NOVEL PIN GEOMETRY IMPROVES TURBINE BLADE TRAILING EDGE COOLING

Nadim Reza

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NOVEL PIN GEOMETRY IMPROVES TURBINE BLADE TRAILING EDGE COOLING

by

Nadim Reza

A Dissertation
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy

Major: Engineering

The University of Memphis
August 2021
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Dedication

This dissertation is dedicated to my Major Advisor Prof. Hochstein, my parents, my sister, and brother-in-law who helped, supported, and encouraged me to successfully complete it.
Acknowledgements

The person who should be mentioned first to accomplish this challenge of my life is my Major Advisor Professor John Hochstein. Special thanks and gratitude to him for his outstanding support and guidance to help me achieve this academic milestone. I had been able to handle the ups and downs of this degree because of his experienced advising. It was Professor Hochstein who motivated me to overcome the distraction and disappointment in the path of research. Other committee members Dr. Marchetta, Dr. Foti, Dr. Hagen and Dr. Blanton also inspired me to reach to this goal. Their suggestions, directions and queries kept me on right track to perform this scholarly activity. I received inspirations from my mother, my sister and my brother-in-law who always tried to motivate me to achieve the target. This work was started during my master’s thesis, and I got many useful information from the book “Fundamentals of Fluid Mechanics” which ultimately directed me to extend the work. I am also grateful to the authors of this book.
Abstract


The motivation for the present research is to improve gas turbine efficiency by improving turbine blade trailing edge cooling. State-of-the-art trailing edge cooling deploys an array of pins inside a cavity in the trailing edge through which air passes to carry away heat. A novel pin shape is proposed in which a slot aligned with the bulk flow direction is added to the conventional pin shape in the center of each pin to increase the rate of heat transfer to the air and to decrease the pressure drop across the cavity.

Computational simulations (using ANSYS FLUENT) of cooling flows through pin arrays of fixed pin diameter and fixed stagger angle with solidities of 25%, 35%, 45%, 55%, 65% were conducted for both the conventional circular cross-section and for the slotted cross-section with slot widths of 5%, 10%, 15%, 20%, and 25% of the pin diameter. The relative performance of these geometries was investigated for both fixed flow rate boundary conditions (Reynolds numbers of 6K, 10K, 20K, and 40K) and for fixed static pressure difference across the array (122 Pa and 1,017 Pa).

For every geometry and boundary condition studied in this investigation, the heat transfer rate for the slotted-pin arrays was significantly higher than for the cylindrical pin arrays. With the exception of a single simulation using the narrowest slot width, the pressure drop across the array was lower for the slotted-pin arrays than for the cylindrical pin arrays. A unique optimal geometry cannot be identified because different figures of merit lead to selection of different geometries. Using heat transfer rate per unit volume as the figure of merit, the slotted-pin array resulted in as much as 15% more heat transfer than the comparable round pin array.
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<td>$A$</td>
<td>Cross-sectional area perpendicular to the flow</td>
</tr>
<tr>
<td>$A_{cyl}$</td>
<td>Curved surface of a cylindrical pin</td>
</tr>
<tr>
<td>$A_{in}$</td>
<td>Circular bottom surface of a cylindrical pin</td>
</tr>
<tr>
<td>$A_{in}$</td>
<td>Inlet area</td>
</tr>
<tr>
<td>$A_{\min}$</td>
<td>Minimum cross-sectional area</td>
</tr>
<tr>
<td>$A_{\text{top}}$</td>
<td>Circular top surface of a cylindrical pin</td>
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<tr>
<td>$A_{\text{wet}}$</td>
<td>Wetted surface area</td>
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<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$C_v$</td>
<td>Specific heat at constant volume</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter of pin fin</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter</td>
</tr>
<tr>
<td>$D_{\max}$</td>
<td>Maximum pin diameter</td>
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<tr>
<td>$f$</td>
<td>Friction factor</td>
</tr>
<tr>
<td>$h$</td>
<td>Convective heat transfer coefficient</td>
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<tr>
<td>$H$</td>
<td>Pin height</td>
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<td>$k$</td>
<td>Thermal conductivity of air</td>
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<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate of coolant</td>
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<td>$N$</td>
<td>Number of pin rows</td>
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<tr>
<td>$\text{Nu}$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>Heat transfer rate</td>
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<td>$\text{Re}$</td>
<td>Reynolds number</td>
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Re_{Dh}  Reynolds number based on hydraulic diameter

S  Solidity of pin array

S_{max}  Maximum array solidity

S_L  Longitudinal (stream wise) spacing

S_T  Transverse (span wise) spacing

SW  Slot width

SPD  Static pressure difference

t  Time to travel longitudinal spacing

T  Static temperature

T_b  Bulk average temperature

T_u  Turbulence intensity

T_w  Wall temperature

u  Axial (stream wise) velocity

V_{in}  Velocity at inlet

V_{max}  Maximum average velocity in the minimum cross-sectional area

α  Stagger angle

ρ  Density of air

μ  Viscosity of air

Δp  Pressure drop
1. Introduction

1.1 Overview of gas turbine system

The technology of gas turbines is one of the innovative inventions of twentieth century. The impact of this technology has changed our lifestyle. Aircraft propulsion and generation of electricity are major applications of gas turbine system. It was first utilized in military jet engines towards the end of Second World War. Although electric power production was a focus of its development, this was not an efficient system at the early stage compared to other prime movers used in electricity generation. Gas turbines had drawbacks like fuel inefficiency, extreme noise, and unreliability etc. Continuous development of this technology helped to remove and reduce these problems and it is still developing to strive for higher efficiency. By the early 21st century, gas turbines were capable of outputs up to 500 MW with thermal efficiencies of over 40 per cent and the gas turbine became widely used in power generation [1].

![Simple open cycle gas turbine system](image)

Figure 1.1. Simple open cycle gas turbine system [1]

As shown in Figure 1.1, the compressor, the combustion chamber, and the turbine (rotor) are the main components of a gas turbine system. The compressor increases the density of the incoming air which is then mixed with fuel in the combustion chamber where the mixture is ignited. The products of combustion are then expanded through the turbine to extract power from
the flow before it leaves the system. For electricity generation, power from the turbine is used to drive the compressor and the generator.

1.2 Need for turbine cooling

Improvement of gas turbine efficiency is one of the objectives of this research work. Unlike reciprocating engines, the processes of compression, combustion and expansion do not take place in a single component. The efficiency of individual components contributes to the performance of gas turbine. Key parameters that influence the efficiency are pressure ratio in the compressor and rotor inlet temperature (RIT). Increasing the number of compressor stages result in relatively high outlet pressure which ultimately raises the efficiency. This method of improving efficiency can be applied to power generating units where there is no restriction on weight. It poses a problem for aircraft propulsion system that demands weight to be kept as low as possible. Therefore, other means of improving cycle efficiency must be pursued.

