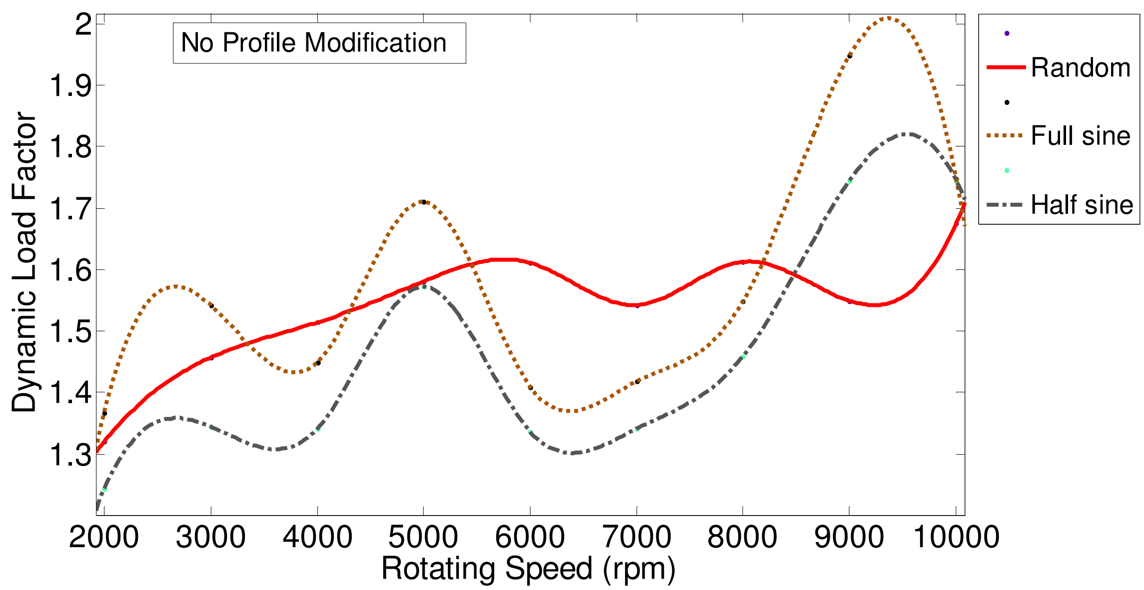

Figure 3.39 Dynamic load factors of sample gears with random, full-sine and half-sine waves of spacing error at 0.0003 in and no profile modification.

3.5.3 Influence of magnitude of tooth spacing error in speed survey

Figure 3.40 to Figure 3.48 display first that Half-sine tooth spacing error create lower dynamic load factor, for the different types of modification. Second, for non-modified ones, for a certain speed range near 6000 rpm, some tooth spacing error even can create higher dynamic load factor, which means the relationship between dynamic load factor and operating speed is irregular. For both linear and parabolic modifications, high spacing error create high dynamic load factor. Third, the linear modification creates lower dynamic load factor.

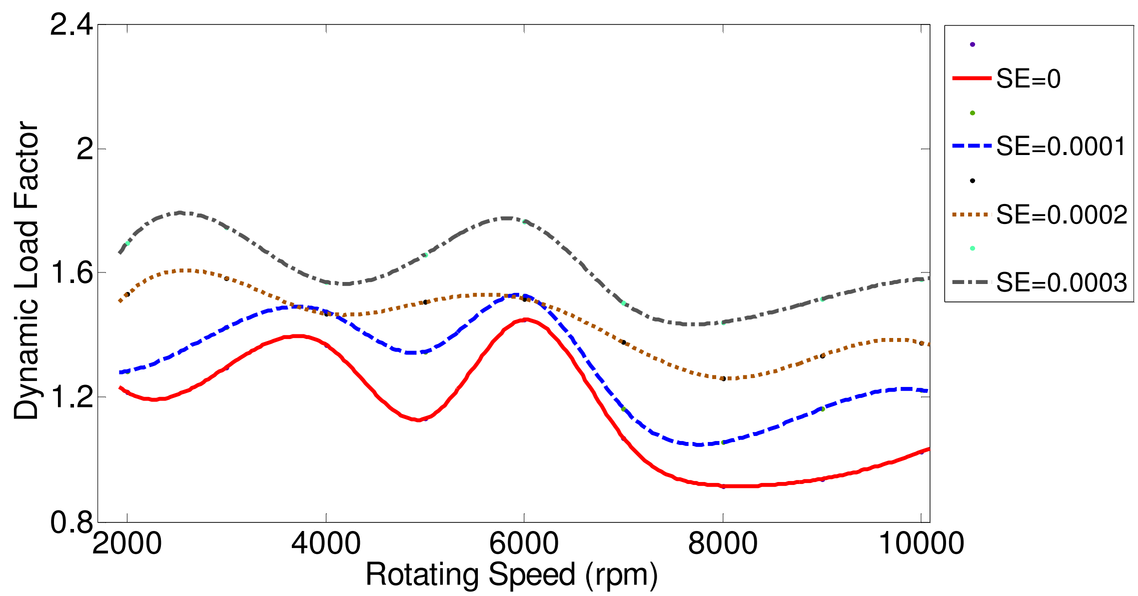

Figure 3.40 Non-modified tooth profile with maximum cumulative random type tooth spacing error from 0 to 0.0003 in.

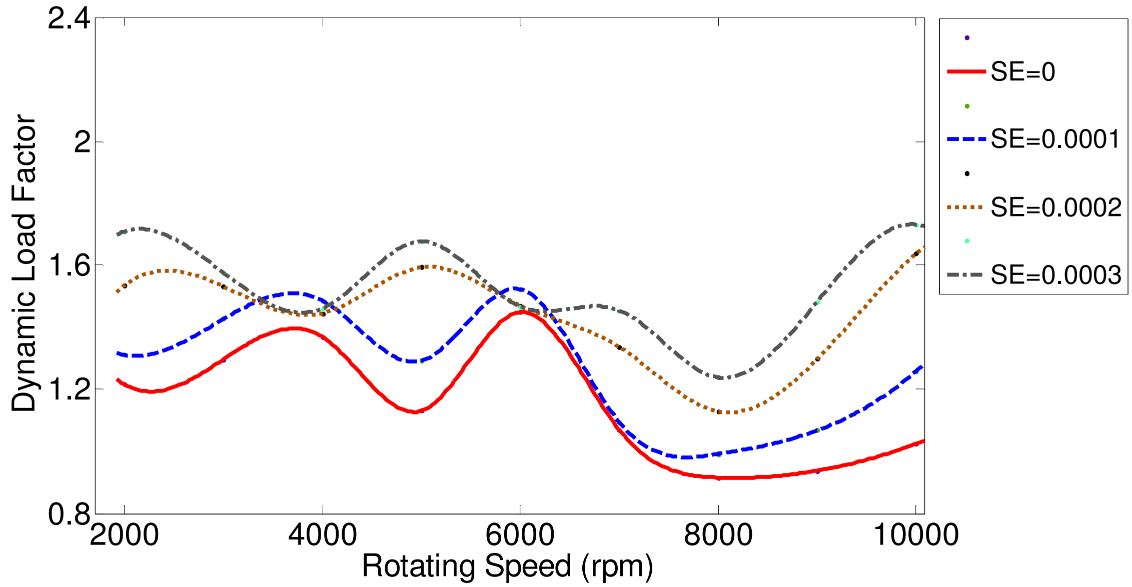

Figure 3.41 Non-modified tooth profile with maximum cumulative full-sine type tooth spacing error from 0 to 0.0003 in.

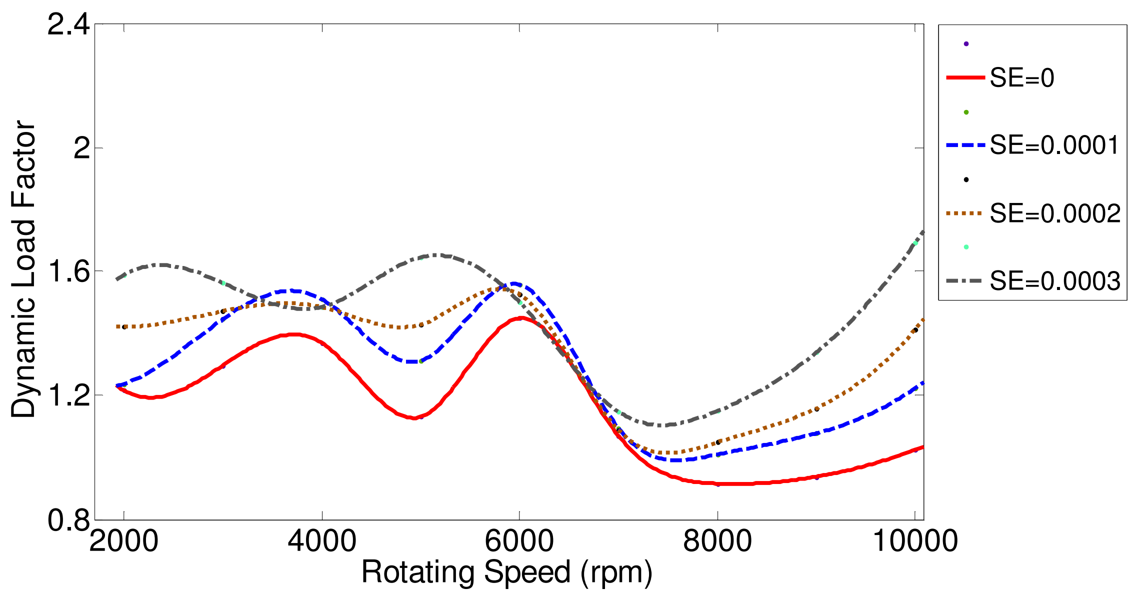

Figure 3.42 Non-modified tooth profile with maximum cumulative half-sine type tooth spacing error from 0 to 0.0003 in.

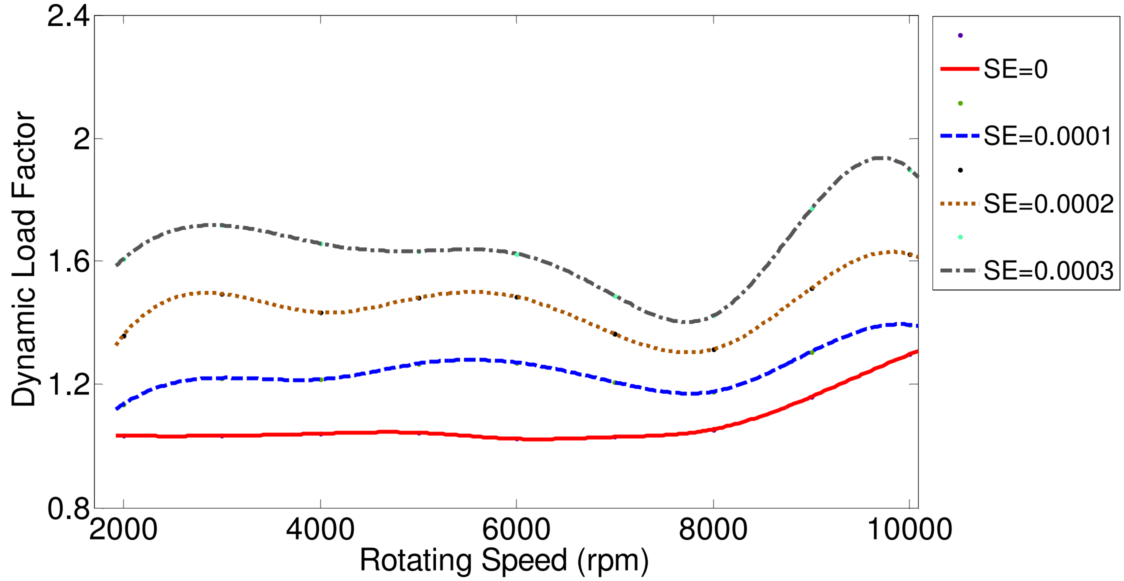

Figure 3.43 Effect of full amount and length of linear tooth profile modification with maximum cumulative random type tooth spacing error from 0 to 0.0003 in.

