Dynamic Rupture Modeling of Induced Earthquakes in Oklahoma

Elizabeth Ann Gilmour

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DYNAMIC RUPTURE MODELING OF INDUCED EARTHQUAKES IN OKLAHOMA

by

Elizabeth Gilmour

A Thesis

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

Major: Earth Sciences

The University of Memphis

August 2018
ACKNOWLEDGEMENTS

This project was made possible by financial support from the United States Geologic Survey and from the Center from Earthquake Research and Information.

I am grateful for many people who helped me with this project and made it possible. First, I would like to thank my advisor, Eric Daub for his assistance and the idea for this project. His advice, whether he was working from here in Memphis or corresponding from overseas, has guided me through my master’s research and towards being a better scientist. I am grateful to the other members of my committee, Chris Powell and Chris Cramer, for their help and expertise. Chris Powell’s knowledge about regional geophysics and geology steered me in the right direction, and Chris Cramer’s expertise in seismic hazards lay the foundation for much of the project.

Thank you to the other graduate students at CERI for your contributions. I’m grateful to Arushi Saxena, not only for research advice, but also for our visits to the gym and adventures at the climbing wall that were a great break from work. Thanks to Seyi Bolarinwa for being a fantastic study partner for my coursework at CERI. Thank you to all my friends at CERI for making grad school a better and brighter place. I would also like to thank a few students for more direct help with this project: Thanks to Khurram Aslam for his code on fault roughness, which was a vital part of this project, and for his help with many questions I had along the way, and thanks to Sabber Ahamed for his help and his advice on neural networks and dynamic rupture models.

Thank you to my parents for their support and seemingly endless interest in earthquakes. Even though I chose a field so different from theirs, they were always happy to talk. Most importantly, I’d like to thank my husband, Bivu Dhungana, for his endless support, both technical and emotional.
ABSTRACT

Since 2008, seismicity has increased in Oklahoma due to high-volume fluid injection. We examine the impact of pore pressure and tectonic factors on the magnitude and the potential for larger earthquakes.

Dynamic rupture models are performed on 10 km faults with varying input parameters. A neural network trained to approximate the results of the models predicts the moment magnitude of a rupture with 74% accuracy. Fault roughness and normal stresses are negatively correlated with moment magnitude, while the pore pressure has no correlation.

We use the trained neural network with a Markov Chain Monte Carlo (MCMC) to find distributions of tectonic conditions. Using the inverse method on induced seismicity data from the Fairview earthquake sequence provides information about frictional parameters and fault geometry that cannot be found using other methods. The results suggest that induced earthquakes are tectonic earthquakes that have been advanced in time, rather than new events.
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The neural network has nodes in the first layer corresponding to each input parameter. Two hidden layers, each containing eight nodes, were used because this architecture produced the most accurate results on the validation dataset. The final layer corresponds to the output, the calculated moment magnitude.

The neural network uses regression to predict the moment magnitude for each set of input parameters. The actual moment magnitude is plotted on the x-axis and the moment magnitude predicted by the neural network is plotted on the y-axis. The line shows the ideal linear relationship between the actual and predicted magnitudes. The neural network appears to be most accurate at predicted the magnitude for larger events.

The trace plots and posterior distributions for the first synthetic test. For each parameter, \((\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \text{the coefficient of dynamic friction, the coefficient of static friction, and the amplitude of fault roughness})\), the trace plot and the sampled distribution are shown. The trace plot shows the values of the parameter for each step by the MCMC. The histogram shows the sampled distribution produced by the MCMC. The solid black line shows the actual value of the parameter and the dashed black line shows the sample mean value. The sampled distributions for \(\sigma_{xx}\), \(\sigma_{xy}\), and dynamic friction are close to the actual parameter values. All values are within the 95% credible interval.

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Plots of 95% credible intervals for the six parameters in each test. The black bar corresponds to the 95% credible interval. The black square shows the mean value from the MCMC. The red circle shows the actual value. In all but two cases, the actual value lies within the 95% confidence interval.

The Fairview sequence earthquakes, located in the center of the map, are located on a partially mapped, reactivated fault. The high-volume injection wells to the northeast of the fault contribute to the increased pore pressure along the fault. The fault is partially unmapped, and may be connected to other faults. The longer rupture length makes larger earthquakes possible.

Map showing the Fairview sequence epicenters and the locations of large injection wells as well as modeled pore pressure. The injection wells are scaled according to the total injection volume in 2016. The high-volume injection wells to the northeast of the Fairview sequence most strongly impact the pore pressure, and earthquakes with hypocentral locations closer to the injection wells have higher pore pressure at the hypocenter.

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Chapter 1
Introduction

Earthquakes are a major concern because of their destructive power. They occur without any warning, and the damage they inflict causes immense economic damage and casualties. For example, the 2004 Indian Ocean earthquake and the resulting tsunami killed 280,000 people [Synolakis and Kong, 2006]. The 2011 Tohoku, Japan earthquake caused $235 billion in damage, making it the most expensive natural disaster in history [Kajitani et al., 2013].

Understanding the risk of earthquake damage, or seismic hazard, allows people to take appropriate precautions. In the United States, large and damaging earthquakes generally occur in known fault zones. These fault zones have been extensively studied, and information about past earthquakes informs our understanding of the size of earthquake that could occur. Because earthquakes have historically occurred in these locations, it has been possible to prepare for future earthquakes by building to certain standards and retrofitting older buildings. Areas that do not have a history of earthquakes have not required buildings to be built to withstand earthquakes and are at particular risk should earthquakes occur.

Despite preparations for large earthquakes and building to seismic standards, earthquakes can still cause fatalities and economic damage. The $M_w$ 6.9 Loma Prieta earthquake occurred in Northern California in 1989. It killed 63 people and caused over $5 billion in damage. The damage included not only houses and commercial buildings on unstable soil, but also the Bay Bridge and an elevated freeway that had been retrofitted to withstand earthquake shaking [Stover and Coffman, 1993].

Since 2008, seismicity in the central United States, not historically an area of great earthquake hazard, has sharply increased. This increase in seismicity is especially significant in
Oklahoma due to induced earthquakes. These earthquakes typically result from the injection of wastewater into the basement rock or deep strata [Ellsworth, 2013; Weingarten et al., 2015]. The injected fluids propagate through pores in the rock or along faults and increase pore pressure both close to the injection site and along faults more than 40 kilometers away from the injection site [Keranen et al., 2013; Goebel et al., 2017].

Particularly in cases where earthquakes occur farther from injection wells, the induced earthquakes are known to occur along pre-existing, optimally oriented faults [Keranen et al., 2014; Yeck et al., 2016; McNamara et al., 2015]. These faults are reactivated by the change in pore pressure due to fluid injection [McNamara et al., 2015]. Continental crust is known to be stressed at near failure, and so small changes in stresses can trigger earthquakes [Townend and Zoback, 2000]. The fault fails when shear stress exceeds the strength of the fault [Ellsworth, 2013]. Pore pressure reduces the effective pressure on the fault, thus it promotes failure. Pore pressure increases as small as 0.01 MPa have been known to trigger induced earthquakes [Keranen et al., 2014]. The increased pore pressure allows rupture nucleation to occur, but the rupture can continue to propagate along pre-existing faults into areas without elevated pore pressure [McGarr and Barbour, 2017; Yeck et al., 2016].

While many of the earthquakes observed in Oklahoma have been small, some have been larger, including a $M_w$ 5.7 earthquake near Prague, Oklahoma in 2011 and another $M_w$ 5.1 earthquake near Fairview, Oklahoma in 2016 [Goebel et al., 2017]. The largest was a $M_w$ 5.8 earthquake near Pawnee, Oklahoma in 2016 [McGarr and Barbour, 2017]. These earthquakes were large enough to cause structural damage [McGarr and Barbour, 2017].