Thermodynamic analysis of the system shows that an increase in RIT increases the gas turbine system efficiency. Numerous blades are mounted on the turbine. The product of combustion is high-temperature and high-pressure gas that flows through the passages between these blades. Turbine firing temperature of large commercial aviation engines is in the range of 1315 – 1650°C whereas the softening temperature of typical blade materials occurs at about 1260°C [2]. This indicates that operating temperatures of modern gas turbines are higher than the melting point of metal. Gas turbines are operated at these higher temperatures with the help of an aggressive cooling of the hot gas path (HGP) components, the use of advanced materials for structural components and protective coatings [2]. The rotating blades are part of the HGP components that are equipped with cooling holes as shown in the Figure 1.2.
In addition to increasing efficiency, these cooling systems extend the turbine blade’s service life. In the absence of a properly designed cooling system, thermal degradation occurs due to high thermal stress which can result in engine failure [4]. Frequent repair, maintenance, and downtime results in financial loss because of interruption in operation.

1.3 Challenges of blade cooling

From a thermodynamic point of view, cooling is a “loss” of energy, and it might appear to be undesirable from that perspective. Because blade cooling enables higher operating temperatures, cooling actually enables an increase in gas turbine thermodynamic efficiency. The primary objective of cooling is heat removal and the secondary objective is to minimize the amount of coolant required. Thus, the target of any cooling system design is enhancement of heat transfer and reduction of coolant flow rate. Turbines may use as much as 20 to 30% of the
compressor air for cooling, purge, and leakage flows, which results in a severe penalty on the thermodynamic efficiency unless the rotor inlet temperature is sufficiently high for the gains to outweigh the losses [2]. Different types of cooling methods are applied to different sections of turbine blade. External cooling and internal cooling are two extensively used methods employed for turbine cooling. The selection of cooling system usually depends on the geometric shape of the section of interest and the flow field in the section. The flow over the airfoil midspan is primarily two dimensional while the flow at the airfoil edges is influenced by the inner hub and outer casing of turbine [5]. Airfoil surfaces contain film cooling holes (Figure 1.2) that are utilized in the external cooling method. On the other hand, internal cooling is accomplished with geometric features like pin fins, ribs, and impingement holes. The design and analysis of cooling schemes is complicated due to the complexity in geometric shapes and the associated flow fields.

1.4 Research objective:

Turbine blade cooling is undoubtedly a challenging task. Trailing edge cooling is particularly challenging because the airfoil shape makes it difficult to provide cooling without jeopardizing the blade’s structural integrity. Although the cooling flow-patch through the airfoil’s mid-section is serpentine, cooling of the trailing edge must be accomplished in a single pass. An array of pins in the trailing edge cooling passage significantly increases the heat transfer rate to the cooling flow by providing many extended surfaces within the passage. The high temperature products of combustion, after being released from combustion chamber, pass over the outer surfaces of the turbine blades. Heat is conducted from both the top surfaces of the blade to the surfaces of the cooling passage from which it is carried away by the coolant. Since the flow follows a tortuous and serpentine path before it reaches the pin array in the cooling passage it is reasonable to assume that the flow in the passage and through the pin array is turbulent. The
The motivation for the present research is to improve turbine blade trailing edge cooling. It is proposed that use of a novel pin with a centered streamwise slot in the middle of an otherwise circular cross-section in the pin array in the blade trailing edge cooling passage will accomplish this objective. The veracity of this assertion is supported by the results of computational simulations using the ANSYS FLUENT commercially available software suite. Validation of these simulations was previous reported [4, 7] using experiment data published [6] for some of the conventional pin geometries of interest in the present investigation.
2. Literature Review

Literature review plays a significant role in research work by documenting the available knowledge on different branches of science. Among the numerous benefits of it, providing chronological record of developments, historical backgrounds, disruptive novel and innovative ideas are most mentionable. Contents of engineering literature are so interesting and intuitive that they can sometimes trigger a new research field or may provide useful guidelines on a research topic. Like other sub-categories of Mechanical Engineering, technological advances of gas turbine systems are also taking place based on research outcomes. The research activities related to gas turbine systems is diverse and vast. Therefore, it is essential to concentrate on a particular area and identify the scope of work. To achieve that, a thorough study of previous work is a useful tool to start the research endeavor. It eliminates the possibility of unnecessary repetition of work and provides a good background of the topic of interest. A target is best selected after the study of relevant literature.

2.1 Geometric features of pin array:

As discussed in section 1.4, the current study aims to improve cooling design for the gas turbine blade trailing edge. Today’s conventional design employs an array of circular cross-section pins extending across a single-pass internal cavity through with cooling air flows. Array solidity, the fraction of volume occupied by pins in the array, is only one of several important parameters used to describe the array. The geometric and phenomenological complexities of the research of interest require clear definitions and specifications to facilitate an effective description of the work and discussion of the results. This chapter begins with a presentation of relevant definitions and conventions that leads to a review of the literature to establish the state-of-the-art on which the present research is built.
2.1.1 Aspect ratio of pin:

Aspect ratio, (pin height/diameter), is an important parameter of design because it affects the amount of heat transfer by controlling the wetted surface area. More specifically, it determines the increase or decrease of exposed surface area due to the incorporation of pins. When a pin that spans the channel is placed inside a rectangular channel, two circular areas are removed from the wetted surface: \( \frac{\pi D^2}{4} + \frac{\pi D^2}{4} = \frac{\pi D^2}{2} \). At the same time, the exposed area is increased by the surface area of the pin: \( \pi DH \). A comparison of these two areas determines whether the exposed area for coolant is increased or reduced. When the two areas are equal, we obtain,

\[
\pi DH = \frac{\pi D^2}{2} \quad \text{or} \quad \frac{H}{D} = \frac{1}{2}
\]

Therefore, any pin with an aspect ratio greater than 0.5 increases the surface area in the cooling channel. Pins of low aspect ratio are called short pins and their height to diameter ratio lies between 0.5 and 4. Jaswal et al. [6] mentioned that a typical pin has height to diameter ratio \((H/D)\) of approximately one. The present investigation has focused on pins of the same diameter \((D = 2.012 \text{ cm})\) and aspect ratio \(\left(\frac{H}{D} = 0.95\right)\) as were studied in Jaswal’s experiments.