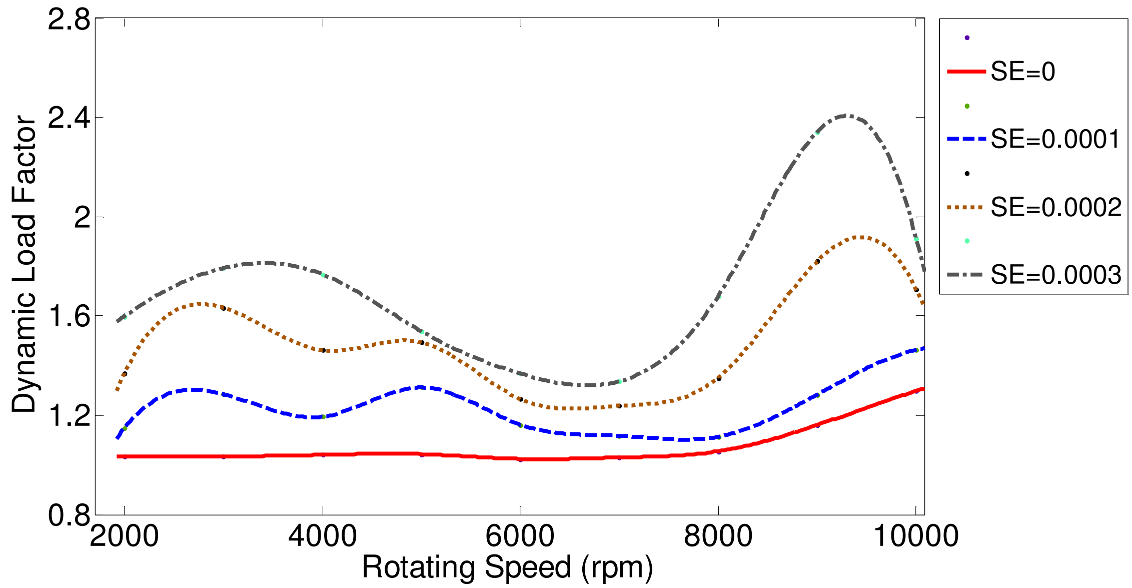

Figure 3.44 Effect of full amount and length of linear tooth profile modification with maximum cumulative full-sine type tooth spacing error from 0 to 0.0003 in.

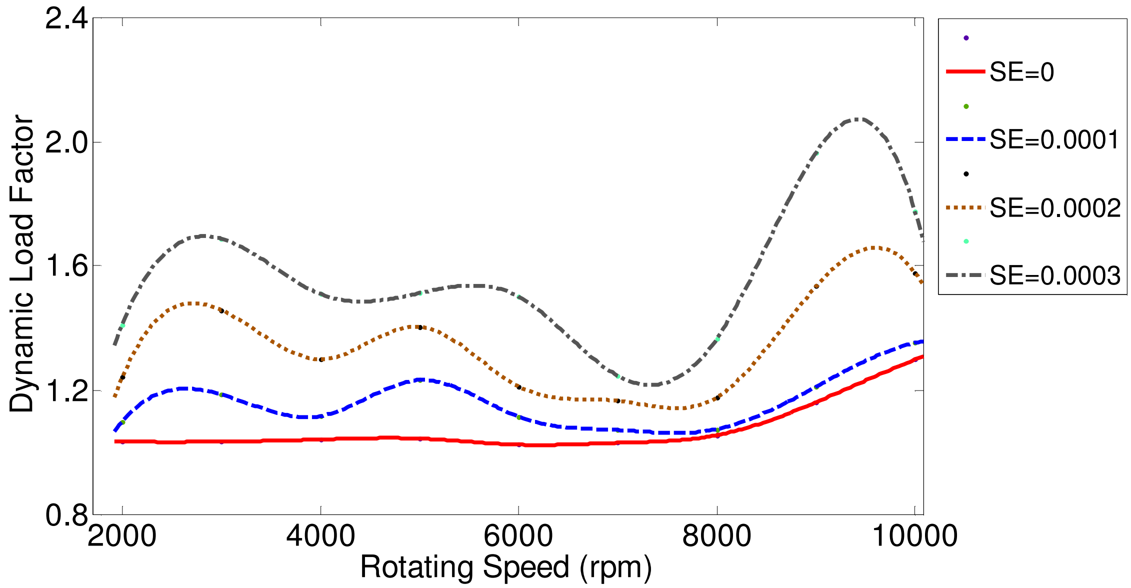

Figure 3.45 Effect of full amount and length of linear tooth profile modification with maximum cumulative half-sine type tooth spacing error from 0 to 0.0003 in.

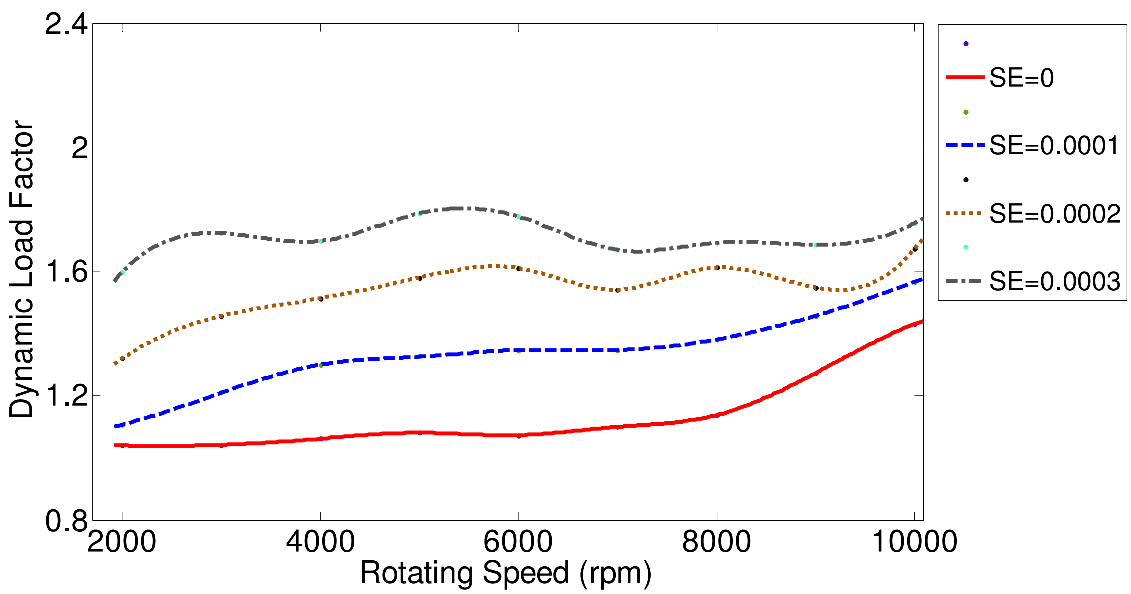

Figure 3.46 Effect of full amount and length of parabolic tooth profile modification with maximum cumulative random type tooth spacing error from 0 to 0.0003 in.

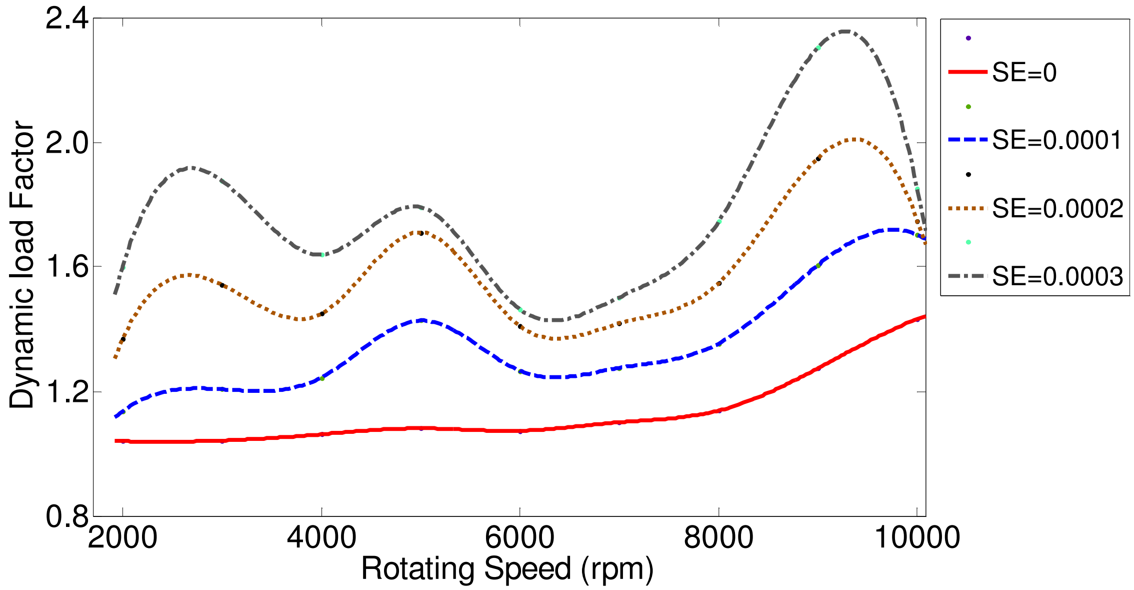

Figure 3.47 Effect of full amount and length of parabolic tooth profile modification with maximum cumulative full-sine type tooth spacing error from 0 to 0.0003 in.

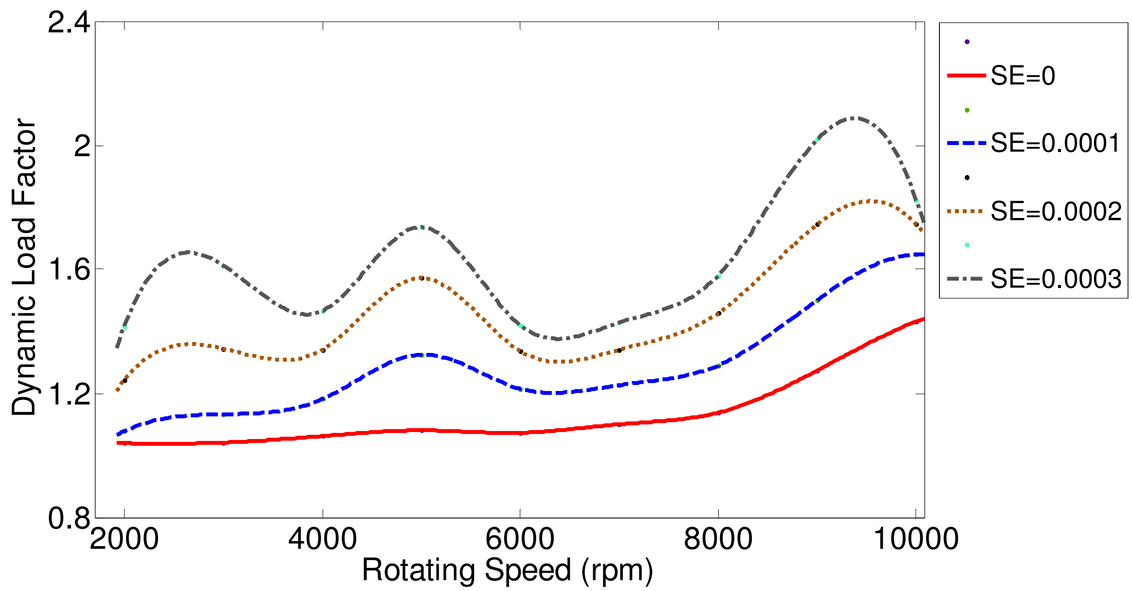

Figure 3.48 Effect of full amount and length of parabolic tooth profile modification with maximum cumulative half-sine type tooth spacing error from 0 to 0.0003 in.

3.5.4 Influence of length of tooth profile modification in speed survey

Figures 3.49 to 3.60 show the influence of profile modification length on gear dynamics in a speed survey. The modification amount Δ is full amount. When the modification length equal to 1.00, the dynamic load factor appears to be lower. For parabolic modification with a length of 65% with a Full-sine spacing error at 0.0001in, the dynamic load factors are the lowest when the operating speed reaches 8000 rpm. That means for Full-sine type spacing error, parabolic modification with shorter length can create a lower dynamic load factor. Normally, when the operating speed reaches the range of 6000 rpm to 7000 rpm, the dynamic load factor is the smallest for both types of modification.