The $M_w$ 5.8 Pawnee earthquake in 2016 damaged unreinforced masonry buildings and collapsed chimneys. As a result of the earthquake, six buildings in nearby Osage County were
declared uninhabitable [Yeck et al., 2017]. A $M_w$ 5.7 earthquake in 2011 destroyed 14 homes, buckled pavement, injured two people, and damaged other buildings [Keranen et al., 2013]. Some of the recent earthquakes in Oklahoma have been near cities and critical infrastructure. The 2016 $M_w$ 5.0 earthquake near Cushing, Oklahoma damaged structures in the nearby city of Cushing [McGarr and Barbour, 2017]. The earthquake was close to a major intersection of critical oil pipelines and the world’s largest oil storage facility. The cities and infrastructure near Oklahoma seismicity could be further damaged by larger events.

The increase both in the number of earthquakes and the occurrence of larger earthquakes impacts the seismic hazard for the region [Peterson et al. 2016]. These earthquakes have all had moment magnitudes of less than 6.0, but the pre-existing faults in Oklahoma are long enough to host larger earthquakes. A larger earthquake presents an especially high seismic risk in Oklahoma because structures are not built to withstand the shaking from a large earthquake.

Seismic hazard is based on the rate of earthquake occurrence, the earthquakes’ magnitudes, and a site’s proximity to the earthquake source [McGuire, 2004]. Induced earthquakes present a challenge for seismic hazard modeling. Typically, the maximum magnitude of an earthquake can be found from records of seismicity in the past hundreds or thousands of years, such as instrumental seismicity or paleoseismicity [McGuire, 2004]. As these induced earthquakes have only been occurring for several decades, a good estimate of their maximum magnitude does not exist.

Statistical methods have been used to explore the maximum magnitude of induced earthquakes. For earthquakes that propagate along pre-existing faults outside the area of elevated pore pressure, the maximum magnitude does not appear to be bound by either the volume of the fluid injected or the pressure of the fluid injected [van der Elst et al., 2016]. Rather, the induced
earthquakes are observed to follow the Gutenberg-Richter law, and are as large as expected [van der Elst et al., 2016]. The Gutenberg-Richter law is a power law that describes the relationship between the number of earthquakes to occur in a region and the number of earthquakes of a certain magnitude or higher [Gutenberg and Richter, 1944]. It states that many more small earthquakes occur than large earthquakes. As a result, it is possible that larger earthquakes, such as earthquakes of $M_w \geq 6.0$, may occur in the future even thought they have not yet occurred in modern-day Oklahoma.

In this study, we take a physics-based rather than statistical approach to estimating the maximum magnitude of an induced earthquake in Oklahoma. These statistical models face limitations in predicting large events. As described by the Gutenberg-Richter law, large events happen much less frequently than small events. Because statistical models perform poorly on rare events, they cannot be expected to determine if events the model has not yet seen can occur.

Physical methods can determine the plausibility of a large earthquake.

However, the necessary inputs for physical rupture models are not well constrained. It is not possible to measure the stresses and the coefficients of friction for a fault "in situ". This study aims to constrain the inputs for the physics-based models, and then use these models to estimate earthquake hazard. We use dynamic rupture models to model earthquakes that occur as the result of combinations of various stresses, friction coefficients, fault roughness profiles, pore pressures, and extent of pore pressure along the modeled fault. As many of these parameters are unknown, we use a Markov Chain Monte Carlo to invert for parameters based on the known value of magnitude and modeled values of pore pressure and pore pressure extent. Through the inversion of magnitude data and pore pressure models from observed earthquake sequences, we are able to probabilistically find the conditions that allow these ruptures to occur.
Chapter 2

Tectonic and Modeling Background

Induced earthquakes

Oklahoma’s geologic history produced critically stressed, optimally-oriented faults on which induced earthquakes can occur. The oldest rocks in Oklahoma, which today make up the crystalline basement, are part of the southern granite-rhyolite province [Shah and Keller, 2017; Johnson, 2008]. Uplift in central Oklahoma, a result of the Ancestral Rocky Mountains orogeny, created a broad region of faults through these rocks, which now make up the crystalline basement [McNamara et al., 2015b]. This uplift occurred during the Pennsylvanian period [Shah and Keller, 2017]. Additionally, during the Pennsylvanian period, marine shale layers were formed in Oklahoma. This shale contains oil and gas reservoirs [Johnson, 2008]. Large amounts of water from the ancient sea were also trapped in the pore space of the oil producing shale. This trapped water, known as produced water when it is captured through oil and gas production, is briny and contains dissolved minerals and other impurities [Rubenstein and Maheni, 2015].

As Oklahoma’s rocks contain reserves of oil and gas, this region has long been important for the oil industry. But by the late 1960s, large scale oil production in Oklahoma was no longer financially rewarding, and oil production declined [Boyd, 2002]. Oil production in Oklahoma increased starting around 2008 when dewatering wells were used to produce oil and gas from previously not profitable areas [Rubenstein and Maheni, 2015]. These wells produce oil and gas from rock formations that also have large amounts of water trapped in the pore space.

This large volume of contaminated water must be disposed of, and is typically injected into the Arbuckle group, a sedimentary formation below the oil-producing shale layers but above the crystalline basement [Rubenstein and Maheni, 2015; Shah and Keller, 2017] (Fig. 1). While
some fluid from hydraulic fracturing operations is disposed of in injection wells, the majority of
the injected fluid in Oklahoma is produced water [Rubenstein and Maheni, 2015]. The rate of
fluid injection increased around 2008 as the use of dewatering production wells, which produce
200 times more wastewater as conventional production wells, became more commonly used
[Keranen et al., 2014].

Fig. 1: Oil and gas as well as water are obtained from the producing layers. The large amounts of
briny, contaminated water, known as produced water, are disposed of through injection into the
Arbuckle formation. The water diffuses through the pore space and moves along other
high-conductivity pathways, such as faults. The injected water increases the pore pressure in the
Arbuckle formation and also in the crystalline basement below it. The crystalline basement is
granite-rhyolite and contains pre-existing faults upon which earthquakes can occur. (Image credit:
Steven Than in Ker Than. "Oklahoma earthquakes linked to oil and gas wastewater disposal

There are 35,000 active wastewater disposal wells in the central and eastern United States
[Rubenstein and Maheni, 2015]. Very large volumes of fluids are disposed of through a small
number of these injection wells. High rate injection wells are considered to be those with
injection volumes over 300,000 barrels per month [Weingarten et al, 2015]. The high-volume injection wells are clustered near dewatering wells in north-central Oklahoma and central Oklahoma [Keranen et al., 2014] (Fig. 2).

![Oklahoma earthquake epicenter locations over time](image)

Fig. 2: Seismicity in Oklahoma hugely increased since 2000. Most of the recent earthquakes are clustered in north-central Oklahoma near the Nemaha fault zone and the high concentration of dewatering wells.

While induced earthquakes can be caused by many different human activities, the induced earthquakes in Oklahoma are caused by increases in pore pressure [Ellsworth, 2013]. The injected fluid causes a pore pressure perturbation around the injection wells [Ellsworth, 2013;
Keranen et al., 2014]. The increase in pore pressure reduces the normal stress on the fault. Its impact can be seen in the Coulomb stress failure change:

\[
\Delta \sigma_f = \Delta \tau + \mu (\Delta \sigma_n + \Delta P).
\]

(2.1)

where \(\Delta \tau\) is the shear stress change on a fault, \(\mu\) is the friction coefficient, \(\sigma_n\) is the normal stress, and \(P\) is the pore pressure. A positive Coulomb stress failure change, \(\sigma_f\), encourages rupture while a negative Coulomb stress failure change makes failure on a fault less likely [Stein, 1999]. The positive pore pressure change, \(\Delta P\), change counteracts the normal stress which is negative in compression. Either a positive pore pressure change or increased shear stress promotes failure on the fault [Stein, 1999]. Many pre-existing faults in Oklahoma are optimally oriented to rupture given the regional stress field, and small changes in pore pressure can induce failure [Alt and Zoback, 2017].