2.1.2 Solidity (S) of pin array:

For an array of pins, solidity is defined as the ratio of volume occupied by the pins to the total volume in the array. For cylindrical pins, the volume ratio is the same as the two-dimensional ratio of the total area occupied by the pins to the area of the wall to which they attach. Solidity describes how closely the pins are packed. In a particular array, it combines the effect of pin diameter and pin spacing along stream wise and span wise directions. Although solidity is a useful parameter, it does not uniquely describe the geometry of the pin array, but it
can be computed from geometric features that do uniquely describe array geometry. Consider the fixed diameter pin array depicted in Figure 4. The selection of a representative area should be in a way such that a multiple of that area equals total end wall area. For example, such area can be a rectangle whose edges pass through the centers of pin bases as depicted in Figure 2.1 by dashed line. Various parameters relevant to staggered array e.g., pin diameter (D), stagger angle (α), longitudinal spacing (S_L), transverse spacing (S_T) are shown in Figure 4. The portions of pin bases contained in the rectangular section add up to an area of a half circle. The dimensions of rectangle are S_L and \( \frac{1}{2} S_T \) respectively along the direction of flow and normal to the flow direction. For an array of cylindrical pins, solidity (S) can be defined as

\[
S = \frac{\text{pin base area}}{\text{rectangular area}} = \frac{1}{2} \left( \frac{\pi D^2}{4} \right) = \frac{\pi D^2}{4 S_L S_T}
\]

where D is pin diameter and S_L and S_T are pin center to center distances in the stream wise (longitudinal) and span wise (transverse) direction, respectively. The expression above clearly shows that solidity is a function of these three parameters. The longitudinal spacing and transverse spacing can be normalized by dividing by pin diameter, suggesting a rearrangement of the equation of interest:

\[
S = \frac{\pi}{4 \frac{S_L S_T}{D D}}
\]
2.1.3 **Stagger angle (α):**

The stagger angle (α) shown in Figure 2.1, can be computed from the depicted geometry:

\[
\tan \alpha = \frac{1}{2} \frac{S_T}{S_L} = \frac{S_T}{2S_L}
\]

Thus, for a staggered array, this angle determines the relative spacing of pin elements in the longitudinal and transverse directions. A stagger angle of 38.66 has been used for all arrays in this study because that is the stagger angle of the array of the experiment used for code validation. [6]. In general, higher solidity increases the rate of heat transfer because the extended surfaces provide more exposed surface for heat transfer to the cooling flow. In competition with this benefit is the higher resistance to flow through the passage because of the presence of so much exposed surface. Therefore, determination of an optimal array geometry must balance these competing influences.

2.2 **Conditions for maximum solidity of inline pin array:**

Although an inline array is not considered in this research work, a little insight into it provides a basis for comparing similar parameters of staggered array. An inline array can be categorized by the ratio of \(S_T\) to \(S_L\).

1. \(S_T\) and \(S_L\) are equal \((S_T = S_L \text{ or } \frac{S_T}{S_L} = 1)\)

2. \(S_T\) is greater than \(S_L\) \((S_T > S_L \text{ or } \frac{S_T}{S_L} > 1)\)

3. \(S_T\) is less than \(S_L\) \((S_T < S_L \text{ or } \frac{S_T}{S_L} < 1)\)

For fixed values of \(S_L\) and \(S_T\), or in other words, if the positions of the pins are kept unchanged, solidity can be maximized by increasing the pin diameter until neighbor pins touch. In the
reverse case, if the pin diameter $D$ is fixed, solidity can be increased by reducing the values of $S_L$ and $S_T$ until neighbor pins touch.

**Case 1: $S_T = S_L$**

Maximum pin diameter yields lowest possible values of spacing ratios when $S_L$ and $S_T$ are fixed. Therefore, solidity is maximum when the spacing ratios are minimum, or pin diameter is maximum. Since in this case, $S_L$ and $S_T$ are equal, maximum pin diameter is found to occur when a pin touches neighbor pins in both the stream wise and span wise direction. Such an inline arrangement is shown in Figure 2.2. From this figure, it is found that maximum diameter ($D_{max}$) is equal to both $S_L$ and $S_T$ for maximum solidity. Thus, the minimum value of either of the spacing ratios is equal to one when longitudinal spacing and transverse spacing are equal $S_L = S_T$. Therefore,

$$\frac{S_L}{D_{max}} = \frac{S_T}{D_{max}} = 1$$

and maximum solidity is

$$S_{max} = \frac{\pi}{4} \frac{1}{S_L S_T} = \frac{\pi}{4} \frac{1}{(1)(1)} = \frac{\pi}{4} = 0.7854 = 78.54\%$$
Case 2: $S_T > S_L$

The transverse spacing is greater than longitudinal spacing ($S_T > S_L$). At a maximum diameter of $D_{\text{max}} = S_L$, neighbor pins touch in the streamwise direction (Figure 2.3).

Thus, the maximum solidity is

$$S_{\text{max}} = \frac{\pi}{4} \frac{1}{S_L} \frac{S_T}{S_T} \frac{D_{\text{max}}}{D_{\text{max}}} = \frac{\pi}{4} \frac{1}{(S_T/D_{\text{max}})} = \frac{\pi}{4} \frac{S_T}{S_T} \frac{D_{\text{max}}}{D_{\text{max}}}$$

$$\frac{S_T}{S_L} > 1 \rightarrow \frac{S_T}{D_{\text{max}}} > 1 \rightarrow S_{\text{max}} < \frac{\pi}{4}$$

Therefore, for an array with $S_T > S_L$, the maximum solidity is a function of $\frac{S_T}{D_{\text{max}}}$ and the maximum solidity is always less than 78.54%.

Case 3: $S_T < S_L$

The maximum diameter is equal to the transverse pin spacing which causes neighbor pins to touch in the transverse direction.
Therefore, with the condition $\frac{S_L}{D_{\text{max}}} > 1$, the maximum solidity is:

$$S_{\text{max}} = \frac{\pi}{4} \frac{1}{\frac{S_L}{S_T}} = \frac{\pi}{4} \frac{1}{\frac{S_L}{D_{\text{max}}}}(1) = \frac{\pi}{4} \frac{S_L}{D_{\text{max}}} < \frac{\pi}{4}$$

The results of this study of the geometry of an inline array are summarized in the following table.

**Table 2.1 Summary of conditions for maximum solidity of inline array**

<table>
<thead>
<tr>
<th>Array Condition</th>
<th>$\frac{S_L}{D}$</th>
<th>$\frac{S_T}{D}$</th>
<th>Solidity, $S$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{S_T}{S_L} = 1$</td>
<td>$\geq 1$</td>
<td>$\geq 1$</td>
<td>$\leq 78.54%$</td>
<td>$D_{\text{max}} = S_L = S_T$, $S_{\text{max}} = 78.54%$</td>
</tr>
<tr>
<td>$\frac{S_T}{S_L} &gt; 1$</td>
<td>$\geq 1$</td>
<td>$&gt; 1$</td>
<td>$&lt; 78.54%$</td>
<td>$D_{\text{max}} = S_L$, $S_{\text{max}} = f\left(\frac{S_T}{D_{\text{max}}}\right)$</td>
</tr>
<tr>
<td>$\frac{S_T}{S_L} &lt; 1$</td>
<td>$&gt; 1$</td>
<td>$\geq 1$</td>
<td>$&lt; 78.54%$</td>
<td>$D_{\text{max}} = S_T$, $S_{\text{max}} = f\left(\frac{S_L}{D_{\text{max}}}\right)$</td>
</tr>
</tbody>
</table>
2.3 Conditions for maximum solidity of staggered pin array

For a typical staggered pin array, the transverse position of pin centers in one row is midway between the pin centers of the preceding row. In such an arrangement the ratio of $S_T$ to $S_L$ is uniquely determined with specification of the stagger angle, $\alpha$. In case of the extreme values of $\alpha$ (i.e., $\alpha = 0, \frac{\pi}{2}$), the array is an inline array. Two cases can be considered to divide the domain of $\alpha$ to find the maximum solidity and the conditions for it:

Case 1: $0 < \alpha < \frac{\pi}{3}$, and Case 2: $\frac{\pi}{3} \leq \alpha < \frac{\pi}{2}$.