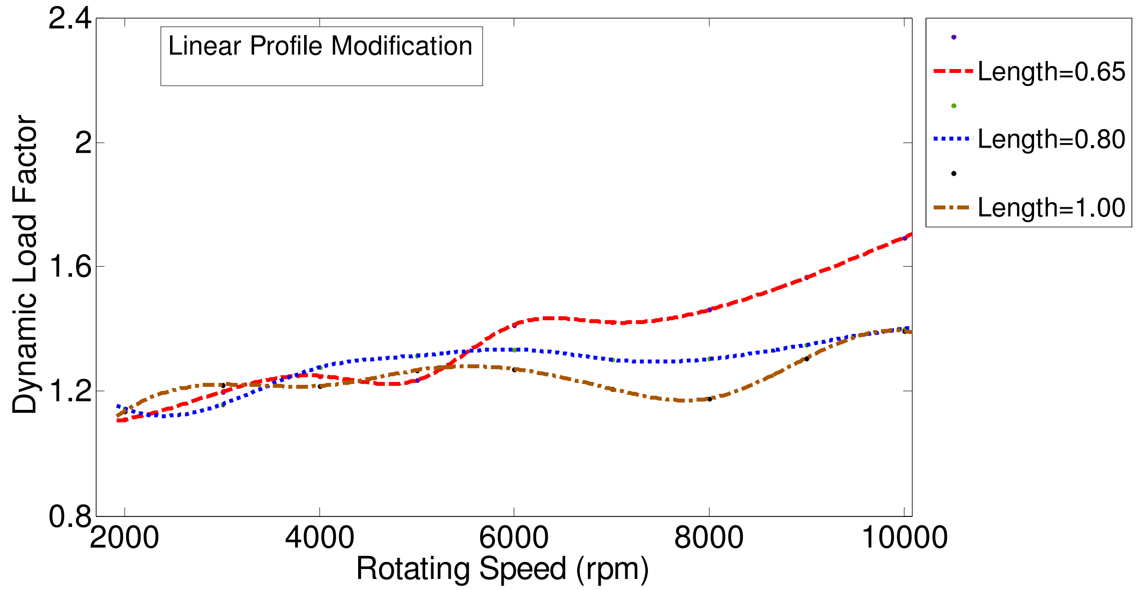

Figure 3.49 Effect of profile modification length on gear dynamic load. Random spacing error with linear profile modification with maximum cumulative tooth spacing error at 0.0001 in (Δ=100%).

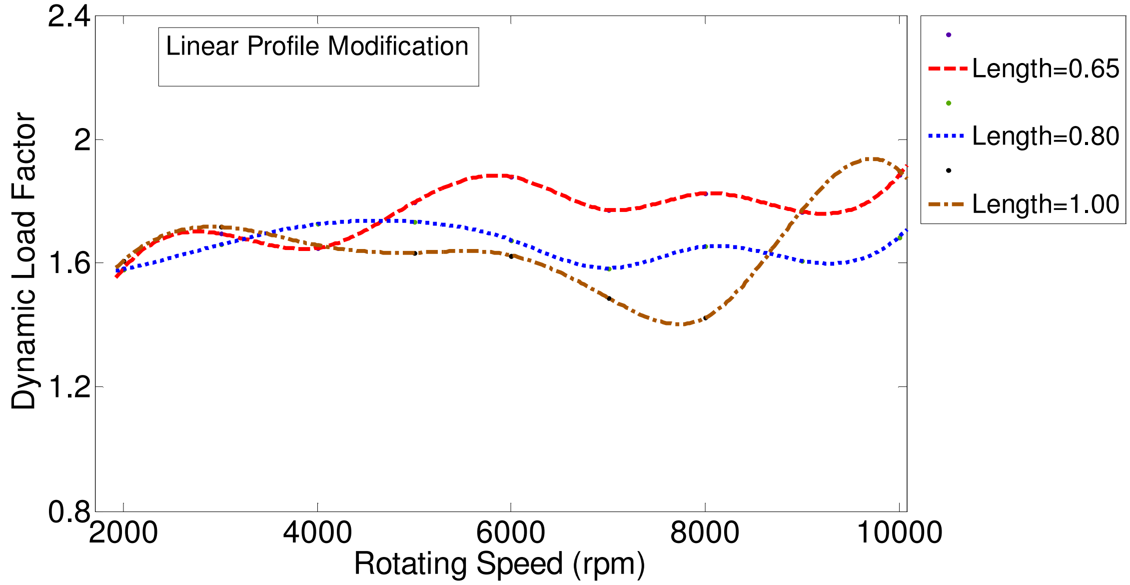

Figure 3.50 Effect of profile modification length on gear dynamic load. Random spacing error with linear profile modification with maximum cumulative tooth spacing error at 0.0003 in (Δ=100%).

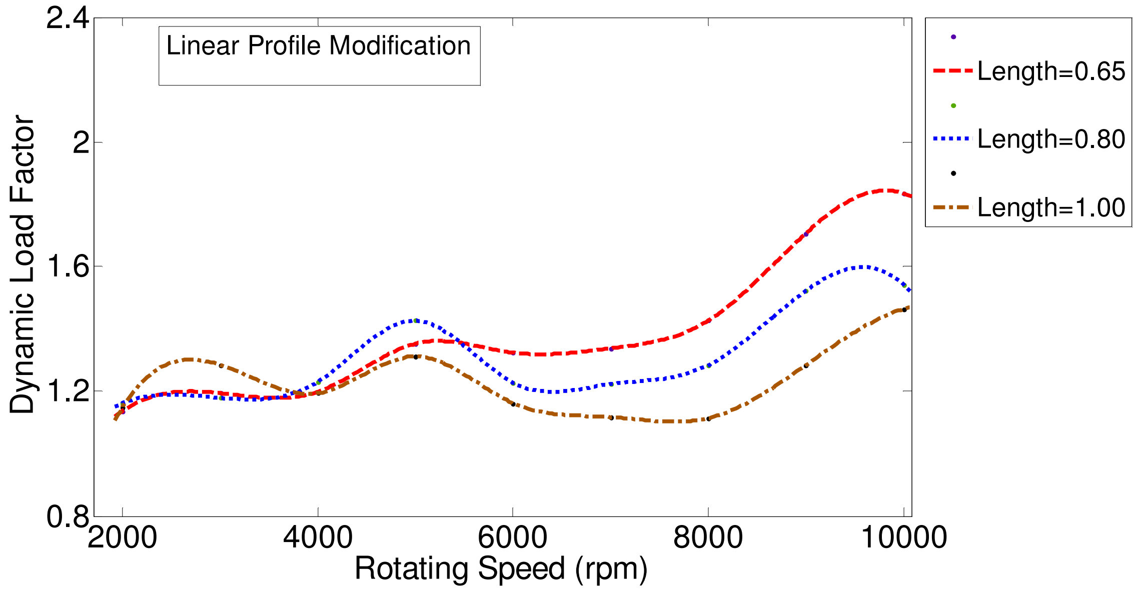

Figure 3.51 Effect of profile modification length on gear dynamic load. Full-sine spacing error with linear profile modification, and maximum cumulative tooth spacing error at 0.0001 in (Δ=100%).

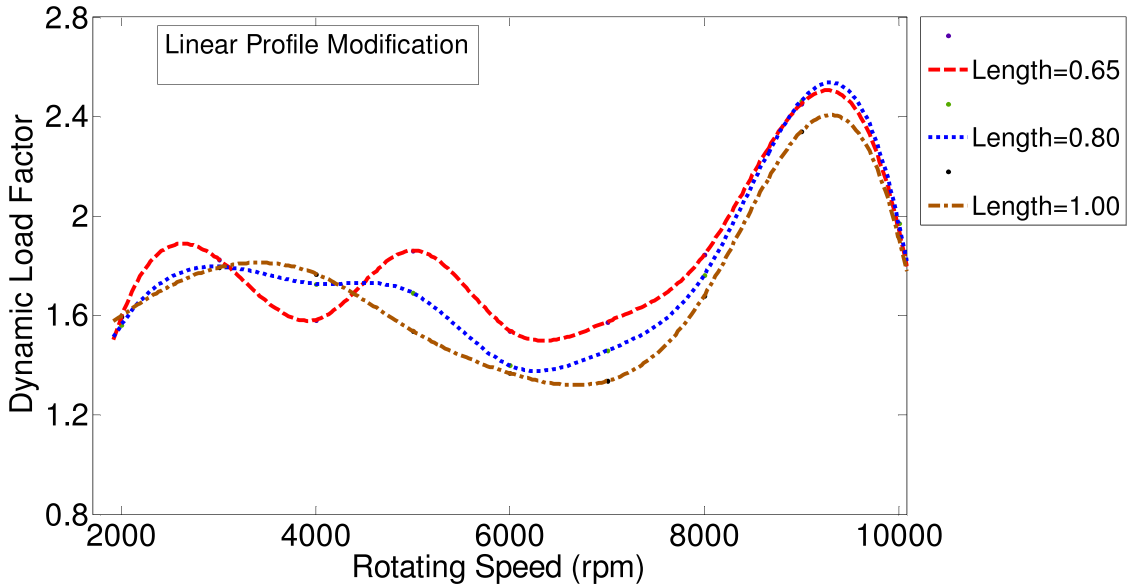

Figure 3.52 Effect of profile modification length on gear dynamic load. Full-sine spacing error with linear profile modification, and maximum cumulative tooth spacing error at 0.0003 in (Δ=100%).

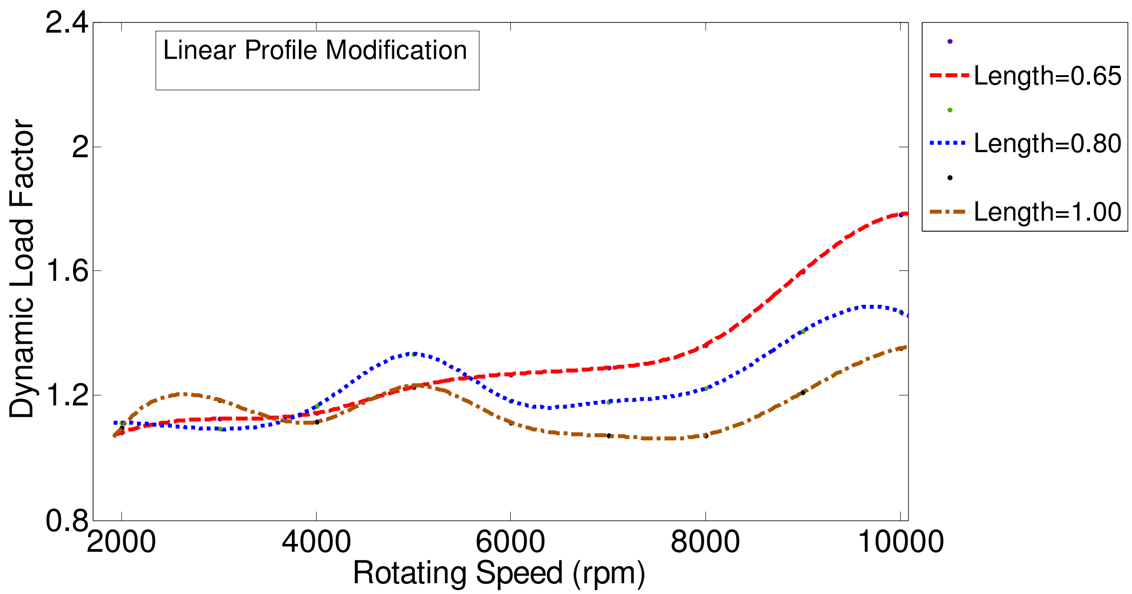

Figure 3.53 Effect of profile modification length on gear dynamic load. Half-sine spacing error with linear profile modification, and maximum cumulative tooth spacing error at 0.0001 in (Δ=100%).