Prior to 2008, earthquakes in the central and eastern United States occurred at a constant rate of about 21 earthquakes each year with magnitude greater than 3.0 [Ellsworth, 2013]. Around 2000, the annual number of earthquakes in the central and eastern United States began to increase, and in 2008 the rate of earthquake occurrence sharply increased, with a pronounced change in Oklahoma. From 2008 to 2013, 45\% of all earthquakes in the central and eastern United States with magnitude greater than 3.0 occurred in Oklahoma [Keranen et al., 2014]. In 2015, 800 earthquakes of magnitude 3.0 or greater occurred in Oklahoma (Fig. 3). Studies have linked the increase in seismicity to human activities, particularly fluid injection [Peterson et al., 2016].

Pore pressure perturbations cause faults to fail, but the impact of changes in pore pressure on the maximum magnitude of earthquakes, and even if such a relationship exists, is unclear.
Fig. 3: Prior to 2008, earthquakes in Oklahoma occurred at a low, constant rate, but the rate of earthquake occurrence drastically increased after 2008.

Initially, when induced earthquakes began occurring in Oklahoma, injection induced earthquakes were considered to be those in the immediate vicinity of injection wells [Llenos and Michael, 2013]. Statistical studies focused on these earthquakes that occurred within the reservoir of increased pore pressure. Shapiro et al. [2011] found that the reservoir geometry controls the maximum magnitude of induced earthquakes. McGarr [2014], on the other hand, concluded that the volume of fluid injected controls the maximum magnitude of induced earthquakes.

After several years, as more fluid was collectively injected into Oklahoma, the areas of increased pore pressure expanded [Keranen et al., 2014] and fluid traveled along highly conductive pathways, such as faults, away from injection wells [Ellsworth, 2013]. Earthquake
sequences were observed not only in the immediate vicinity of high-volume injection wells, but also many kilometers away. For example, the Fairview earthquake sequence occurred on a partially-mapped, reactivated fault in the crystalline basement. The earthquakes were more than 12 km and as much as 40 km away from high-volume injection wells [Yeck et al., 2016; Goebel et al., 2017].

An earthquake sequence is a series of earthquakes that are related to each other spatially and temporally. Much of the seismicity in Oklahoma is part of earthquake sequences. A sequence is typically dominated by a mainshock, which may be a magnitude unit larger than the other earthquakes. The Fairview sequence is one such earthquake sequence. The largest earthquake in the Fairview sequence was a $M_w$ 5.1 earthquake that occurred in February of 2016. This earthquake shows that large earthquakes can occur away from injection wells. The Fairview sequence began in 2015 and continued through 2016, and included several other earthquakes with $M_w \geq 4.0$ [Yeck et al., 2016]. The sequence occurred along and on a partially-mapped, 14 km fault that is optimally oriented to rupture given the regional stress field [Yeck et al., 2016; Goebel et al., 2017]. Goebel et al. [2017] also examined the role of pore pressure in the Fairview sequence. They found that changes in pore pressure as small as 0.01 MPa can cause earthquakes.

Van der Elst et al. [2016] argue that induced earthquakes are not statistically distinct from natural, tectonic earthquakes. They explain that previous research into the limits of induced earthquakes studied only earthquakes that exist within the reservoir of increased pore pressure, rather than those that occur on critically stressed faults farther from active injection wells. They found that induced earthquakes follow the Gutenberg-Richter law, and are not distinct from natural earthquakes. This analysis leads them to believe that induced earthquakes that propagate
along faults into areas outside the reservoir are limited by tectonics rather than fluid injection characteristics.

McNamara et al. [2015a] explain that induced earthquakes on long, pre-existing faults have the potential to damage important infrastructure. He argues that earthquakes are occurring on unmapped, reactivated faults, and that due to the regional stresses, these faults are critically stressed. They point out that due to the length of these faults and the stresses on them, there is a potential for a large, damaging earthquake.

Yeck et al., [2016] discuss the role far-field pressurization plays in the occurrence of earthquakes on pre-existing faults away from injection wells. They argue that high-rate injection into the Arbuckle group, which increased seven-fold in the months before the earthquake sequence began, led to increased pore pressure as much as 12km away from the injection wells. They claim the Arbuckle group is hydraulically connected to the crystalline basement, so fluids have been able to travel into pores in the basement. Yeck et al. found the moment tensors for the Fairview sequence and confirmed that these are strike-slip earthquakes occurring on pre-existing, optimally oriented faults (Fig. 4).

This study takes a physics-based approach by modeling induced earthquakes that occur on existing faults and propagate along these faults outside the area of elevated pore pressure to determine a probabilistic maximum magnitude. The previous research shows that induced earthquakes can occur along these faults and that small changes in pore pressure can cause these earthquakes to nucleate. The physics-based model can distinguish whether induced earthquakes are new events that would never have occurred if not for fluid injection, or if they are tectonic earthquakes whose nucleation is accelerated by the increased pore pressure. Most importantly,
this approach provides physical insight into the mechanics of earthquake failure and has the potential to constrain the likelihood of larger events occurring.
Fig. 4: a) A map of faults in Oklahoma. Due to its complex geologic history, Oklahoma is crisscrossed by faults. In north-central Oklahoma, where many of the injection wells and induced earthquakes are found, the faults strike northeast and are optimally oriented given the regional stress field. b) Focal mechanisms for recent Oklahoma earthquakes show that they are predominantly strike-slip events.
Computational methods

Dynamic rupture models

Dynamic rupture models are computational models that, based on initial conditions, calculate the results of an earthquake including slip on a fault as well as motion off the fault. Dynamic models do so using the elastodynamic wave equation coupled to a friction law describing fault failure [Andrews, 1976; Cocco and Bizzarri, 2002; Day, 1982; Daub and Carlson, 2008, 2010, 2010]. In this study we use the finite difference method to accurately calculate slip and other results both spatially and temporally [Kozdon et al., 2012]. Dynamic rupture models have been used as forward modeling techniques to solve various problems in earthquake mechanics [Kozdon et al., 2012].

Ruptures are modeled on a regular grid of blocks, and faults are modeled as frictional interfaces between the blocks. The blocks can slip on the interfaces, and the blocks can elastically deform (Fig. 5). These models can be conducted in two or three dimensions. For two-dimensional in-plane models, such as those used in this study, the model requires three components of the stress tensor, \( \sigma_{xx} \), \( \sigma_{yy} \), and \( \sigma_{xy} \), and two velocity components, \( u_x \) and \( u_y \). [Daub, 2017] The velocities are set to zero at the start of the rupture model, and the initial stresses are provided based on the problem [Daub, 2017]. As the model uses the finite difference method, the stresses and velocities are updated for each time step.