Case 1: $0 < \alpha < \frac{\pi}{3}$

When the stagger angle is between zero and $\frac{\pi}{3}$, pin diameter can be increased to reach highest solidity until the pins touch in the transverse direction. Maximum diameter then becomes equal to transverse spacing $S_T$ i.e., $D_{\text{max}} = S_T$. In the longitudinal direction, the pins can be brought closer when pins touch diagonally in which case $\alpha = \frac{\pi}{6}$ as shown in Figure 2.5 (b). Coolant flow is entirely blocked by the presence of pins in this case.
Figure 2.5. Staggered pin array (a) $0 < \alpha < \frac{\pi}{3}$ (b) $\alpha = \frac{\pi}{6}$

It is obvious from the figure,

$$AD = \frac{S_T}{2}, \quad BC = 2BD = 2S_L, \quad \tan \alpha = \frac{AD}{BD} = \frac{S_T}{2S_L}$$

For maximum solidity,

$$\frac{S_T}{D_{\text{max}}} = 1, \quad \frac{S_L}{D_{\text{max}}} = \left( \frac{S_L}{S_T} \right) \left( \frac{S_T}{D_{\text{max}}} \right) = \left( \frac{1}{2\tan \alpha} \right) = \frac{1}{2 \tan \frac{\pi}{6}} = \frac{\sqrt{3}}{2}$$

$$S_{\text{max}} = \frac{\pi}{4} \frac{1}{S_L} \frac{S_T}{D_{\text{max}}} = \frac{\pi}{4} \frac{1}{\left( \frac{\sqrt{3}}{2} \right)} = \frac{\pi}{2 \sqrt{3}} = 0.907 = 90.7\%$$
In the domain of $\alpha$ ranging between zero and $\frac{\pi}{3}$, maximum solidity occurs when,

$$\alpha = \frac{\pi}{6} \text{ or } \frac{S_T}{S_L} = 2\sqrt{3}, \text{ then } \frac{S_L}{S_{max}} = \frac{\sqrt{3}}{2}, \frac{S_T}{S_{max}} = 1 \text{ and maximum solidity is } 90.7\%.$$

**Case 2: $\frac{\pi}{3} \leq \alpha < \frac{\pi}{2}$**

When stagger angle varies between $\frac{\pi}{3}$ and $\frac{\pi}{2}$, maximum solidity occurs when the pins touch its neighboring pins in the diagonal, longitudinal and transverse direction. This special situation is shown in Figure 2.6 (a). Since the three circles touch each other, pin centers essentially create an equilateral triangle for which stagger angle is equal to $\frac{\pi}{3}$. In this type of configuration, backflow will occur since the pins completely restricts the coolant flow.

From the Figure 2.6 (b),

$$AD = \frac{S_T}{2}, \ BC = 2BD = 2S_L, \ \ \ tan\alpha = \frac{AD}{BD} = \frac{S_T}{2S_L}$$

![Diagram showing staggered pin array with equations and descriptions.](attachment:image.png)
For maximum solidity,

\[
\frac{S_L}{D_{\text{max}}} = \frac{1}{2} \frac{S_T}{D_{\text{max}}} = \left( \frac{S_T}{S_L} \right) \left( \frac{S_L}{D_{\text{max}}} \right) = \left( 2 \tan \alpha \right) \left( \frac{1}{2} \right) = \frac{\pi}{3} = \sqrt{3}
\]

\[
S_{\text{max}} = \frac{\pi}{4} \frac{1}{S_L} \frac{S_T}{D_{\text{max}}} D_{\text{max}} = \frac{\pi}{4} \left( \frac{1}{2} \right) \left( \frac{1}{\sqrt{3}} \right) = \frac{\pi}{2 \sqrt{3}} = 0.907 = 90.7\%
\]

In the domain of \( \frac{\pi}{3} \leq \alpha < \frac{\pi}{2} \), maximum solidity occurs when, \( \alpha = \frac{\pi}{3} \) or \( \frac{S_T}{S_L} = 2\sqrt{3} \), then \( \frac{S_L}{D_{\text{max}}} = \frac{1}{2} \frac{S_T}{D_{\text{max}}} = \sqrt{3} \) and maximum solidity is 90.7%.

<table>
<thead>
<tr>
<th>Stagger angle, ( \alpha )</th>
<th>( \frac{S_L}{D_{\text{max}}} )</th>
<th>( \frac{S_T}{D_{\text{max}}} )</th>
<th>Solidity, ( S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 &lt; \alpha &lt; \frac{\pi}{3} )</td>
<td>( \sqrt{3} ) ( 2 )</td>
<td>( 1 )</td>
<td>( \leq 90.7% )</td>
</tr>
<tr>
<td>( \frac{\pi}{3} \leq \alpha &lt; \frac{\pi}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \sqrt{3} )</td>
<td>( \leq 90.7% )</td>
</tr>
</tbody>
</table>

2.4 Literature review:

A large amount of research work has been carried out on turbine cooling since it leads to performance gain in gas turbine system. The amount of literature on the cooling of turbine blade is itself huge. Different types of cooling methods are applied to different sections of blade to achieve desired cooling. For example, impingement cooling is suitable for leading edge of rotor blade [8]. This region is subjected to high-speed rotation. There are many tiny holes on the blade to apply a jet on the surfaces of the blade. Impingement of the jet through these holes is an efficient method of cooling for the first stage of vanes. Rib turbulators are employed inside the midsection internal cooling passage to increase the rate of heat transfer to the cooling flow. The
design of cooling for the trailing edge is challenging because of space limitations and aerodynamic shape. Structural integrity under high thermal load is a major concern. Cooling by air flow through a single pass cooling passage containing a pin array to enhance heat transfer is today’s typical design. These arrays use short pins due to geometric constraints [10].

Solidity and stagger angle are very useful parameters for describing the pin array. As with a typical extended surface deployed to enhance heat transfer, providing both a conduction path for heat outward from the passage walls and a greater surface exposed to the cooling flow result in significantly more heat transfer than would occur if the pin array was not present in the passage. Metzger et al. [11] studied the effect of longitudinal spacing using an arrangement of circular cross-section pins with $S_T/D = 2.5, H/D = 1$, and longitudinal spacing $S_L/D$ ranging from 1.0 to 5.0.