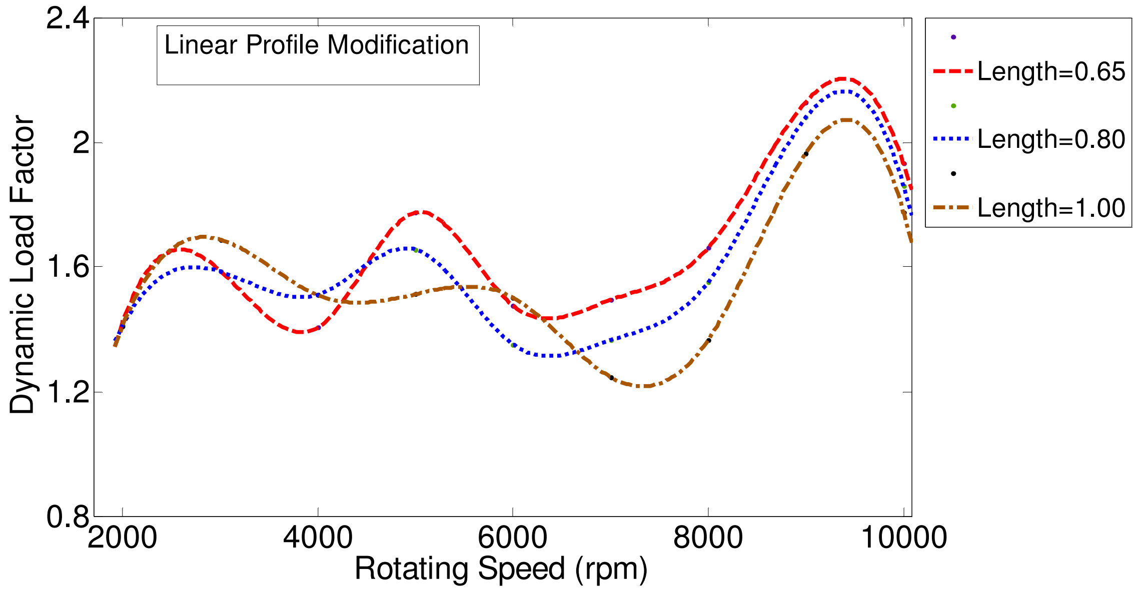

Figure 3.54 Effect of profile modification length on gear dynamic load. Half-sine spacing error with linear profile modification, and maximum cumulative tooth spacing error at 0.0003 in (Δ=100%).

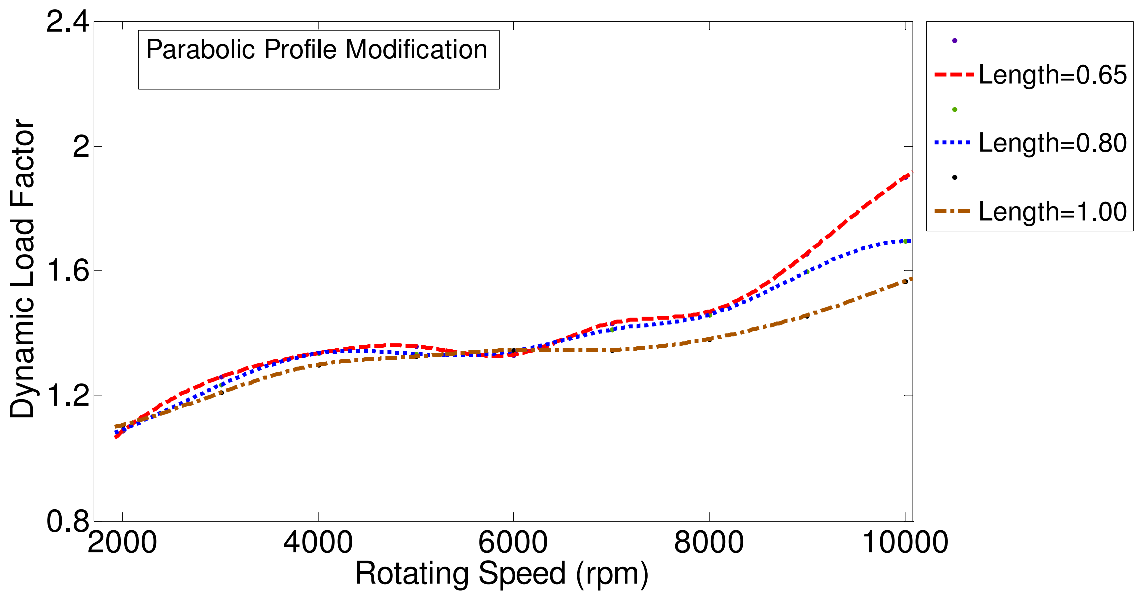

Figure 3.55 Effect of profile modification length on gear dynamic load. Random spacing error with parabolic profile modification, and maximum cumulative tooth spacing error at 0.0001 in (Δ=100%).

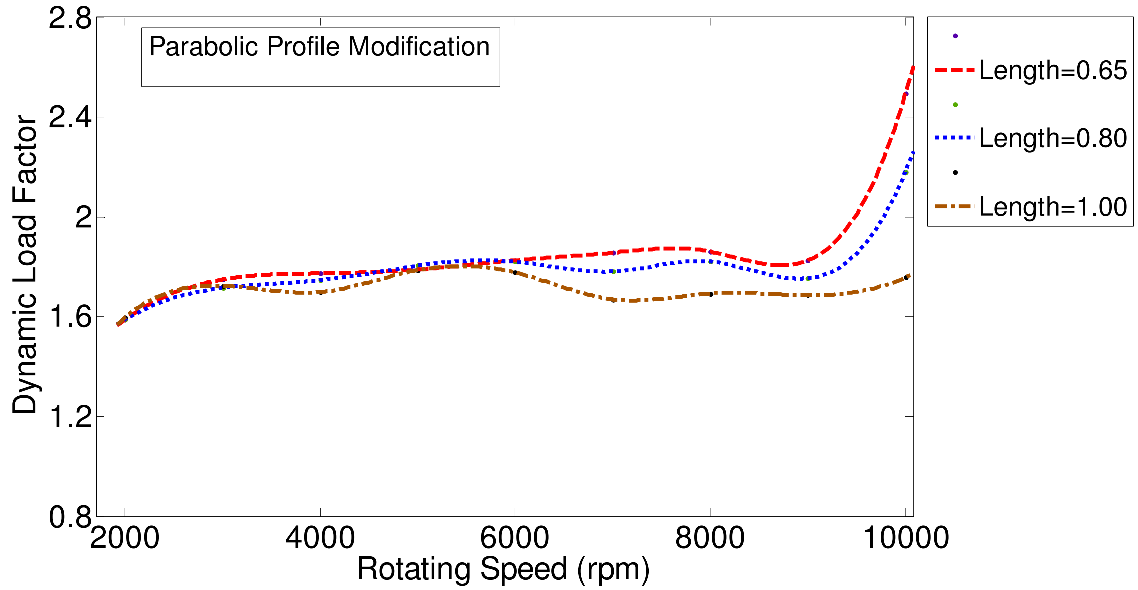

Figure 3.56 Effect of profile modification length on gear dynamic load. Random spacing error with parabolic profile modification, and maximum cumulative tooth spacing error at 0.0003 in (Δ=100%).

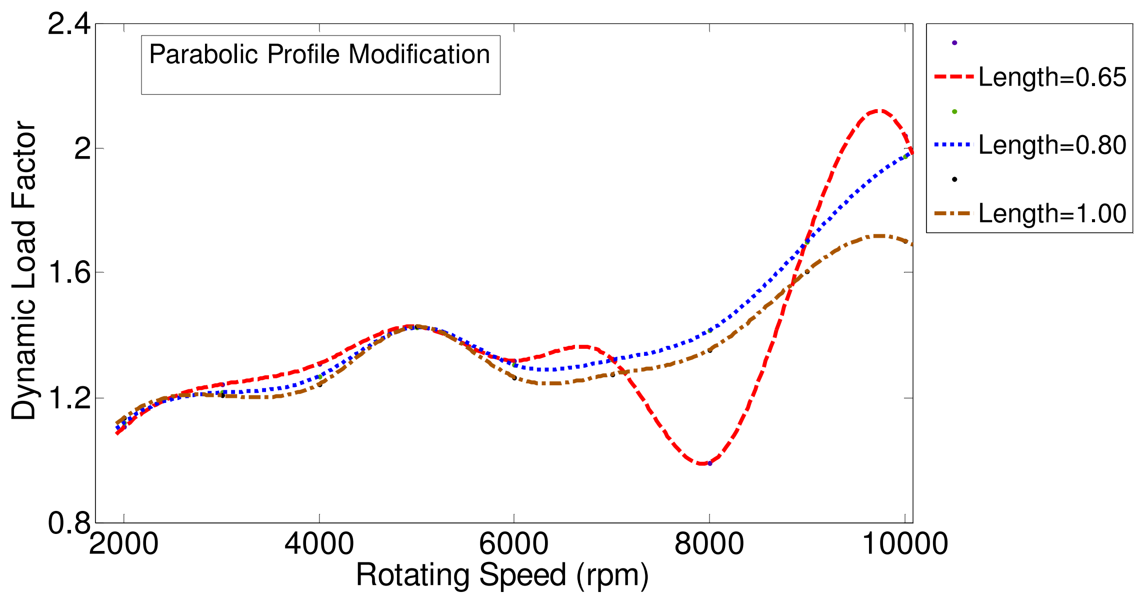

Figure 3.57 Effect of profile modification length on gear dynamic load. Full-sine spacing error with parabolic profile modification, and maximum cumulative tooth spacing error at 0.0001 in (Δ=100%).

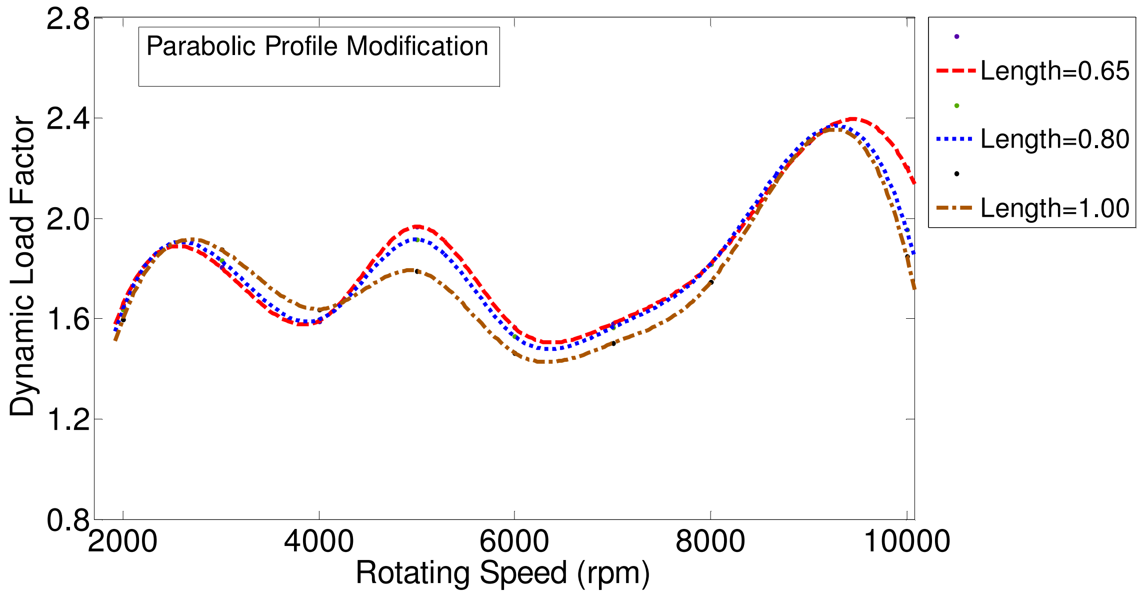

Figure 3.58 Effect of profile modification length on gear dynamic load. Full-sine spacing error with parabolic profile modification, and maximum cumulative tooth spacing error at 0.0003 in (Δ=100%).