The dynamic rupture code is governed by the wave equation, equations describing deformation of the medium, and friction on the fault. The elastodynamic wave equation in two dimensions is given by:
Fig. 5: Dynamic rupture models at four snapshots in time showing particle velocity in the x (fault-parallel) direction. The rupture initiates at a point 2.5 km along the strike of the fault, and the rupture propagates along the fault in both directions from that point. The limiting velocity of the rupture tip is near the shear wave velocity in the medium. The models have a strong barrier at the domain edges to prevent the rupture from reaching the edge of computational space.

\[
\rho \frac{\partial v_x}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}
\]  
(2.2)

\[
\rho \frac{\partial v_y}{\partial t} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}
\]  
(2.3)

where \( \rho \) is the density of the medium, and \( \sigma_{ij} \) is a component of stress [Daub, 2017].
Another constitutive law relates the stresses to the deformations. In a homogenous, elastic material, it takes the form of:

\[
\frac{\partial \sigma_{ij}}{\partial t} = L_{ijkl} \dot{\varepsilon}_{kl}\]

(2.4)

where \( L_{ijkl} \) is the elastic tensor and \( \dot{\varepsilon}_{kl} \) is the strain rate [Daub, 2017]. This is given by Hooke’s Law for a homogenous isotropic material:

\[
L_{ijkl} \dot{\varepsilon}_{kl} = \lambda \delta_{ij} \frac{\partial v_k}{\partial x_k} + G \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)
\]

(2.5)

In the elastic tensor, \( \lambda \) is the first Lamé parameter and \( G \) is the shear modulus [Daub, 2017]. The Kronecker delta, \( \delta_{ij} \) is one when \( i = j \), and it is zero otherwise. Summation is implied over the repeated indices in the elastic tensor.

The dynamic rupture code used in this study uses linear slip-weakening friction to describe the failure on the fault [Andrews, 1976]. The coefficient of friction is a function of slip distance (Fig. 6). The frictional coefficient linearly reduces from a higher coefficient of static friction, \( \mu_s \) to a lower coefficient of dynamic friction, \( \mu_d \), over a slip weakening distance, \( d_c \), as the fault slips [Daub, 2017].

\[
\mu(U) = \begin{cases} 
(\mu_s - \mu_d) \left(1 - \frac{U}{d_c}\right) + \mu_d & (U < d_c) \\
\mu_d & (U \geq d_c).
\end{cases}
\]

(2.6)

The model incorporates band-limited, self-similar fractals to represent realistic fault geometry based on the study by Candela et al. (2012) on fault roughness. The frictional interface
between the blocks has a profile described by this complex geometry (Fig. 7). These rough faults introduce complex shear and normal tractions along the fault (Fig. 8). The model rotates the coordinate system along the fault to account for the irregular geometry.

The tectonic conditions, including the stress tensor, coefficients of friction, and fault geometry, are necessary inputs for dynamic rupture models. These conditions, however, are difficult to measure in the earth. Our limited knowledge of these conditions reduces the utility of dynamic rupture models for making predictions of slip or earthquake magnitude. The computational expense of running these models limits their ability to be used for inverse problems. Here we use an artificial neural network to approximate the results of the dynamic rupture model to speed up calculations and make an inverse problem possible. To date, physical models have not been used to invert for parameters in a systematic way due to their computational expense.
Artificial neural networks

Artificial neural networks are machine learning algorithms that approximate other functions, classify data, and detect patterns [Rosenblatt, 1958; Rumelhart et al., 1986; Cybenko, 1989]. One use of neural networks in geophysics is to approximate other functions, or in other words, produce a meta-model. This use of artificial neural networks reduces the computational expense of repeatedly running large computational models. DeVries et al. [2017] explain that the artificial neural network approximates the computational model by mapping model inputs $x$ onto model outputs $y = f(x)$ through adjusting its weights and connections. The artificial neural network can be used to approximate the same calculations as the original model more quickly and on a less powerful computer. According to the universal approximation theorem, a sufficiently complex neural network can approximate any other function [Cybenko, 1989].
Neural networks consist of layers of interconnected artificial neurons [Rosenblatt, 1962]. Each artificial neuron, or node, has an activation function and each connection has a numerical weight. The artificial neurons are arranged in layers. The first layer contains a neuron that corresponds to each of the model inputs, and the final layer contains an artificial neuron for each model output. Between the input and output layers are hidden layers. A neural network can have different numbers of hidden layers and different numbers of neurons in each hidden layer depending on the complexity of the problem [Géron, 2017].

The weights in an artificial neural network are trained rather than explicitly programmed. This study uses the multilayer perceptron, a method for supervised learning. It is a feed-forward neural network that uses a supervised learning technique called backpropagation for training [Géron, 2017]. The multilayer perceptron has been used successfully to fit both linear and nonlinear functions to complex data sets. To train the neural network, the neural network is given a data set with known target values. The neural network iteratively predicts results for a set of

Fig. 8: Normal and shear stresses on the fault at the first time step. The rough fault introduces large normal stress variations. The large perturbation in shear stress at 2.5 km along the strike of the fault is used to nucleate the rupture.
inputs and then compares the predicted results to the known, correct results. Through backpropagation, the weights for the nodes are adjusted to reduce error [Rumelhart, 1986].

Neural networks can be used for classification or regression. Classification problems in machine learning are those in which the algorithm predicts which category an observation belongs to based on features of that observation. Regression problems are those in which the machine learning algorithm predicts a target value based on the features of the observation [Géron, 2017]. Prior work using neural networks for classification of dynamic rupture model results have shown that they can be effective in predicting the results of modeled ruptures. A classification artificial neural network that was used to predict whether a rupture would break a barrier and continue to propagate along a fault predicted the result with 83.5% accuracy [Ahamed, 2018]. This study builds off of the work by Ahamed in its use of a regression neural network to predict the moment magnitude of a rupture using the input parameters of the dynamic rupture model. The artificial neural network can do the calculations in fractions of a second, rather than the hours needed to run a dynamic rupture model.

**Markov Chain Monte Carlo**

We develop a new application of an inverse method to constrain the tectonic conditions using the trained neural network along with a Markov Chain Monte Carlo. The Markov Chain Monte Carlo (MCMC) is a Bayesian method to solve inverse problems. This method estimates parameter values by taking a random walk in parameter space, sampling from the posterior distribution, and comparing the model parameters with the data.

As the MCMC moves through parameter space, it samples from the posterior distribution. In this case, the MCMC uses the trained neural network to check sets of parameters. The MCMC accepts a step with certainty if the new step is more likely than the previous step. If the new step
is less likely, it accepts with a certain probability. The step size and the probability with which a
new step is accepted is determined so that the MCMC will spend more time in high-probability
regions while fully exploring the parameter space. In this way, the MCMC samples more from
high-probability regions than from low-probability regions. The MCMC records each step in
parameter space, and these results provide a probability distribution for each parameter. The
neural network estimates the magnitude of a rupture, and then the MCMC samples parameter
space to find sets of parameters that could produce the ruptures.

MCMC was chosen for this inverse problem because it finds probabilistic distributions of
parameters, rather than solving for a single value. MCMC methods depend on the forward model
and a prior distribution describing additional constraints and knowledge about the solution. These
methods can be applied to nonlinear as well as linear problems [Aster et al., 2011]. The MCMC is
effective in high-dimensional problems, such as this one [Cowles and Carlin, 1996]. The MCMC
will attempt to find distributions for the tectonic conditions that allow the induced earthquakes to
occur.

Earthquake sequences are crucial to this study. We assume the tectonic parameters
including stresses, friction coefficients, and fault roughness, are constant for each earthquake in
an earthquake sequence while the pore pressure varies due to varying distances from the injection
wells and other hydrologic factors. While it is difficult to measure the tectonic conditions,
hydrologic modeling makes obtaining accurate pore pressure values possible. Based on the pore
pressure values, the inverse problem will solve for the tectonic conditions. Our ability to model
pore pressure from freely available injection well data allows us to invert for the other parameters.
Chapter 3

Methods and data

Dynamic rupture dataset

A set of dynamic rupture models is simulated to explore how a rupture’s moment magnitude is related to various tectonic conditions. The model is based on what we know about induced earthquakes in Oklahoma, and the resulting slip on the fault is used to calculate moment magnitude. These rupture models are used to produce a dataset of ruptures representing induced earthquakes that nucleate in the area of elevated pore pressure and then, depending on the conditions in which they occur, may or may not propagate along the fault outside the reservoir.