<table>
<thead>
<tr>
<th>Test No</th>
<th>$S_L/D$</th>
<th>$S_T/D$</th>
<th>$H/D$</th>
<th>Solidity, $S$ (%)</th>
<th>$S_T/S_L$</th>
<th>Stagger Angle, $\alpha$ (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>2.5</td>
<td>1.0</td>
<td>31.4</td>
<td>2.50</td>
<td>68.2</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
<td>1.0</td>
<td>12.6</td>
<td>1.00</td>
<td>45.0</td>
</tr>
<tr>
<td>3</td>
<td>3.0</td>
<td>2.5</td>
<td>1.0</td>
<td>10.5</td>
<td>0.83</td>
<td>39.7</td>
</tr>
<tr>
<td>4</td>
<td>5.0</td>
<td>2.5</td>
<td>1.0</td>
<td>6.3</td>
<td>0.50</td>
<td>26.6</td>
</tr>
</tbody>
</table>

Experimental data was used to propose a heat transfer correlation with a typical mathematical structure.

$$\text{Nu}_D = 0.135 \text{Re}_D^{0.69} \left(\frac{S_L}{D}\right)^{-0.34} \quad \text{for } 10^3 < \text{Re}_D < 10^5$$
Both longitudinal spacing and stagger angle were varied to produce pin arrays of different solidities. In this process, transverse distance remained constant as the longitudinal spacing was increased, in a different stagger angle and solidity for the new array. The investigated range of solidity is less than 35% while the stagger angle varied from 25 to 70 degrees.

The effect of stream wise spacing and span wise spacing on heat transfer and pressure loss of short pin arrays were studied by Lawson et al. [10]. Two values for each of the two spacing ratios were chosen and the height of pin was equal to its diameter for all the cases. The parameter space along with corresponding solidities and stagger angle are summarized in the following table.

<table>
<thead>
<tr>
<th>Test No</th>
<th>$S_L/D$</th>
<th>$S_T/D$</th>
<th>$H/D$</th>
<th>Solidity, $S$ (%)</th>
<th>$S_T/S_L$</th>
<th>Stagger Angle, $\alpha$ (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.73</td>
<td>2.0</td>
<td>1.0</td>
<td>22.7</td>
<td>1.16</td>
<td>49.2</td>
</tr>
<tr>
<td>2</td>
<td>3.46</td>
<td>2.0</td>
<td>1.0</td>
<td>11.3</td>
<td>0.58</td>
<td>30.0</td>
</tr>
<tr>
<td>3</td>
<td>1.73</td>
<td>4.0</td>
<td>1.0</td>
<td>11.3</td>
<td>2.31</td>
<td>66.6</td>
</tr>
<tr>
<td>4</td>
<td>3.46</td>
<td>4.0</td>
<td>1.0</td>
<td>5.7</td>
<td>1.16</td>
<td>49.2</td>
</tr>
</tbody>
</table>

Four test cases provided three different solidities and three different stagger angles where maximum solidity of cylindrical pin array is less than 25%. Case 1 and case 4 have the same stagger angle but solidity of case 1 is four times that of case 4. This had been accomplished by reducing transverse spacing to one-half and doubling the longitudinal spacing ratio. Case 1 was found to perform better than case 4 in terms of heat transfer. A comparison of case 2 and case 3 revealed that the spacing ratios have been changed in such a way so that the resulting solidities
are same whereas stagger angles were changed. It was reported that the friction factor is much more strongly dependent on the transverse spacing of the pins in the array than on their longitudinal spacing. The best thermal performance was found in the array of tightest longitudinal and transverse spacing.

The computational simulations of Rao et al. [12] revealed that adding dimples to the walls of the trailing edge cooling passage increased the rate of heat transfer to the colling fluid. The Nusselt number increase with increasing dimple depth but so did the friction factor. The experiments of Kirsch et al. [13] were conducted to study the influence of altering a circular pin array geometry by removing four rows in the middle of the array or by inserting larger non-circular pins in the fourth row of the array. The flow returned to fully developed channel flow in the gap between pin rows. The heat transfer rate increased downstream of the fourth row containing the larger pins of non-circular geometry.

Ames et al. [6] studied the heat transfer and pressure drop characteristics in constant height channel, and in a converging channel, equipped with an array of conventional circular pins and with an array of diamond shaped pins. A high solidity (45%) pin array for both types of pins was tested for flows with a Reynolds number ranging from 3,000 to 60,000. The friction factor for flow through both the constant height channel and the converging channel was reported to be comparable but the heat transfer rate for the converging channel was 6% lower than the rate for the constant height channel.

2.5 Scope of research:

The literature review revealed that studies typically varied both solidity and stagger angle from case to case in the search for a better understanding of the heat transfer in a typical gas turbine blade trailing edge cooling passage. Although a high solidity conventional pin array of
45% was studied [6], most arrays studied had a solidity of less than 35%. This review suggested that it might be useful for the present study to compare the cooling effectiveness of arrays of fixed stagger angle but different solidities and to extend the range of investigation to arrays of higher solidity than had been studied. To investigate the relative cooling performance of conventional solid pin arrays and slotted pin arrays, a set of arrays were defined with a stagger angle of 38.66° for a range of solidities: 25%, 35%, 45%, 55%, 65%. A slot to the pin adds an additional parameter to the design space. Therefore, for each solidity the performance of slotted pin arrays of slot widths equal to 5%, 10%, 15%, 20%, and 25% of pin diameter was investigated. Array geometries of both conventional pins and slot pins are shown in Figure 2.7. The stagger angle is fixed and there are eight pin rows for all the array configurations. All arrays are shown to scale. Although the computational domain for this research extended further upstream and downstream, those portions of the domain have been truncated to better reveal the differences between the array geometries. Thus, the pin array is more focused and the relative spacing of various solidities become apparent. No geometry is shown, and no simulations were conducted for arrays with a solidity of 65% and slot widths of 0.20D and 0.025D because these specifications result in the overlap of adjacent pins.
Figure 2.7. Conventional and slot pin geometry of different solidity and different slot widths.
The blade trailing edge is thin leaving little space for an internal flow cooling passage. It is observed from Figure 2.7 that a relatively high solidity pin array is a more compact array configuration that occupies less space than that of a low solidity array. For example, the space required to accommodate a 25% solidity array is approximately 2.5 times greater than is required for a 65% solidity array. For a specific solidity, the size of the array is fixed for all pin geometries because the slot formed by splitting the solid pin in half and moving the halves away from each other. The pin center is therefore the same for all slot widths as is the solid volume within the array.

Investigation of the influence on the thermal performance of the cooling passage of each independent variable, throughout their ranges as defined in the preceding paragraphs, requires extraction of data from a large number of simulations and interpretation of that data. Inclusion of all of that information in the body of this dissertation would make it very large and damage the readability of it. Therefore, the body contains only enough sample data to show the influence of interest and to draw relevant conclusions. For example, if all arrays of the same solidity show a similar relationship between Reynolds number and Nusselt number, a figure showing this relationship for a representative solidity is included in the body and figures for other solidities are included in Appendices C and D.
3. Research Methodology

3.1 Computational model

Computational simulations of the Ames experiments [6] include eight rows of pins with the same array geometry as was used for the experiments. The computational domain is a streamwise slice of the array for which symmetry conditions along the sides of the slice present at boundary simulating an array of infinite width.