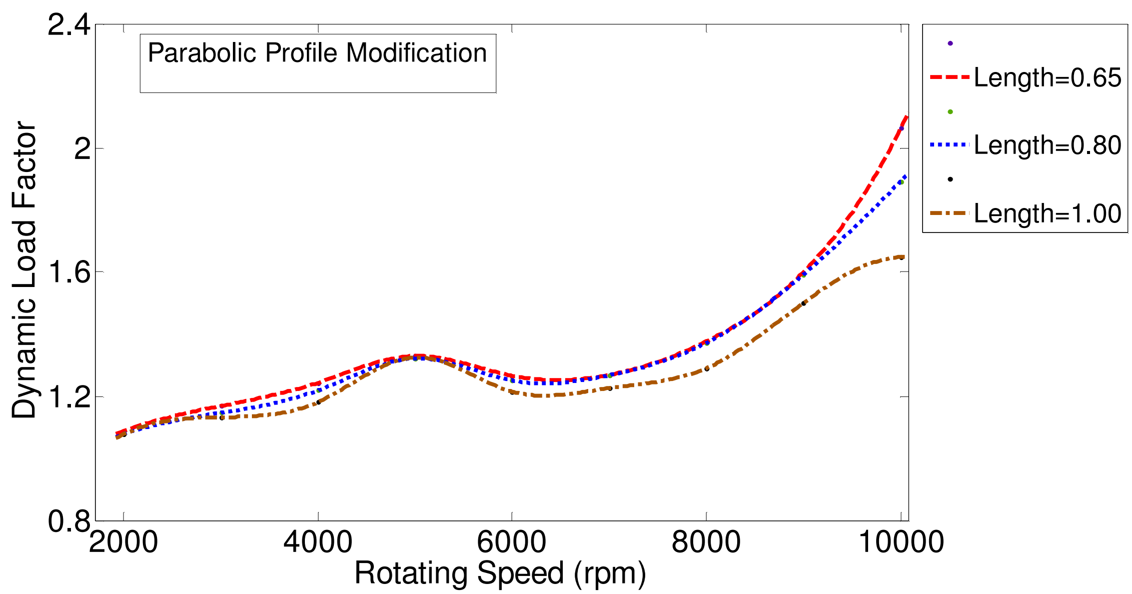

Figure 3.59 Effect of profile modification length on gear dynamic load. Half-sine spacing error with parabolic profile modification, and maximum cumulative tooth spacing error at 0.0001 in (Δ=100%).

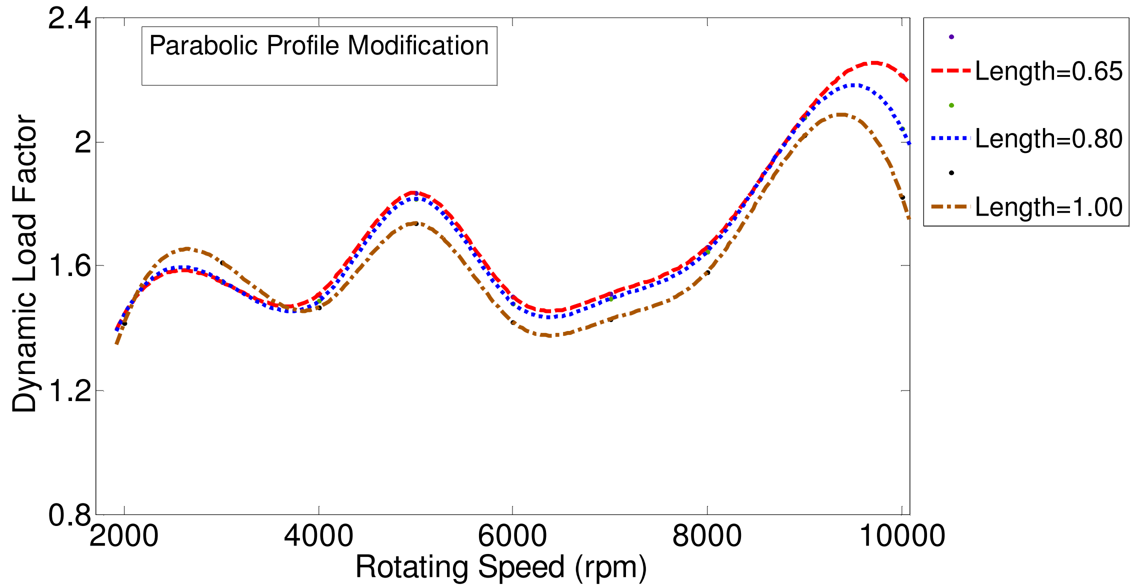

Figure 3.60 Effect of profile modification length on gear dynamic load. Half-sine spacing error with parabolic profile modification, and maximum cumulative tooth spacing error at 0.0003 in (Δ=100%).

3.5.5 **Influence of amount of profile modification in speed survey**

Figures 3.61 to 3.78 show the influence of profile modification amount on the dynamics of gears in speed surveys. This group displays a comparison of dynamic load factor subjected to different amounts of profile modification with different values of spacing error. For full-sine and half-sine type of spacing error, the dynamic results are intuitive and regular. Generally, lower amount of profile modification would create lower dynamic load factor at operating speeds of 6000 rpm and above. Higher amount of profile modification would create lower dynamic load at the speed of 6000 rpm and below. Because the random type of spacing error is irregular, so its dynamic curves are more sensitive to the amount of profile modification and their variations are also irregular. Its dynamic load factor is lower than that of full-sine and half-sine types of spacing error in certain speed range. For all type of spacing error, the trend of reduced dynamic response near 7000 rpm becomes very significant when the value of maximum spacing error increases.

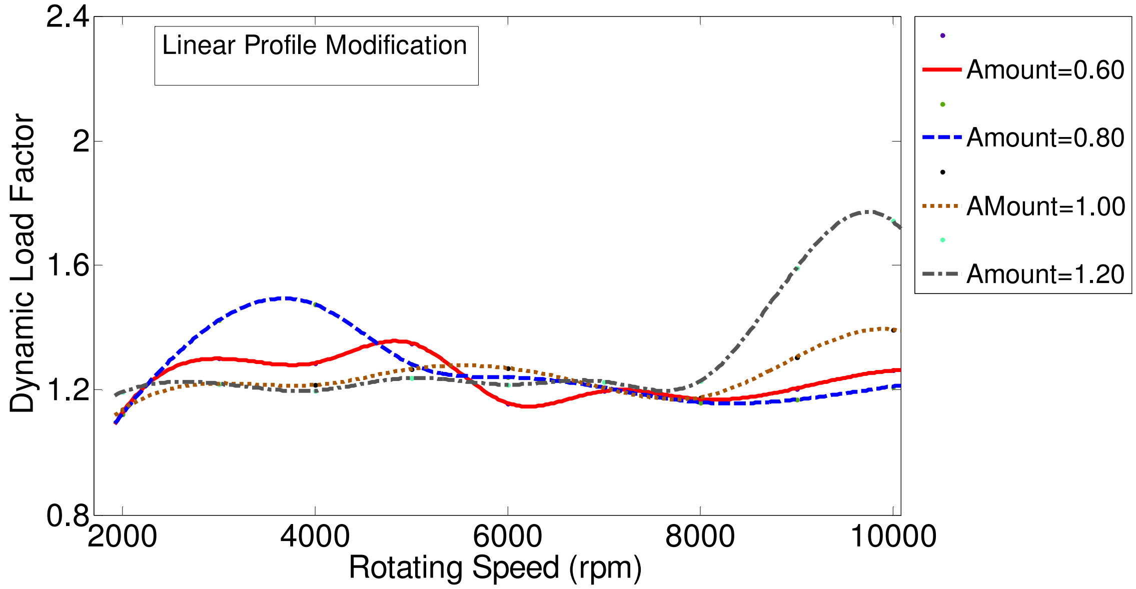

Figure 3.61 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative random type tooth spacing error of 0.0001 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

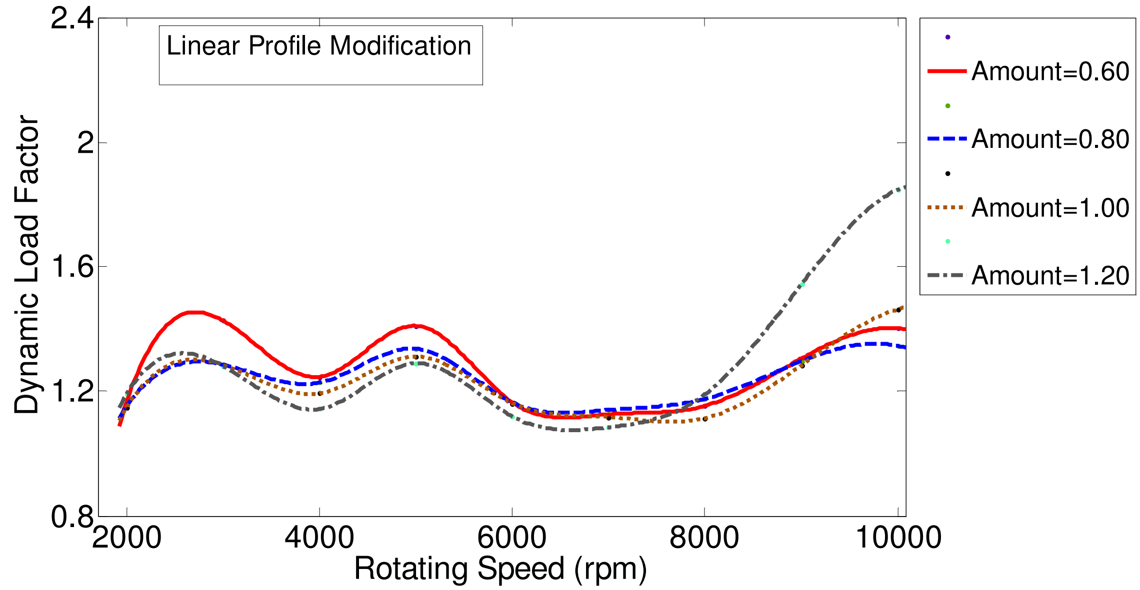

Figure 3.62 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative full-sine type tooth spacing error of 0.0001 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

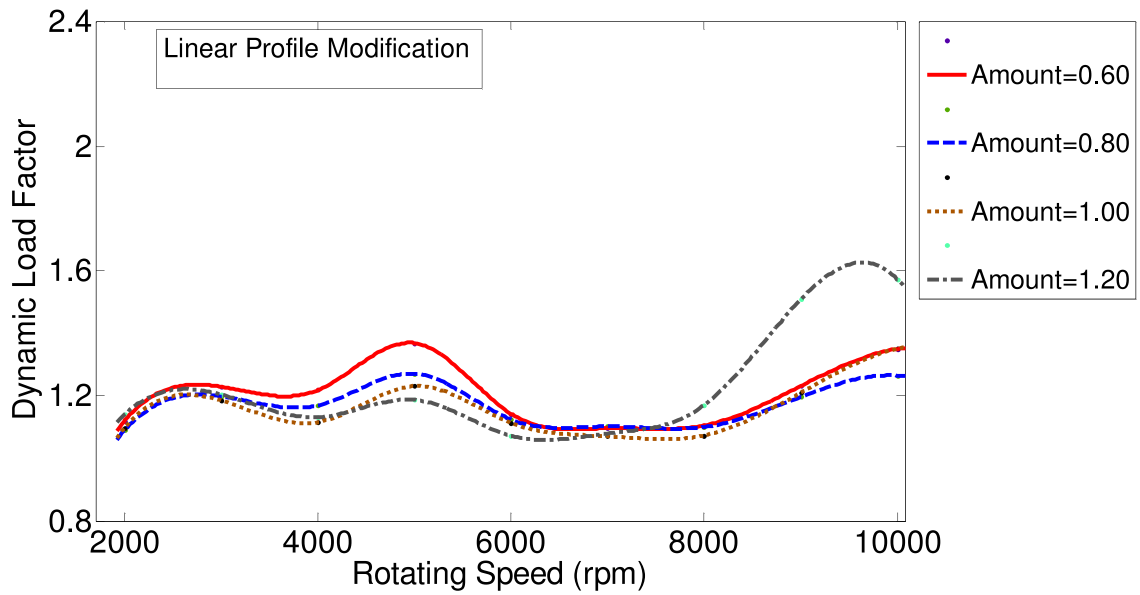

Figure 3.63 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative half-sine type tooth spacing error of 0.0001 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