Induced earthquakes in north-central Oklahoma are known to be strike-slip earthquakes that occur on near-vertical faults in the crystalline basement [McNamara et al., 2015b]. These re-activated ancient faults are optimally oriented given the regional stress field [Goebel et al., 2017]. We represent these earthquakes as two-dimensional ruptures because they occur at constant depth in the crystalline basement. Each rupture is modeled on a ten-kilometer fault. This fault length is chosen because faults of this length can host earthquakes up to approximately $M_w 6.0$. The off-fault velocity structure is a uniform elastic material.

Dynamic rupture models require the stress tensor, the coefficients of dynamic and static friction, and the fault geometry. The input parameters for all models are varied randomly within a selected range of values. Because these rupture models are two-dimensional, they require three components of the stress tensor. The range of stresses is chosen based on the depth of the earthquake hypocenters, which is around 6 km [McNamara et al., 2015b]. The model uses slip weakening friction, in which the friction coefficient is reduced from the higher coefficient of static friction to the lower coefficient of dynamic friction over a characteristic length scale [Ida,
Both the coefficient of static friction and the coefficient of dynamic friction are randomly varied. The dynamic friction ranges from 0.2 to 0.4, and the static friction ranges from 0.6 to 0.8. For each rupture model, the static friction is always at least 0.2 higher than the dynamic. The fault geometry is based on Candela et al.’s [2012] research into realistic fault geometry. Each fault has a self-similar, fractal profile. This roughness profile is represented as the root mean square roughness amplitude. The RMS roughness amplitude is varied from 0.001 to 0.03 to represent a range of possible fault geometries. The wavelength of the fractal is cut off at 20 times the grid spacing so that there are no fractals with wavelength less than 125 meters in this model. The code fdfault is used for the dynamic rupture modeling [Daub, 2017].

Because these ruptures represent earthquakes beginning in the reservoir of elevated pore pressure and propagating along a pre-existing fault into areas without elevated pore pressure, these models include two addition parameters: the pore pressure and the pore pressure extent. The pore pressure represents the increase in pore pressure at the earthquake hypocenter, ranging from 0 to 1.0 MPa. These values are based on research by Keranen et al. [2014] and Goebel et al. [2017]. The pore pressure extent represents the portion of the fault with high pore pressure, and this parameter ranges from 0.3 to represent 30% of the fault to 1.0 to represent elevated pore pressure along the entire fault.

The dynamic rupture model calculates the slip on the fault (Fig. 9). The slip can then be used to calculate the moment magnitude of the ruptures as:

\[ M_w = \frac{2}{3} \log_{10}(\mu AD) - 10.7 \]  

(3.1)

where \( \mu \) is the shear modulus in Pa, \( A \) is the area of rupture in \( m^2 \), and \( D \) is average
Fig. 9: A rupture as propagating along a fault as modeled by the dynamic rupture model. The model shows the propagation of the rupture in space and time, and the rupture can be seen moving along the 10 km fault. The y-axis shows slip on the fault.

Correlation plots show that the fault roughness is the most important parameter for determining moment magnitude, and it has a high negative correlation. Rougher faults produce larger normal stress variations that are likely to cause the rupture to arrest before rupturing the entire fault. The normal stresses also have a negative correlation with moment magnitude. More negative values of compressive stress give more stored energy that can be released when the fault
slips, leading to larger slips. The pore pressure and the pore pressure extent have noticeably small impacts on moment magnitude, and the coefficients of friction also have little correlation with the moment magnitude. This correlation agrees with the hypothesis that pore pressure impacts the nucleation of induced earthquakes, but the propagation and magnitude are controlled by tectonic factors. (Fig. 10)
Correlations between parameters and magnitude

Fig. 10: A correlation plot for the dynamic rupture model dataset. The plot shows which features are correlated. The stresses are all correlated, with the normal stresses positively correlated with each other and negatively correlated with the shear stress. The friction coefficients are correlated with each other. The magnitude is strongly negatively correlated with the fault roughness and the normal stresses, and has a small positive correlation with the shear stress.
Neural network regression

The suite of completed dynamic rupture models provides a dataset to train, validate, and test the artificial neural network. The dynamic rupture model inputs – stresses, coefficients of friction, fault roughness, pore pressure, and pore pressure extent – are the inputs to the artificial neural network regressor, and the calculated moment magnitude is the output. The neural network reduces the computational time needed to find the moment magnitude based on inputs to the dynamic rupture model.

The full set of dynamic rupture model results includes 1,500 sets of input parameters each paired with the resulting moment magnitude. The set of dynamic rupture model results is broken into three sets: a training dataset, a validation dataset, and a testing dataset. The training set, which comprises 2/3 of the data, is used to train the artificial neural to calculate the moment magnitude. Neural networks adapt quickly to the current dataset when training. It is important to develop a model that also translates to new, unseen datasets. In this study, the neural network is implemented with scikit-learn.

The architecture of the neural network and the hyperparameters must be chosen to maximize the predictive accuracy on new datasets. It is also important to avoid underfitting or overfitting. Overfitting occurs when a neural network memorizes the noise in the dataset in addition to modeling the underlying patterns. When an overfitted neural network encounters new data, it makes poor predictions. An underfitted neural network is not complex enough for the problem it is being used to solve and thus suffers from poor performance because it is not able to represent all of the robust aspects of the data. Thus, it is crucial to strike an appropriate balance when setting up a model.
A hyperparameter is a parameter of the machine learning algorithm that determines the complexity of the model. It is set by the user and modified to improve the accuracy of the machine learning model [Géron, 2017]. Examples of hyperparameters include the number of neurons in each hidden layer and the regularization parameter. The validation dataset is used to select the hyperparameters and check that the neural network architecture is appropriate for the complexity of the problem. The regularization parameter helps avoid overfitting by penalizing neuron weights that are excessively high. The appropriate value for the regularization parameter is determined by testing the predictive accuracy of the model on the validation dataset.

Similarly, two additional hyperparameters are the number of hidden layers and the number of neurons in each hidden layer. Just like the regularization parameter can vary, models can be made with different numbers of hidden layers and numbers of neurons. The numbers of hidden layers and neurons per hidden layer that provide the highest accuracy on the validation data are chosen as the hyperparameter for the model. The neural network used in this study has two hidden layers each containing eight nodes (Fig. 11). These hyperparameters as well as the regularization hyperparameter are chosen so the neural network can most accurately predict the magnitudes for ruptures in the cross-validation dataset.

The testing dataset is a new dataset previously unseen by the neural network. It is used to check the predictive accuracy of the neural network. On the testing dataset, the neural network predicted magnitudes with 74% accuracy. The neural network was able to explain 73.8% of the variance observed in the testing dataset. The R squared value is 0.739 (Fig. 12).
Fig. 11: The neural network has nodes in the first layer corresponding to each input parameter. Two hidden layers, each containing eight nodes, were used because this architecture produced the most accurate results on the validation dataset. The final layer corresponds to the output, the calculated moment magnitude.
Fig. 12: The neural network uses regression to predict the moment magnitude for each set of input parameters. The actual moment magnitude is plotted on the x-axis and the moment magnitude predicted by the neural network is plotted on the y-axis. The line shows the ideal linear relationship between the actual and predicted magnitudes. The neural network appears to be most accurate at predicted the magnitude for larger events.
**Markov Chain Monte Carlo method**

The Markov Chain Monte Carlo is a Bayesian method that estimates parameter values by sampling from a posterior distribution. The MCMC uses a random walk to move through parameter space. The true values of the posterior distribution are unknown, but by moving through parameter space and sampling from the posterior distribution, the MCMC can produce a probability distribution for each of the unknown parameters. The MCMC is used in this study to invert to find the unknown coefficients of friction, stresses, and amplitude of fault roughness based on the known or modeled values of pore pressure, pore pressure extent, and magnitude.