![Three-dimensional model of 35% solidity with solid cylindrical pin.](image)

The 3D model geometry consists of inlet, outlet, side walls, eight rows of cylindrical pins, and top and bottom walls as shown in Figure 3.1. An upstream distance of 5D and a downstream distance of 5D are used before and after the pin array, respectively where D is the pin diameter. The model shown in Figure 3.1 is referred to as three-dimensional full width geometry. A half width geometry is created by splitting along the surface at mid width as shown in Figure 3.2.
The side walls have the same symmetry boundary condition for full width and half width geometries. The representative 3D models can be further simplified based on assumptions and idealizations. If end effects are neglected at top wall and bottom walls, this 3D model geometry becomes a 2D surface which imposes much smaller computational demands for simulation of the processes of interest than would be imposed by a 3D model. Like the three-dimensional models, 2D surface geometries can also be full width and half width. The model geometries for slotted pin arrays have been created in a similar fashion.

Figure 3.2. Split half width model from the 3D full width model (Solidity = 35%).

Figure 3.3. Two-dimensional full width and half width geometries (Solidity = 35%).
3.2 Validation of simulations

From the previous paragraphs, it can be concluded that four different types of model geometries have been created to simulate flow through non-slotted and slotted pins. These are:

1. Three-dimensional Full Width Geometry (3DFWG)
2. Three-dimensional Half Width Geometry (3DHWG)
3. Two-dimensional Full Width Geometry (2DFWG)
4. Two-dimensional Half Width Geometry (2DHWG)

Simulation results using all the four computational domains were conducted to validate the computational simulations using the experimental data of [6]. As previously reported [4], simulations using the commercial ANSTS FLUENT software suite were in good agreement with the Ames data. The simplest among them is the last one (2DHWG) requires the least computational time and resources for a simulation. Therefore, after a step-by-step validation from complex 3D geometry to simple 2D geometry, 2DHWG type has been used in most of the simulations reported in the present research.

ANSYS FLUENT is a widely used software package applied to numerous flow visualization and flow modeling situations. It has been used in this computational study because a campus license was readily available for execution on the university’s High-Performance Computing (HPC) platform. Several turbulence models are available in this software. Validation simulations were successful with the k-epsilon model, the k-omega model, and the LES model [4]. After weighing the advantages and disadvantages of the models it was decided to use the k-epsilon model with an enhanced solid wall boundary treatment for the simulations in the present research.
Two kinds of flow boundary conditions are applied to simulate the flows of interest. The literature typically reports the thermal performance of the cooling passage as a function of the Reynolds number in which the pin diameter is the characteristic length and the characteristic velocity is the mass average velocity through a transverse plane in the array for which the area open to flow is a minimum, $v_{\text{max}}$. The minimum area open to flow can be computed from specification of the pin diameter, stagger angle, and solidity. The ratio of this minimum flow area to the cross-sectional area at the channel inlet is used to compute the value of the inlet velocity boundary condition used to simulate flow through the cooling passage with a desired Reynolds number. The ratio of the inlet area to this minimum flow area in the array is a function of pin diameter, stagger angle, and array solidity. Therefore, the flow rate through arrays of different solidities will be different even though the same Reynolds number is used to characterize flow through both of the arrays.

Some simulations have been conducted to produce a specified static pressure difference across the channel. The motivation is to better understand the relative cooling performance for a typical turbine design process in which the pressure difference available to drive the cooling flow through the channel is imposed by other choices in the gas turbine system’s design. Symmetry flow and thermal boundary conditions are applied at the side walls (3D geometry) or line surfaces (2D geometry) parallel to the direction of flow. A no-slip boundary condition has been imposed at all solid surfaces. For all simulations, the inlet flow temperature is 300 K, and all solid surfaces have a specified temperature boundary condition of 400K.

Among many different parameters, heat transfer and pressure drop are the two most significant items in the study of different pin arrays. Following standard practice, non-dimensional parameters like Reynolds number (Re), Nusselt number (Nu) and friction factor (f)
have been used to characterize the performance of the arrays. Of primary importance is the amount of heat that can be transferred with the constraints imposed by the geometry of a gas turbine blade trailing edge. In addition to Nusselt number, other relevant parameters associated with heat transfer have examined such as heat transfer rate per unit height, heat transfer rate per unit volume, and heat transfer per unit mass flow. As is standard practice in the literature, the characteristic length used in the reported Reynolds numbers and Nusselt numbers is the pin diameter. The influence of Reynolds number of the cooling performance of the arrays has been studied using a specified inlet velocity boundary condition to simulate flows with Reynolds numbers of 6,000, 10,000, 20,000, and 40,000. A specified static pressure difference of 122 Pa and 1017 Pa was imposed across the channel for the two “design cases.” A pressure difference of 122 Pa was chosen because that is the pressure difference for the simulation with Re= 6,000 and an array of solid pins with a solidity of 65%. A difference of 1017 Pa matches the pressure difference for the simulation with Re= 40,000 and a solid pin array with a solidity of 45%.

An approximate value for friction factor is computed as a figure of merit because lower values will result in increased flow through the passage and therefore increased heat transfer from the passage. Although the flow through the first few rows is still developing, an average friction factor per row is computed to provide an estimate of the influence on the mass flow rate of adding or deleting rows from the array.

3.3 Sample calculation procedure:

Because the characteristic length for both the Reynolds number and Nusselt number is the pin diameter D, a value of 2.012 cm was used for all pins in all arrays in all simulations in this study. The pin array geometry is uniquely defined with specification of the pin diameter, D,
the array solidity, $S$, and the stagger angle, $\alpha$. Values for the longitudinal spacing, $S_L$, and the transverse spacing, $S_T$, can be computed as follows.

Figure 3.4 shows the geometric relationship between $\alpha$, $S_L$, and $S_T$.

The following table presents values for the longitudinal pitch ($S_L$) and transverse pitch ($S_T$) for the solidity values
included in the parameter space of this study. These values are used for both the solid conventional pin arrays and novel slotted pin arrays.

Table 3.1 Longitudinal and transverse spacing for different solidities.

<table>
<thead>
<tr>
<th>Solidity</th>
<th>0.25</th>
<th>0.35</th>
<th>0.45</th>
<th>0.55</th>
<th>0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_L (cm)</td>
<td>2.80</td>
<td>2.38</td>
<td>2.10</td>
<td>1.90</td>
<td>1.75</td>
</tr>
<tr>
<td>S_T (cm)</td>
<td>4.51</td>
<td>3.81</td>
<td>3.36</td>
<td>3.04</td>
<td>2.80</td>
</tr>
</tbody>
</table>

A sample calculation to determine values for longitudinal and transverse spacing is shown in Appendix A. After running the simulation, the step-by-step procedure to determine Nusselt number and friction factor is described in the next sub-sections.