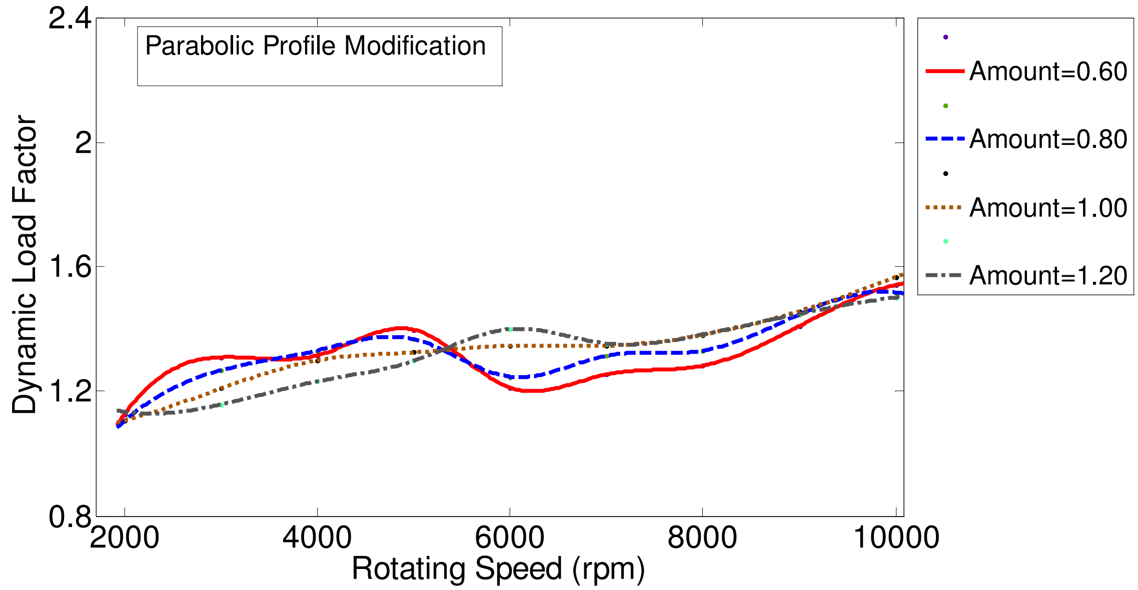

Figure 3.64 Effect of profile modification amount on gear dynamics, for parabolic tooth profile modification with maximum cumulative random type tooth spacing error of 0.0001 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

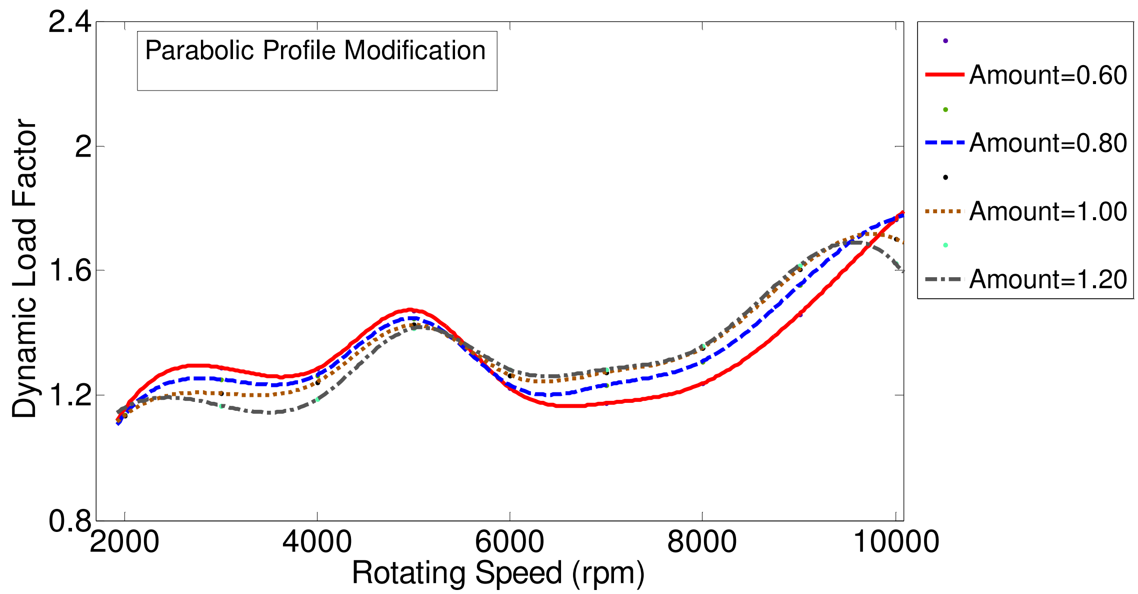

Figure 3.65 Effect of profile modification amount on gear dynamics, for parabolic tooth profile modification with maximum cumulative full-sine type tooth spacing error of 0.0001 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

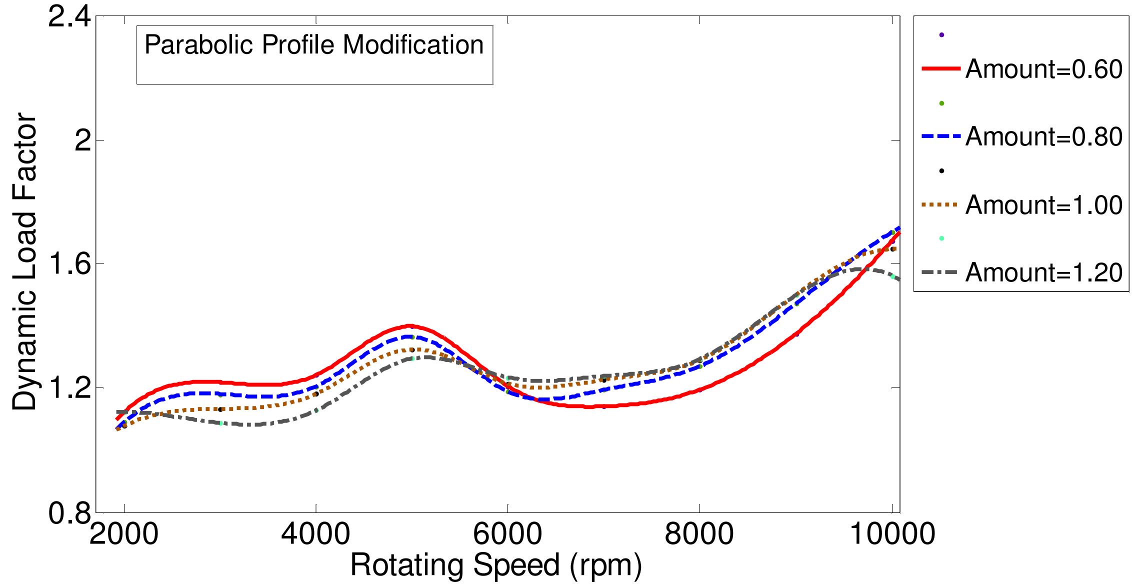

Figure 3.66 Effect of profile modification amount on gear dynamics, for parabolic tooth profile modification with maximum cumulative half-sine type tooth spacing error of 0.0001 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

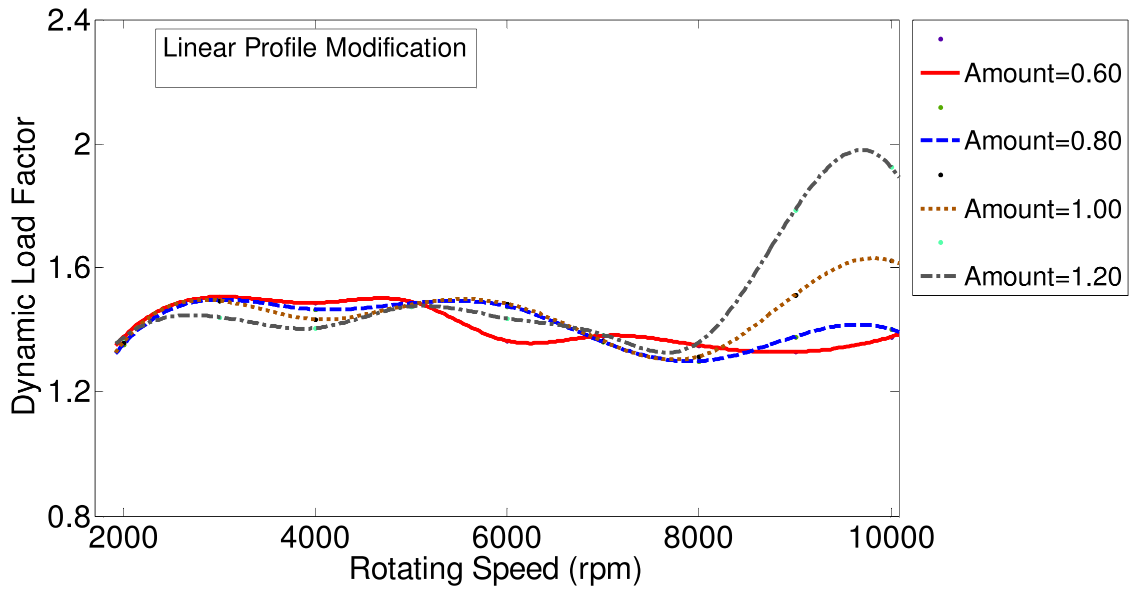

Figure 3.67 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative random type tooth spacing error of 0.0002 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

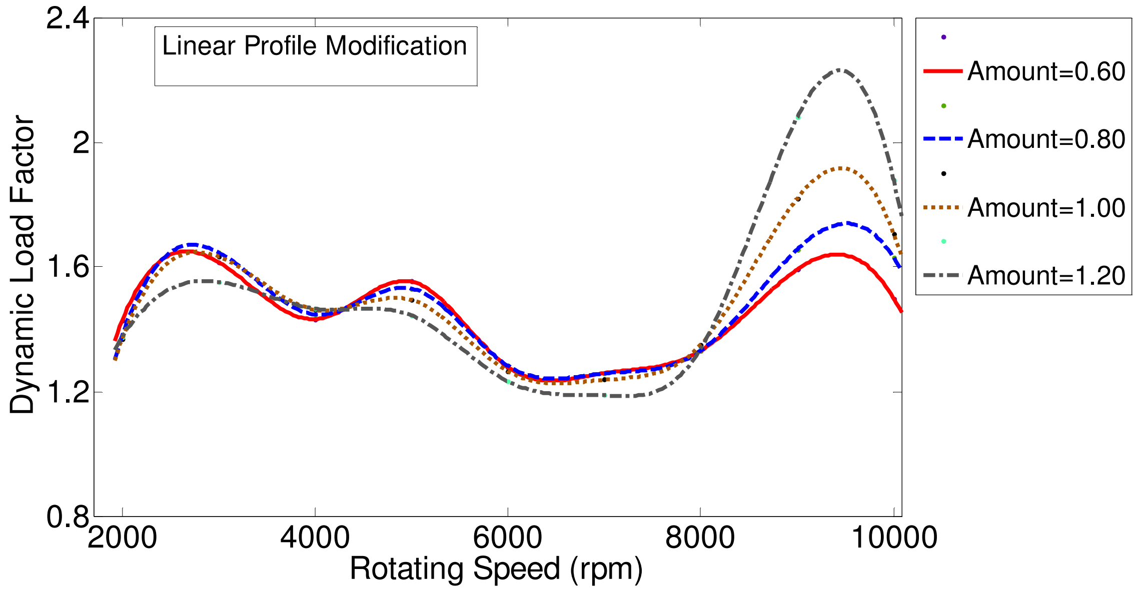

Figure 3.68 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative full-sine type tooth spacing error of 0.0002 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

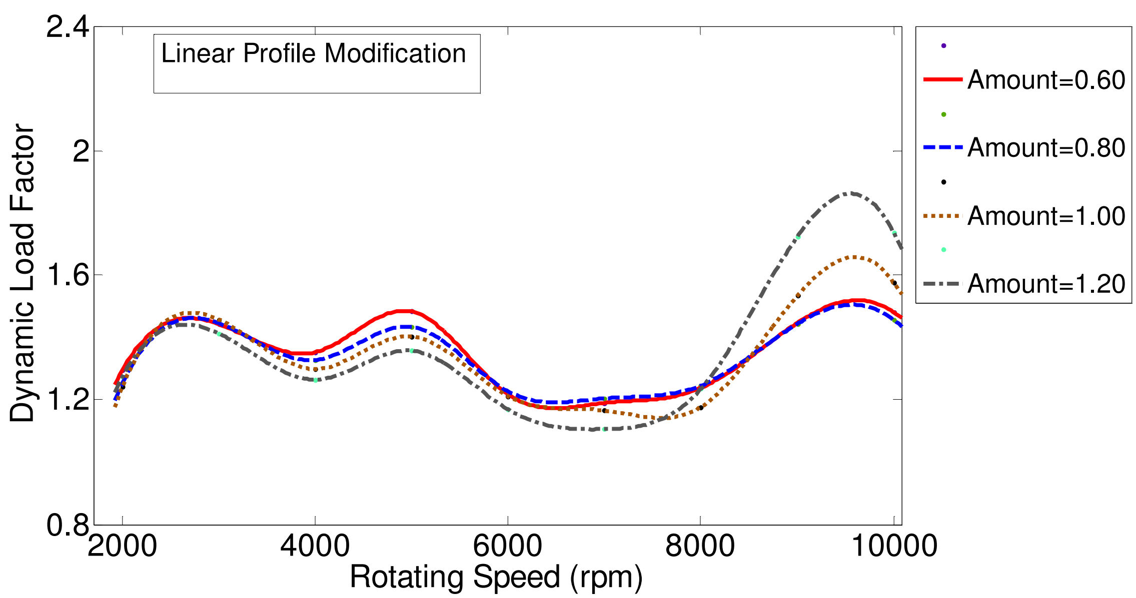

Figure 3.69 Effect of profile modification amount on the gear dynamics, for linear tooth profile modification with maximum cumulative half-sine type tooth spacing error of 0.0002 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