While the MCMC is the inverse method, the forward model for this inversion is the artificial neural network that has been trained to calculate moment magnitude from sets of parameters. Paired with the neural network, which can quickly estimate moment magnitudes, an MCMC model can be used to find the parameters that create an earthquake catalog while the neural network approximates the forward model to compare the sets of parameters to the data. In this study, the MCMC was implemented with the PyMC library [Patil, 2010].

To test whether the inverse method can yield satisfactory results, we performed three synthetic tests. The synthetic test models an earthquake sequence. Each synthetic test consists of 100 dynamic rupture models. The models for a test have known, uniform values for the stresses, coefficients of friction, and the amplitude of fault roughness, but the pore pressure and pore pressure extent are varied for each rupture depending on the distance from the injection wells. The hypocentral location is different for each earthquake in an earthquake sequence, and the pore pressure is different at each hypocentral location. The moment magnitude was calculated using
the dynamic rupture model for each rupture in a set, in the same way the moment magnitude was calculated for each rupture in the training data.

A synthetic test is analogous to the inversion that will be performed on real earthquake data, and serves as a test of whether this method can produce accurate results. The results of the MCMC inversion include trace plots and probability distributions for each unknown parameter. Trace plots show how the MCMC has moved through parameter space and show the sampled value of the parameter for each step of the chain. The trace plots show how the MCMC has explored parameter space. The histogram shows the probability distribution that is created by sampling from the posterior distribution. If the MCMC fully explores parameter space and the samples chosen by the MCMC are not correlated, the sampled distribution should reflect the posterior distribution.

The MCMC is given sets of moment magnitude, pore pressure, and pore pressure extent for each rupture, and it finds the remaining six tectonic parameters through its random walk in parameter space. For each parameter, the MCMC is given a prior distribution. Here the prior distributions are uniform distributions encompassing the possible values for each parameter. The uniform prior distribution means that we do not consider any values more probable than others; instead, the prior distribution is only a range of possible values. Information about the tectonic parameters is found from laboratory experiments of slip [Scholz, 1998], as well as studies of exhumed faults, such as the study of fault roughness by Candela, et al. [2012]. These studies help constrain a general range for the parameters. Not enough information exists about tectonic conditions in Oklahoma to provide the MCMC with more precise probability distributions for the parameters.

For each iteration, or step in parameter space, that the MCMC takes, the neural network
calculates the magnitude for each rupture based on the provided values of pore pressure and pore pressure extent and the sampled values of the tectonic parameters. By comparing this estimated magnitude value to the known magnitude, the MCMC can check the accuracy of this set of parameters. The MCMC then takes another step in parameter space and tests a new set of parameters.

The subplots in Figure 13 show the results of the first synthetic test. Subplot (a) shows the trace for $\sigma_{xx}$, a component of normal stress. The MCMC widely explored parameter space, but tended to stay towards more negative values. Subplot (b) shows the sampled distribution for $\sigma_{xx}$. The actual value was 27.5 MPa, and the sampled mean value was slightly lower at 25.2 MPa.

Subplot (c) shows the trace for $\sigma_{yy}$, the other component of normal stress. Here the MCMC also widely explored parameter space but tended towards higher numbers. Subplot (d) shows the sampled distribution for $\sigma_{yy}$. The sampled mean, -18.7 MPa is higher than the actual value of -22.5 MPa. Subplots (e) and (f) are the trace plot and sampled distribution for $\sigma_{xy}$, the shear stress. The MCMC roughly produces a Gaussian distribution, and the sampled mean, 12.4 MPa is somewhat higher than the actual value of 9.75 MPa. Subplots (g) and (h) show the trace plot and sampled distribution for $\mu_d$, the coefficient of dynamic friction. Here the MCMC also produces a roughly Gaussian distribution, and the sampled mean is very close to the actual parameter value.

Subplots (i) and (j) show the trace and the sampled distribution for $\mu_s$, the coefficient of static friction. Subplots (k) and (l) show the trace plot and the sampled distribution for the amplitude of fault roughness. The MCMC tends toward much higher values for the amplitude of fault roughness. The actual parameter value is 0.015, while the sampled mean is 0.028. The distribution appears to show two overlapping, roughly Gaussian distributions.

Figure 14 shows the MCMC trace plots and sampled distributions for the second synthetic
Fig. 13: The trace plots and posterior distributions for the first synthetic test. For each parameter, \( (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \text{the coefficient of dynamic friction, the coefficient of static friction, and the amplitude of fault roughness}) \), the trace plot and the sampled distribution are shown. The trace plot shows the values of the parameter for each step by the MCMC. The histogram shows the sampled distribution produced by the MCMC. The solid black line shows the actual value of the parameter and the dashed black line shows the sample mean value. The sampled distributions for \( \sigma_{xx}, \sigma_{xy}, \) and dynamic friction are close to the actual parameter values. All values are within the 95% credible interval.
test. This test used entirely different values of the stresses, the coefficients of friction, and the amplitude of fault roughness than the first test in order to simulate a different earthquake sequence. Subplot (a) and (b) show the trace plot and the sampled distribution for $\sigma_{xx}$. For this parameter, the MCMC explored the full range of possible values and produced what appears to be a roughly Gaussian distribution. However, the distribution skews more negative than the actual parameter value, producing a distribution with a mean value of -16.9 MPa versus the actual parameter value of -12.05 MPa. Subplots (c) and (d) show the trace plot and sampled distribution for $\sigma_{yy}$. As in the first test, the MCMC widely samples parameter space but fails to produce a satisfactory distribution. The sampled mean is -21.1 MPa versus the actual parameter value of -14.6 MPa. Subplots (e) and (f) show the trace plot and the sampled distribution for $\sigma_{xy}$. The trace shows that the MCMC tended towards the lower values, but the sampled mean, 5.2 MPa is very close to the actual parameter value of 4.36 MPa. Subplots (g) and (h) show the trace plot and the sample distribution for $\mu_d$. The sampled distribution appears to be approximately a Gaussian distribution, but it is lower than the actual value. Subplots (i) and (j) show the trace plot and the sampled distribution for $\mu_s$. In this case, the MCMC fails to sample the full range of values and instead is caught in a particular location in the posterior distribution. The actual value for $\mu_s$ lies outside the 95% credible interval. Subplots (k) and (l) are the trace plot and the sampled distribution for the amplitude of fault roughness. Once again, the MCMC finds a sampled mean that is significantly higher than the actual parameter value for the fault roughness.

Figure 15 shows the MCMC trace plots and sampled distributions for all parameters in the third synthetic test. This is another synthetic test with entirely new values for the tectonic parameters. Subplots (a) and (b) are the trace plot and the sampled distribution for $\sigma_{xx}$. As in the first test, the MCMC stays towards the more negative values, but the sampled mean, -30.6 MPa is
Synthetic test 2 MCMC results

Fig. 14: The trace plots and posterior distributions for the second synthetic test. For each parameter, \((\sigma_{xx}, \sigma_{yy}, \sigma_{xy},\) the coefficient of dynamic friction, the coefficient of static friction, and the amplitude of fault roughness), the trace plot and the sampled distribution are shown. The trace plot shows the values of the parameter for each step by the MCMC. The histogram shows the sampled distribution produced by the MCMC. The solid black line shows the actual value of the parameter and the dashed black line shows the sample mean value. The sampled mean values for \(\sigma_{xy}\) and dynamic friction are close to the actual parameter values. In this test, the sampled value of static friction is outside the 95\% credible interval.
still close to the actual parameter value, -33.5 MPa. Subplots (c) and (d), the trace plot and sampled distribution of $\sigma_{yy}$, once again show that the MCMC widely sampled parameter space but failed to produce a useful distribution or an accurate sampled mean. Subplots (e) and (f), the trace plot and sampled distribution for $\sigma_{xy}$ show that the MCMC stayed in the lower values in parameter space. It produced a Gaussian distribution, but the distribution is skewed to lower values. The actual parameter value is 12.6 MPa while the sampled mean is 7.9 MPa. Subplots (g) and (h), which correspond to the trace plot and sampled distribution for $\mu_d$, show that the MCMC fully samples parameter space, but the sampled mean is lower than the actual parameter value. Subplots (i) and (j) show the trace plot and the sampled mean for $\mu_s$. Here, the sampled mean exactly corresponds to the actual parameter value, 0.77. The final two subplots, (k) and (l) show the trace plot and sampled distribution for the amplitude of fault roughness. As in the first two tests, the sampled mean is significantly higher than the actual parameter value, but the actual value is still within the 95% credible interval.