3.3.1 Step 1: Inlet Velocity (V_in)

The characteristic velocity used in the Reynolds number, V_max, is the mass average velocity computed using the area open to flow between adjacent pins in a pin row. For example, V_max is calculated using the defining equation of Reynolds number Re_D:

\[ V_{\text{max}} = \frac{Re_D \times \mu}{\rho \times D} \]

where, \( \mu \) and \( \rho \) are coolant viscosity and density, respectively, and \( D \) is the pin diameter.

Figure 3.5. Top view of part of the full width 2D model geometry of no-slot solid pin array.
For full width model geometry,

\[
\frac{A_{\text{min}}}{A_{\text{inlet}}} = \frac{(S_T - D)H}{S_T H} = \frac{(S_T - D)}{S_T}
\]

where \(H\) is the height in the third dimension.

For the half width model geometry,

\[
\frac{A_{\text{min}}}{A_{\text{inlet}}} = \frac{(S_T - D)H}{\frac{S_T}{2} H} = \frac{(S_T - D)}{S_T}
\]

For slotted pin configuration, the minimum cross-sectional area is distributed because of the slot. For each row, this area consists of three parts, the summation of those is the area open to flow. This area is still the same as it is for solid pin array since the slot is introduced by moving the two half cylindrical pins away from each other in the crossflow direction and without losing any pin mass or changing the solidity of array.
\[ A_{\text{min}} = (S_T - D)H \]

Thus, the area ratio remains identical for no slot and slot configuration whether it is full width or half width model geometry.

\[ \frac{A_{\text{min}}}{A_{\text{inlet}}} = \frac{(S_T - D)}{S_T} \]

Inlet velocity is determined by applying the continuity equation between inlet and minimum cross-section area,

\[ V_{\text{in}} A_{\text{inlet}} = V_{\text{max}} A_{\text{min}} \]

\[ \Rightarrow V_{\text{in}} = \frac{A_{\text{min}}}{A_{\text{inlet}}} \times V_{\text{max}} \]

\[ \Rightarrow V_{\text{in}} = \frac{(S_T - D)}{S_T} \times V_{\text{max}} \]

A table included in Appendix B contains the maximum average velocity, the inlet velocity, and values of parameters required for the computational simulation’s turbulent flow model for each array simulated in the present research.

3.3.2 Step 2: Mass flow rate (\( \dot{m} \))

![Figure 3.8. Planes and regions in the half width slotted pin geometry.](image)

Cutting planes that pass through the center of each pin row are defined to enable computation of integral variables such as mass flow rate and Nusselt number. An upstream plane is specified one longitudinal pitch upstream from the first-row pin center as depicted in the
Figure 3.8 and a downstream plane is specified one longitudinal pitch downstream of the last-row pin center, respectively. These planes (p0, p1, p2, p3, p4, ...) are used to conduct a fundamental check of the simulation by computing the mass flow rate through each. Unless there is almost perfect agreement between all values computed, there must be an error in the simulation. For example, for the 2D-simulation of flow through half-width geometry, the mass flow rate per unit height through each plane should agree with the basic algebraic computation for flow at the inlet:

\[ \dot{m} = \rho \times \frac{S_T}{2} \times V_{in} \]

### 3.3.3 Step 3: Heat transfer rate

To determine the rate of heat transfer to the fluid within a region bounded by the cutting planes, the bulk average fluid temperature at each plane is computed. The mass flow rate through a cutting plane of unit height is:

\[ \text{Mass flow rate} = \dot{m} = \int \rho u (dy \cdot 1) = \int \rhoudy \]

The energy per unit mass of flow through the cutting plane is:

\[ \text{Energy per unit mass} = C_v T \]

The energy flow rate through the cutting plane is:

\[ \text{Energy flow rate} = \frac{\text{Energy}}{\text{time}} = \frac{\text{Mass}}{\text{time}} \times \frac{\text{Energy}}{\text{Mass}} = (\int \rho u C_v T dy) \]

A single bulk average temperature can be defined to compute heat transfer to the fluid as it passes through a region in the channel.

\[ \dot{m} C_v T_b = (\int \rho u dy) C_v T_b = (\int \rhoudy) (C_v T) \]

\[ \Rightarrow \text{Bulk Avg. Temp} = T_b \equiv \frac{\int \rho u C_v T dy}{\int \rho u C_v dy} = \frac{\int uT dy}{\int u dy} \]
The custom field function feature of Fluent is utilized to calculate \( T_b \) at each of the planes. The heat transfer rate per unit height becomes,

\[
\frac{\dot{Q}}{H} = \frac{\dot{m}}{H} \times C_p \times \Delta T_b
\]

Where, \( \Delta T_b \) is the bulk average temperature difference between two adjacent planes by which the region is bounded.

3.3.4 Step 4: Wetted surface area

The wetted surface area is computed per unit height for a two-dimensional geometry because this value will be required to compute the convection coefficient that is itself required to compute the Nusselt number. It has been mentioned in section 3.2 that four types of model geometries have been considered in the validation process. The expression of wetted surface area assumes different forms in accordance with the model geometry. For each case, the formula of wetted surface area of a typical row is shown in the Table 3.2. For the three-dimensional model geometry, the wetted surface for a typical row consists of three areas – top wall, bottom wall, and curved surface of cylindrical pin. Top or bottom surface area is found by subtracting the circular area from the rectangular area whose edges are \( S_L \) and \( S_T \). Two-dimensional model geometries are either the top view or the bottom view of corresponding three-dimensional models. In the two-dimensional case, only the curved surface of the pin is a wetted surface. Since the height of pin is not considered in 2D geometries, the wetted surface is computed on a per unit height basis. The curved surface of a cylindrical pin in 3D is \( \pi DH \), which becomes \( \pi D \) for the 2D models. For the half width model, each row contains half of a pin. Therefore, the wetted surface area per unit height is half of the value for full width geometries.
Table 3.2 List of formulae of wetted surface area for different types of model geometry

<table>
<thead>
<tr>
<th>Type of Model Geometry</th>
<th>Formula</th>
<th>Figure of a typical row</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-dimensional Full Width Geometry (3DFWG)</td>
<td>( A_{\text{wet}} = A_{\text{top}} + A_{\text{bottom}} + A_{\text{cyl}} = 2\left(S_L S_T - \frac{\pi}{4} D^2\right) + \pi DH )</td>
<td><img src="image" alt="Figure of a typical row" /></td>
</tr>
<tr>
<td>Three-dimensional Half Width Geometry (3DHWG)</td>
<td>( A_{\text{wet}} = A_{\text{top}} + A_{\text{bottom}} + A_{\text{cyl}} = 2\left(S_L S_T - \frac{1}{2} \frac{\pi}{4} D^2\right) + \frac{1}{2} \pi DH )</td>
<td><img src="image" alt="Figure of a typical row" /></td>
</tr>
<tr>
<td>Two-dimensional Full Width Geometry (2DFWG)</td>
<td>( \frac{A_{\text{wet}}}{H} = \pi D )</td>
<td><img src="image" alt="Figure of a typical row" /></td>
</tr>
<tr>
<td>Two-dimensional Half Width Geometry (2DHWG)</td>
<td>( \frac{A_{\text{wet}}}{H} = \frac{\pi D}{2} )</td>
<td><img src="image" alt="Figure of a typical row" /></td>
</tr>
</tbody>
</table>

### 3.3.5 Step 5: Convection coefficient

The convection coefficient computed using the upstream bulk temperature \( (T_{b,p1}) \) for Region2 is calculated as:
\[
h_{\text{Region2}} = \frac{\dot{Q}}{H} = \frac{\dot{Q}}{A_{\text{wet}} \times (T_W - T_{b,p1})} = \frac{\dot{Q}}{A_{\text{wet}} \times (T_W - T_{b,p1})}
\]

Here, \(\dot{Q}/H\) is calculated in step 3, \((A_{\text{wetted}})/H\) is found from step 4 according to the type of model geometry and \(T_W\) is the wall temperature fixed at 400 K in this study.