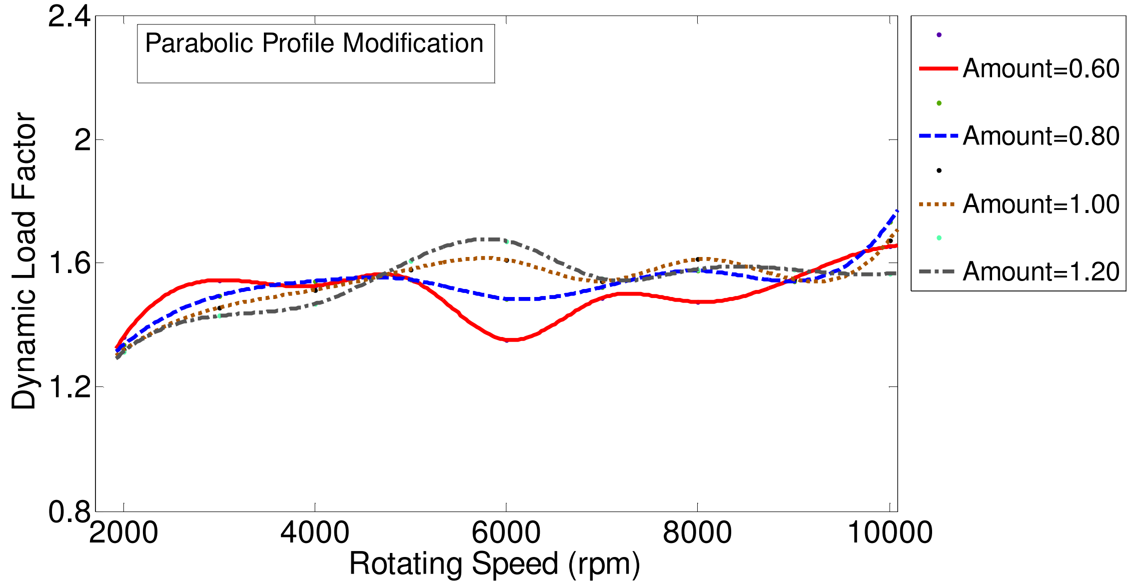

Figure 3.70 Effect of profile modification amount on the gear dynamics, for parabolic tooth profile modification with maximum cumulative random type tooth spacing error of 0.0002 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

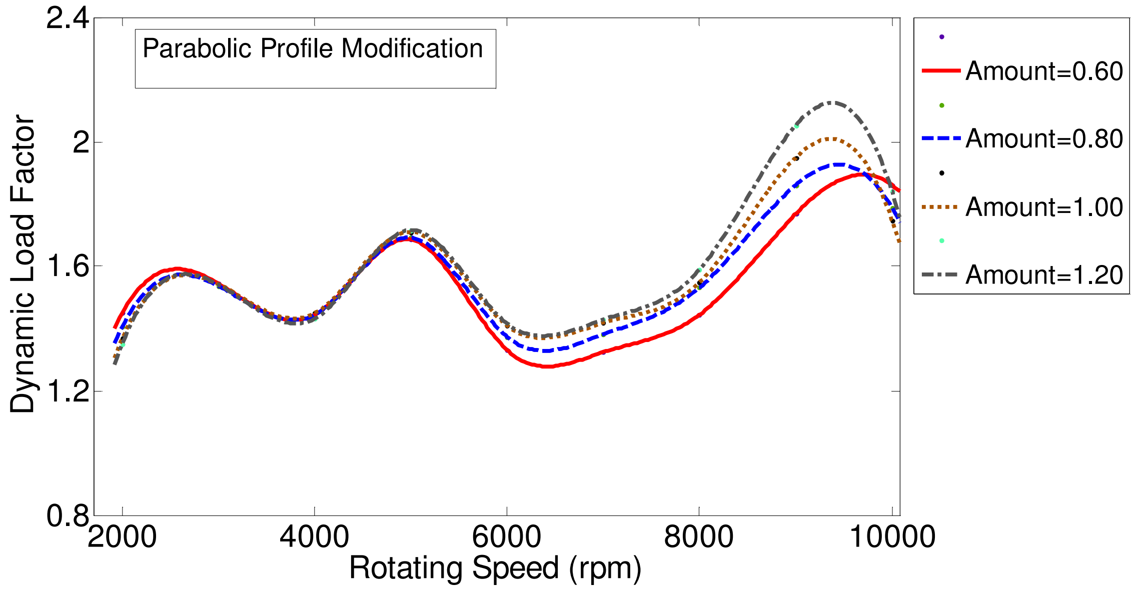

Figure 3.71 Effect of profile modification amount on gear dynamics, for parabolic tooth profile modification with maximum cumulative full-sine type tooth spacing error of 0.0002 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

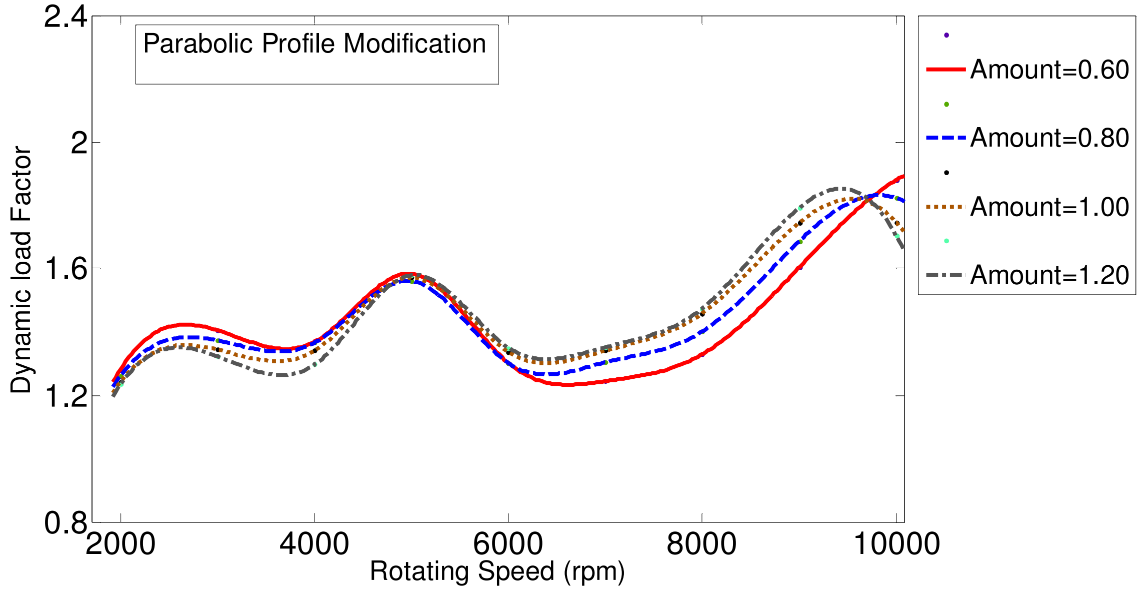

Figure 3.72 Effect of profile modification amount on the gear dynamics, for parabolic tooth profile modification with maximum cumulative half-sine type tooth spacing error of 0.0002 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

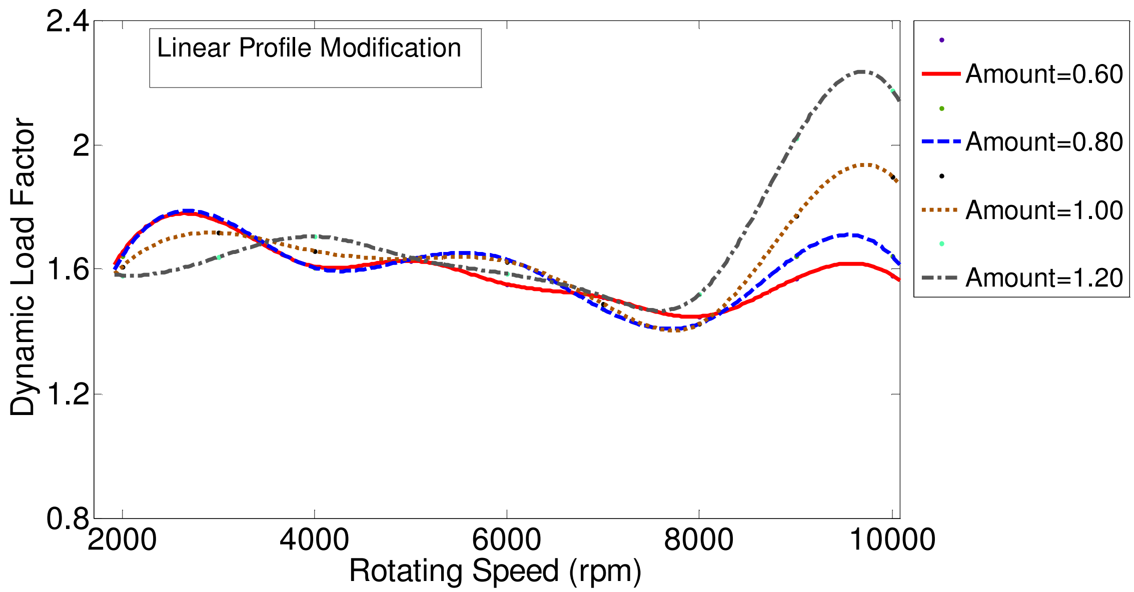

Figure 3.73 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative random type tooth spacing error of 0.0003 in. The modification varies from 0.60 to 1.20, and the length is 100%.