Information can be learned about MCMC results by inspecting the trace plot and sampled distribution, as well as using convergence metrics to determine if the train converges. According to convergence metrics, in all cases, the MCMC converges. In the three synthetic tests, the model results are all physically plausible. In all instances, the coefficient of static friction is realistically higher than the coefficient of dynamic friction. Also, in each test, the normal stresses are negative and the shear stress is positive, as are actual stresses in compression. For many of the parameters, the sampled mean is very close to the actual value. The model’s sampled distributions for $\sigma_{xx}$ are consistently close to the real parameter value, and the model produces reasonable mean values for the coefficients of friction and the shear stress. As Figure 16 shows, for nearly all values, the actual parameter value lies within the 95% credible interval. Still, there are several issues with the
Synthetic test 3 MCMC results

Fig. 15: The trace plots and posterior distributions for the third synthetic test. For each parameter, \((\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \text{the coefficient of dynamic friction, the coefficient of static friction, and the amplitude of fault roughness})\), the trace plot and the sampled distribution are shown. The trace plot shows the values of the parameter for each step by the MCMC. The histogram shows the sampled distribution produced by the MCMC. The solid black line shows the actual value of the parameter and the dashed black line shows the sample mean value. The sampled distributions for \(\sigma_{xx}\) is close to the actual parameter value. The sampled mean for static friction is exactly the value of the actual parameter. In this test, the sampled value of dynamic friction is outside the 95% credible interval.
MCMC results. The MCMC consistently fails to find a Gaussian distribution for $\sigma_{yy}$, but it still yields distributions for which the actual parameter value lies within the 95% credible interval. Additionally, values for the amplitude of fault roughness are too high in each test. Erratic parameter values in the synthetic tests may result from the MCMC remaining in one part of parameter space and not fully exploring and sampling from the parameter space, or the values may be inaccurately constrained. The MCMC inversion has limitations and can produce inconsistent results, but it is still able to constrain all parameters with a degree of accuracy.
95% Credible intervals for synthetic tests

Fig. 16: Plots of 95% credible intervals for the six parameters in each test. The black bar corresponds to the 95% credible interval. The black square shows the mean value from the MCMC. The red circle shows the actual value. In all but two cases, the actual value lies within the 95% confidence interval.
**Fairview sequence**

The Fairview sequence is an earthquake sequence that occurred in north-central Oklahoma from 2015 to 2016. The largest event was a $M_w$ 5.1 earthquake in 2016, and several other events had moment magnitudes of 4.0 or larger [Goebel et al., 2017]. These earthquakes occur along a pre-existing fault that is optimally-oriented given the regional stress field [Yeck et al., 2016]. These events are triggered by increased pore pressure from an area with high-volume injection wells to the northeast of the fault. The events primarily occur on a 6 km section of a 14 km fault [Yeck et al., 2016], but the 14 km fault may be connected to the other nearby faults (Fig. 17). If this is the case, the potential exists for larger earthquakes [Alt and Zoback, 2017]. The Fairview earthquake sequence is an interesting case study for this method because the earthquakes occur along a single fault, so we assume they have the same stresses, coefficients of friction, and amplitude of fault roughness. These earthquakes nucleate in an area of elevated pore pressure and then rupture along the fault into areas with lower pore pressure.

**Pore pressure modeling**

The pore pressure and pore pressure extent can be modeled from freely available injection well data. The change in pore pressure, $\Delta \rho$, from fluid injection is governed by the diffusion equation:

$$c \Delta^2 \Delta \rho = \frac{\partial \Delta \rho}{\partial t} + \text{source}$$

(3.2)

where $c$ is the hydraulic conductivity and equals $\frac{\kappa}{\mu S}$. $\kappa$ is the intrinsic permeability of rock, $S$ is
Fig. 17: The Fairview sequence earthquakes, located in the center of the map, are located on a partially mapped, reactivated fault. The high-volume injection wells to the northeast of the fault contribute to the increased pore pressure along the fault. The fault is partially unmapped, and may be connected to other faults. The longer rupture length makes larger earthquakes possible.

the storage coefficient, and $\mu$ is the viscosity of the fluid [National Research Council, 2013]. This equation can be solved for the induced pore pressure at a point distance $R$ from an injection well:

$$\Delta \rho = \frac{V\mu Q_0}{\pi R^2 H^2 S \kappa}$$

(3.3)

where $V$ is the volume injected, $H$ is the layer thickness, and $Q_0$ is the injection rate. We assume the contributions to induced pore pressure from each injection well sum linearly, so the induced pore pressure at a given location is the sum of the induced pore pressure from each injection well. Figure 18 shows the modeled pore pressure in the area around the Fairview earthquake sequence. Injection wells to the northeast of the fault increase pore pressure throughout the area.

The pore pressure extent is found by calculating the induced pore pressure on modeled 10
km faults. The pore pressure extent is considered to be the portion of the fault where the induced pore pressure is at least 75% of the induced pore pressure at the hypocenter.

Fig. 18: Map showing the Fairview sequence epicenters and the locations of large injection wells as well as modeled pore pressure. The injection wells are scaled according to the total injection volume in 2016. The high-volume injection wells to the northeast of the Fairview sequence most strongly impact the pore pressure, and earthquakes with hypocentral locations closer to the injection wells have higher pore pressure at the hypocenter.

**Fairview sequence inversion**

With the earthquake magnitude data as well as the modeled pore pressure value and pore pressure extent, we perform an inversion for the tectonic conditions. We use the same method as for the synthetic test. As the earthquake sequence occurs along a single fault, we assume the unknown tectonic parameters are the same for each observed rupture. This method assumes that all earthquakes in the sequence are subjected to the same regional stress tensor.
Fig. 19: Trace plots and histograms for the traces found in by inverting for the unknown parameters. The dashed black line corresponds to the mean value for each histogram. The sampled distributions for $\sigma_{xy}$, static friction, dynamic friction, and the amplitude of fault roughness help constrain the parameters, while the distributions for $\sigma_{xx}$ and $\sigma_{yy}$ are less informative.
Figure 19 shows the results for the inversion using the Fairview sequence data. Figure (a) and (b) show the trace plot and the sampled distribution for $\sigma_{xx}$. In this case, the MCMC samples from the full range of parameter space, but the probability distribution appears to be a uniform distribution. The uniform distribution provides essentially no information about $\sigma_{xx}$. The mean of this distribution is -22.5 MPa. Figure (c) and (d) are the trace plot and normal distributions for $\sigma_{yy}$. The sampled distribution for $\sigma_{yy}$ skews towards higher values. This plot is similar to the sampled distributions for $\sigma_{yy}$ in the synthetic tests. As in the synthetic tests, the results provide little useful information about $\sigma_{yy}$. Here the mean value for the sampled distribution is -21.2 MPa. Subplot (e), the trace plot for $\sigma_{xy}$, shows that the MCMC sampled from the entire range of values, but spent more time in the lower values. Subplot (f) shows the sampled distribution for $\sigma_{xy}$, and it shows a distribution that similarly skews towards the low values. The mean of the distribution is 11.4 MPa. Subplot (g) and (h) are the trace plot and sampled distribution for the coefficient of dynamic friction. The inversion roughly produces a Gaussian distribution with a mean value of 0.45. Subplot (i) and (j) show the trace and sampled distribution for the coefficient of static friction. The samples tend towards higher values, as is expected for the coefficient of static friction. The mean of this distribution is 0.54. Subplots (k) and (l) show the trace plot and sampled distribution for the amplitude of fault roughness. The mean of this distribution, 0.026, is close to the values found in the synthetic tests. In the synthetic tests, however, the mean values found for fault roughness were consistently higher than the actual parameter values.