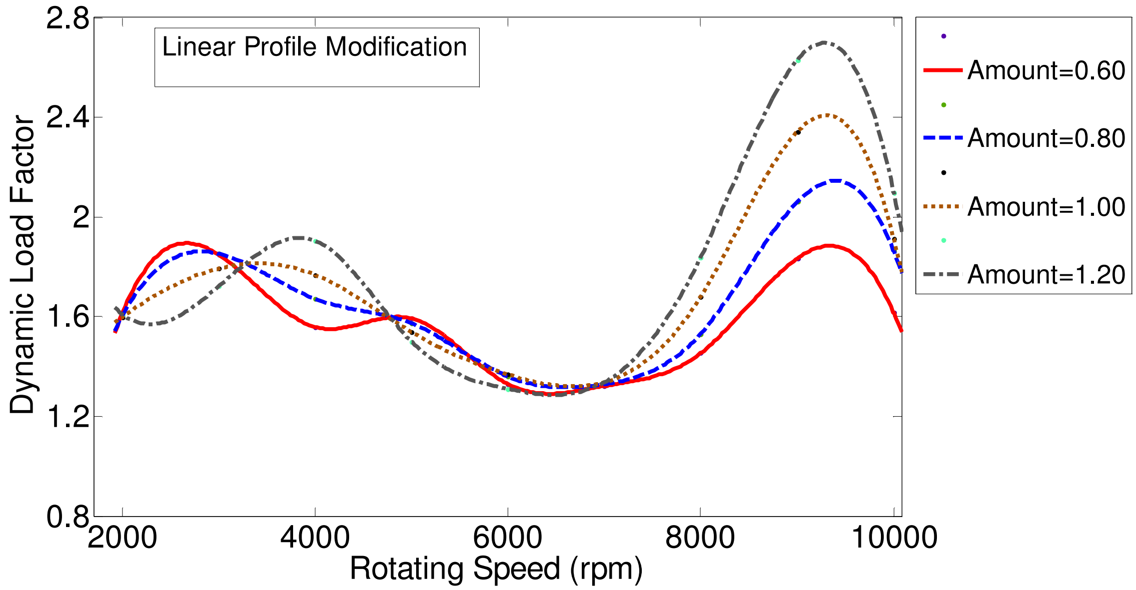

Figure 3.74 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative full-sine type tooth spacing error of 0.0003 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

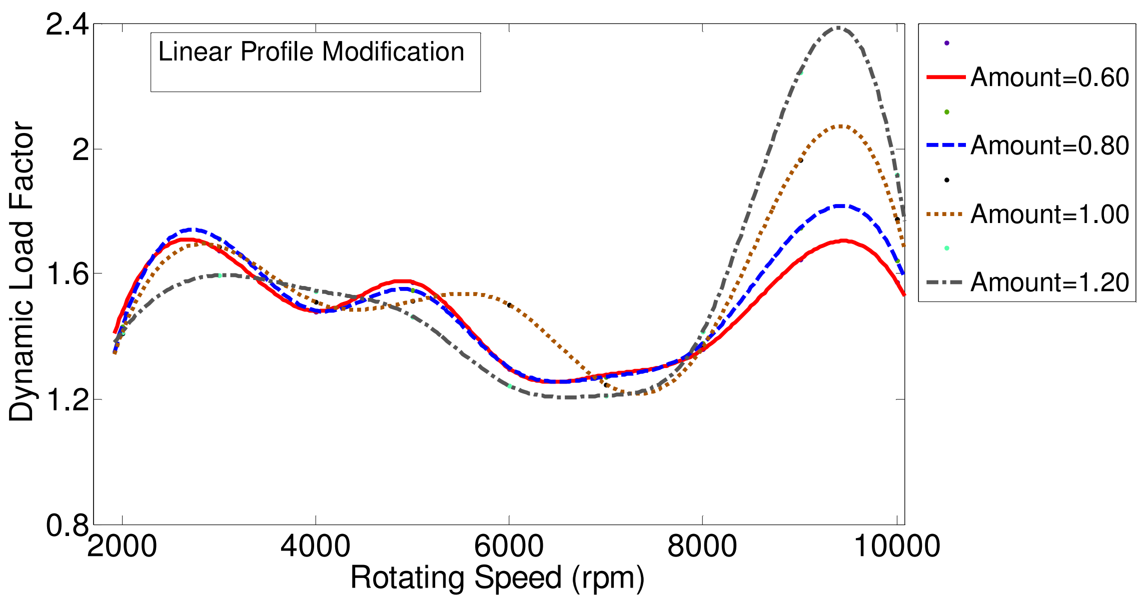

Figure 3.75 Effect of profile modification amount on gear dynamics, for linear tooth profile modification with maximum cumulative half-sine type tooth spacing error of 0.0003 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

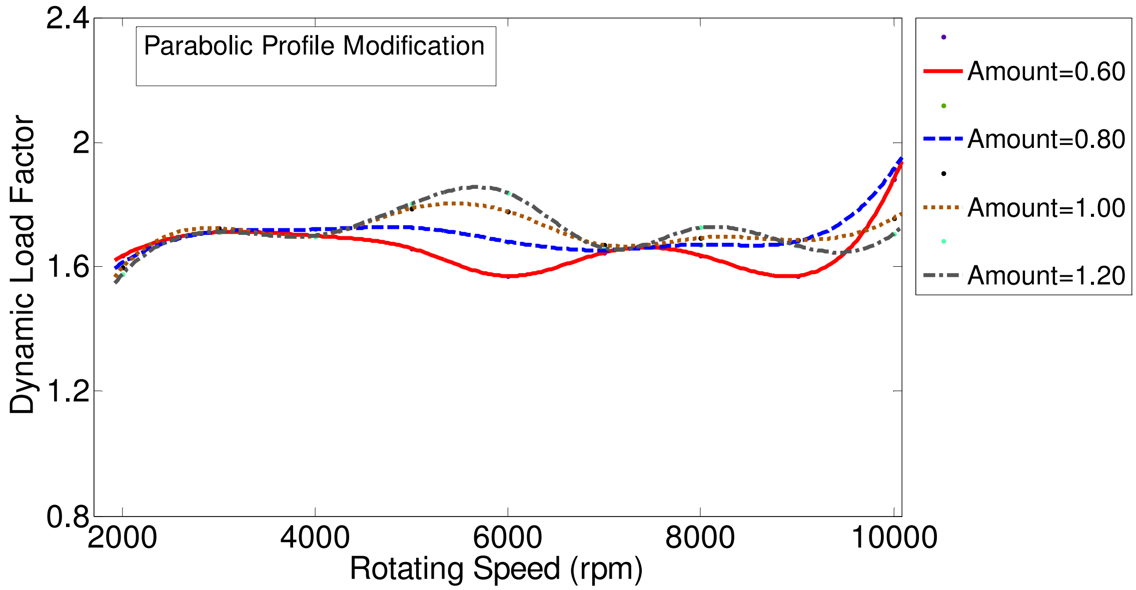

Figure 3.76 Effect of profile modification amount on the gear dynamics, for parabolic tooth profile modification with maximum cumulative random type tooth spacing error of 0.0003 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

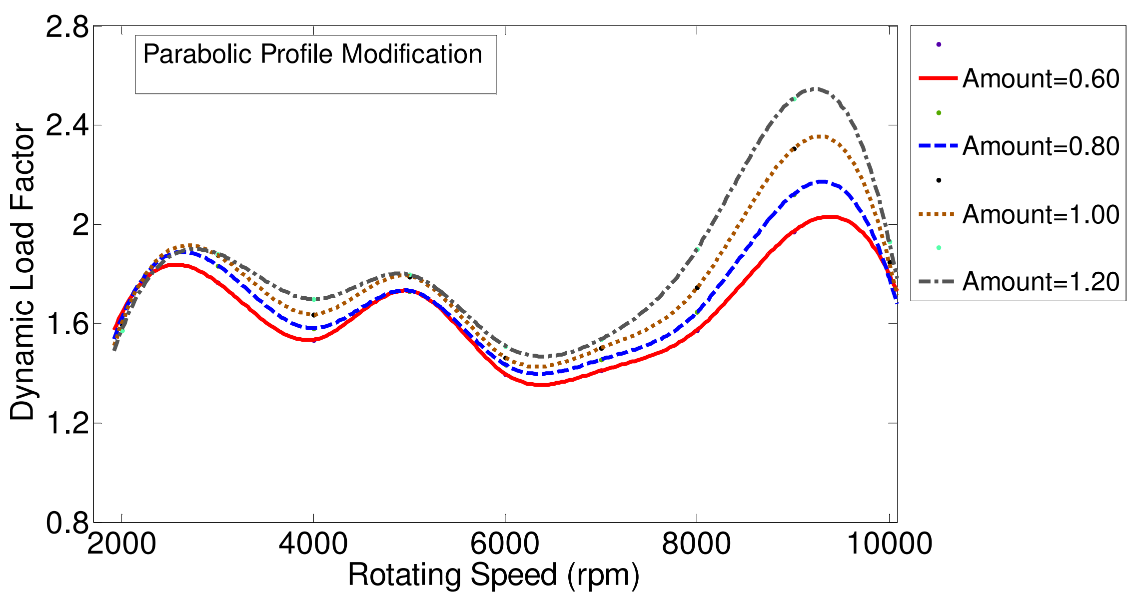

Figure 3.77 Effect of profile modification amount on the gear dynamics, for parabolic tooth profile modification with maximum cumulative full-sine type tooth spacing error of 0.0003 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

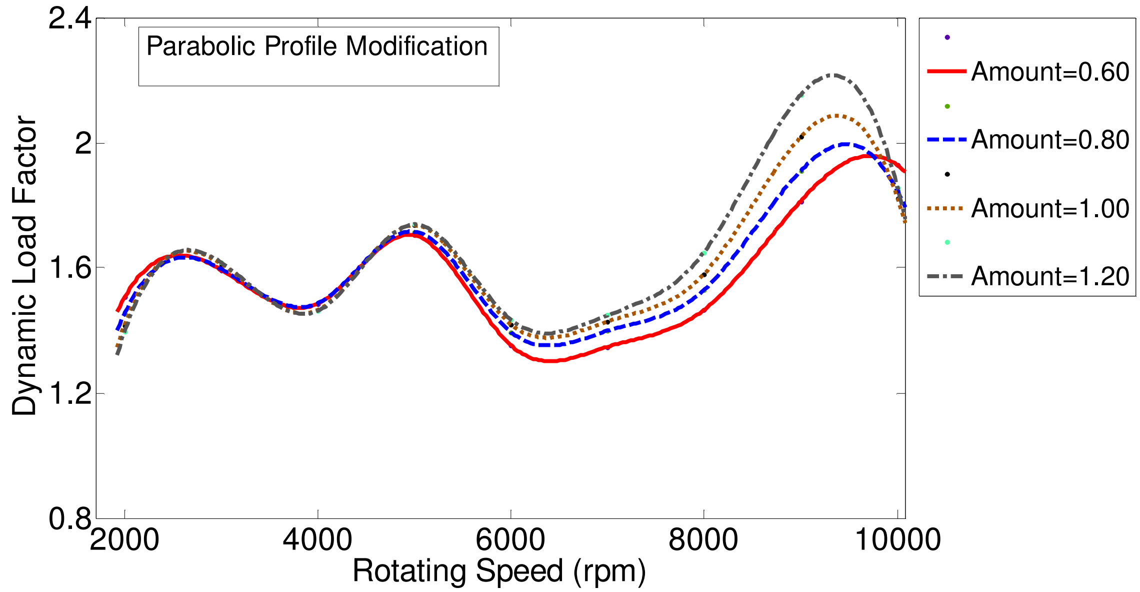

Figure 3.78 Effect of profile modification amount on the gear dynamics, for parabolic tooth profile modification with maximum cumulative half-sine type tooth spacing error of 0.0003 in. The modification amount varies from 0.60 to 1.20, and the length is 100%.

CHAPTER 4

4. CONCLUSIONS AND RECOMMENDATIONS

Dynamic analysis of low contact ratio gears was conducted using computer program DANST to investigate the effect of full-sine, half-sine and random wave forms of tooth spacing error with maximum cumulative spacing error of 0.0001 in., 0.0002 in., and 0.0003 in. on gear dynamic load factor. The effect of tooth profile modifications on the dynamic response of gears with different forms of spacing errors was also investigated. Both linear and parabolic profile modifications were applied to the sample gear sets. The amount and length of tooth profile modification were changed systematically for each type of spacing error. The results obtained from all cases were studied carefully and compared to each other. The following conclusions were drawn from the above investigations:

1. The amount of maximum cumulative spacing error has a very significant effect on the dynamic load factor. The higher the amount of maximum cumulative spacing error, the higher the dynamic load factor.
2. Half-sine form spacing error creates lower dynamic load factor than do full sine and random forms of spacing error. The random form of spacing error typically produces higher dynamic load than the other type of spacing errors.
3. Gears with parabolic tooth profile modification generally have lower dynamic load factor than those with linear profile modification.
4. Linear tooth profile modification is more sensitive to the amount of cumulative spacing errors. Gears with linear profile modification have higher dynamic

load increase when the cumulative spacing errors increase.

5. For most of cases studied, gears with tooth profile modifications create smaller dynamic load than the unmodified gears when the cumulative spacing error is lower.

6. In some cases studied, tooth profile modifications show very little or even detrimental effects on the dynamic response of the gears. This does not mean that tooth profile modification is not effective in reducing the dynamic load factor, but rather that the modification amount and length should be carefully controlled for specific gear configurations for better results.

7. Excessive profile modification leads to extremely high dynamic load factor. Therefore, over-modification should be avoided when applying profile modification to gears.

This study analyzes the dynamic load response of low contact ratio gear systems. It is recommended that same parameters and factors be used to evaluate high contact ratio gear trains with the application of a wide range of load with more variation of gear ratio. Similar approach of using linear and parabolic tooth profile modification can be adopted to determine better tooth profile modification to improve dynamic performance of gear sets with higher contact ratios.

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