The results for the inversion of the Fairview sequence are inconsistent: some probability distributions seem to be informative about the actual value of the unknown parameter, but probability distributions for other parameters provide no information about the posterior distribution. The method appears to constrain $\sigma_{xy}$, $\mu_s$, and $\mu_d$, and may provide some information
about the amplitude of fault roughness. On the other hand, the method does not seem to produce any useful results for $\sigma_{xx}$ and $\sigma_{yy}$. Neither the three synthetic tests nor the Fairview sequence inversion yielded useful distributions for $\sigma_{yy}$. There are several explanations for why the inversion for $\sigma_{xx}$ did not yield a useful distribution. First, this component of normal stress may not have a significant impact on the moment magnitude for these ruptures, and it is possible that any value of $\sigma_{xx}$ in the distribution could create the same magnitude in combination with different other parameters. Alternatively, the values of $\sigma_{xx}$ may have varied for the different ruptures in the Fairview sequence, leading to a broad posterior distribution as is shown in subplot (b).

Additionally, the modeled ruptures were nucleated by a large shear stress perturbation, unlike the real earthquakes. The method of rupture nucleation may impact the results. Based on the distributions of $\sigma_{xx}$ and $\sigma_{yy}$, the model seems to prefer higher values of the normal stresses and lower values of the shear stress.
Chapter 4
Discussion

This study develops a new application of an inverse method in which we use MCMC paired with a neural network trained to estimate rupture magnitude. The objective is to estimate the values of unknown parameters related to induced earthquakes. These parameters can then be used for dynamic rupture modeling to estimate the maximum magnitude of an induced earthquake. The method developed in this study provides information about the friction coefficients, shear stress, and fault geometry of ruptures in the Fairview earthquake sequence that cannot be measured by seismic or geodetic methods. In three synthetic tests, the actual value for the majority of parameters falls within the 95% credible interval, and the inverse method finds many of the parameters with a higher degree of accuracy. The artificial neural network approximates the dynamic rupture model and magnitude calculation with 74% accuracy. This study also explores which factors control the magnitude of an induced earthquake. The dynamic rupture model dataset shows that the moment magnitude of the modeled earthquake has essentially no correlation with the pore pressure or pore pressure extent.

Previous research explored the impact of fluid injection on the maximum magnitude of induced earthquakes. Some previous research concluded that fluid injection controls the maximum magnitude, suggesting either that the volume of fluid injected limits the maximum magnitude [McGarr, 2014] or that the reservoir geometry controls the maximum magnitude [Shapiro et al., 2011]. Other studies found that the magnitudes of induced earthquakes are no different statistically than natural, tectonic earthquakes [van der Elst et al., 2016], which suggests that the magnitude depends on tectonic factors.

This study finds that pore pressure does not control the magnitude of induced earthquakes;
instead, pore pressure allows the ruptures to nucleate on pre-existing faults, but tectonics control the propagation and maximum magnitude. The magnitude of the pore pressure is small compared to regional stresses and does not determine rupture propagation. However, fluid injection may impact the maximum magnitude of induced earthquakes because injection controls earthquake nucleation. Based on the Gutenber-Richter law, it is expected that there will be higher magnitude events in a sample containing more earthquakes, but injection does not directly control the rupture magnitude. These results further suggest that induced earthquakes in Oklahoma are tectonic earthquakes that are accelerated by the increased pore pressure rather than purely induced earthquakes that would never have occurred otherwise. These results agree with van der Elst et al. [2016] in finding that induced earthquakes are not distinct from tectonic earthquakes.

This study demonstrates that neural networks can be used to approximate dynamic rupture models in regression as well as classification problems. Neural networks had previously been used to predict whether a rupture would propagate past a barrier in a classification problem [Ahamed, 2018]. They had also been used to accelerate other models in geophysics and reduce computation time [DeVries et al., 2017].

Understanding friction on faults and measuring stresses “in situ” continue to be major problems in geophysics. Tectonic parameters including stresses and coefficients of friction cannot be measured on real faults. More precise prior estimates of the stresses and friction parameters for Oklahoma could be gained from laboratory experiments. Fault roughness could be better estimated through studies of exhumed faults in Oklahoma, if exhumed faults exist. In this study, it is necessary to use uniform distributions as the prior distributions for all parameters. Further research into any of the tectonic parameters for earthquakes in Oklahoma would improve our prior distributions by allowing us to use more accurate normal distributions rather than uniform
distributions, and doing so would produce more accurate posterior distributions for the unknown parameters.

Many of the earthquake sequences in Oklahoma have occurred on mapped or partially mapped faults. These faults can be long enough to host large and potentially dangerous earthquakes. As the Gutenberg-Richter law explains, far more small earthquakes occur than large earthquakes. Due to the limited sample of earthquakes that have occurred in Oklahoma since 2008, it is reasonable that no larger earthquakes have yet occurred even if the tectonic conditions and faults could produce a very large event [van der Elst et al., 2016].

This method can be used to estimate the maximum magnitude of an earthquake along a certain fault. Using real seismic data, as in the Fairview sequence case study, allows us to estimate the stresses, frictions, and fault roughness on a fault. For known and mapped faults, the ruptures can be modeled on the actual fault lengths. These longer faults can host larger earthquakes than the 10 km faults used in the initial rupture modeling. To obtain the input parameters for these models, we will sample from the posterior distribution found by the MCMC.

Because the input parameters are sampled from the posterior distribution, it is necessary to run many models to find a probability distribution for rupture magnitude. As these models will be run on a longer fault, the computation time will increase. Longer faults require larger computational space, and also must be run for more time steps. To do so, the grid spacing must be changed. Doing so creates challenges with respect to scaling the fractal faults. As the current code cuts off short wavelength features, simply scaling up the fault length will alter the fault roughness.

Due to their computational expense, dynamic rupture models have not previously been used in systematic inverse problems. Instead, past inversions using dynamic rupture models have used trial and error methods [Peyrat et al., 2001]. The use of neural networks to approximate
dynamic rupture models makes it possible to use a neural network as the forward model in this study.

While statistical models have limited utility in constraining the likelihood of events that have not yet been observed, dynamic rupture models can estimate the maximum magnitude if certain parameters are known. With the magnitude data, modeled values of pore pressure and pore pressure extent along the fault, and the inversion results for the stresses, the coefficients of friction, and the fault geometry, it is possible to run additional rupture simulations and accurately determine if these conditions are such that a larger earthquake could occur. Rupture simulations run with the actual fault lengths can potentially be used to estimate the maximum magnitudes of induced earthquakes using physics, rather than historical data. The maximum magnitude of a potential earthquake is necessary for accurate hazard estimates. Hazard models for induced earthquakes in Oklahoma are based on statistical methods, which perform poorly on rare events such as major earthquakes. This means that if the potential for a large earthquake exists in Oklahoma, hazard maps may underestimate their likelihood or magnitude. Continued work with this physics-based method can provide additional estimates of seismic hazard.


Townend, John, Mark D. Zoback (2000), How faulting keeps the crust strong, Geology, May 2000; v. 28; no. 5; p. 399–402.