### 3.3.6 Step 6: Nusselt number

By definition, the Nusselt number is the ratio of the rate of heat transfer by convection to the rate of heat transfer to a stagnant fluid. For Region2, it is calculated using the following formula.

\[
\text{Nu}_{\text{Region2}} = \frac{h_{\text{Region2}} D}{k}
\]

The Nusselt number for every region is computed using the heat transfer coefficient computed for the corresponding region. For purposes of comparison to data reported row-by-row in the literature, the value of Nu is computed for a row as the mean value of its adjacent regions. For example,

\[
\text{Nu}_{\text{row2}} = \frac{1}{2} (\text{Nu}_{\text{Region2}} + \text{Nu}_{\text{Region3}})
\]

Here, row2 is comprised of half of Region2 and half of Region3 and similarly for other rows.

### 3.3.7 Friction factor:

The static pressure for a cutting plane is computed using an area-weighted average of the static pressures in that plane. The static pressure difference across the array is the difference in static pressure between the first plane located upstream of the array, \((p0)\), and the last plane located downstream of the last row, \((p9)\). The friction factor per row is calculated using the following equation [2] for both solid and slotted pin array:
\[ f = \frac{\Delta p}{2N\rho V_{\text{max}}^2} \]

Where, \( N \) is the number of pin rows, which is equal to 8 (eight) for the current study.

### 3.4 Code validation

For all research using computational models to simulate components and systems, it is important to show that the simulations are adequate representations of the physics of interest. This validation process typically relies on comparison of simulations of configurations and boundary conditions for which data from experiments is available. This section presents a brief summary of the simulation validation study. A much more detailed report including descriptions of mesh convergence studies and accuracy comparisons between modeling choices, (e.g. for turbulent flow models, standard k-epsilon .vs. k-omega .vs. LES) can be found in Reza [4].

Ames et al. [6] reported measurements of heat transfer for flow through an array of circular pins with a solidity of 45% in a channel of rectangular cross section that are well represented by a power law dependence of Nusselt number on Reynolds number.

\[ \text{Nu}_D = 0.508\text{Re}_D^{0.548} \]

This correlation is shown as the curve labeled “Ames Nu” in Figure 3.9. The steady incompressible flow model in ANSYS FLUENT, with the standard k-epsilon turbulent flow model, was used to simulate the experiment configuration for flow at Reynolds numbers of 6,000, 10,000, 20,000 and 40,000. The Nusselt numbers predicted by these validation simulations are depicted as markers in Figure 3.9. It is clear from Figure 3.9 that there is excellent agreement between the Nu predicted by the Ames correlation and the computational simulations over the entire range of Reynolds numbers of interest.
Ames et al. [6] reported that a power law dependence of friction factor on Reynolds number well represented measurement for flow through the 45% solidity array.

\[ f = 1.72 \text{Re}_D^{-0.297} \]

Figure 3.10 shows that again there is excellent agreement between the friction factor predicted by the Ames correlation and the computational simulations over the entire range of Reynolds number of interest. It is concluded that the using the ANSYS FLUENT steady incompressible flow model, with its standard k-epsilon turbulent flow model, in a mesh of density and geometry similar to those of the validation simulations should produce simulations that well represent the flows and heat transfer processes of interest for the present research.
Figure 3.10. Comparison of simulated friction factor with Ames data for 45% solidity array
4. Conventional Pin Array

The conventional pin arrays in this study are comprised of eight rows of pins of circular cross-section arranged with a fixed stagger angle spanning the height of the cooling channel. The width of the channel is determined by the transverse pitch of the array, \( S_T \), which is a function of the array solidity, \( S \), and the array’s stagger angle, \( \alpha \). For half width geometry, width of the model geometry is half of transverse pitch. The length of the model geometry is determined by the longitudinal pitch, \( S_L \), and the number of rows. The model geometry includes an unobstructed channel length five pin diameters upstream and downstream of the pin array.

Simulations are performed using four different Reynolds numbers (6,000; 10,000; 20,000; 40,000). Two different types of boundary conditions have been used to simulate the coolant flow through the pin arrays. A specified inlet velocity boundary condition is used to provide simulation results as a function Reynolds number because that is the most common independent variable for presenting data for these flow in the literature. Other simulations are performed with a specified static pressure difference across the channel to better simulate the design process in which flow rate through the cooling channel is a consequence of design choices made for other components in the gas turbine system which result in a static pressure difference across the channel.

4.1 Results of velocity inlet boundary condition:

The pin array acts like a heat exchanger sitting in the cavity of turbine blade trailing edge. In fact, the principles of a cross flow heat exchanger can be applied here since heat is pumped from top and bottom while coolant air flows across the pin array. The following subsections describe the simulations for which the Reynolds number of the flow is specified, and the results of those simulations.
4.1.1 Bulk average temperature of coolant:

As mentioned in the previous chapter, the entire pin array is divided into several regions using cutting planes that pass through the center of the pins in a row. For each simulation, the bulk average fluid temperature is computed for each of these planes. For example, Figure 4.1 shows the increase in temperature as the air flows through the cooling passage for an array solidity of 35%. The pattern of curves is similar for other solidities. Results of other solidities of the conventional pin arrays are presented in Appendix C. For all Reynolds numbers, there is little increase in bulk average temperature in the upstream region of pin array, because there is relatively little exposed surface from which heat can be transferred to the cooling flow. An almost linear increase of bulk average temperature is observed as coolant moves through the rows. Coolant exit flow temperatures are higher for the lower Reynolds number flows because the coolant has more time to interact with the exposed surfaces in the channel than does the relatively high-speed flow associated with the higher Reynolds number flows. The rate of temperature increase, and therefore the rate of heat transfer, at the beginning and in the end of pin array is different from the rate between rows because the coolant interacts with only half of the curved surface of pin in these instances. Bulk average temperatures are calculated at the cutting planes whereas other quantities are evaluated for regions and/or rows whichever is relevant and appropriate to comply and compare with the literature. Considering this anomaly of initial and exit sections, rows 2-7 or regions except the first and last ones have been considered to calculate the average value of a variable.
As compared to Figure 4.1 in which the variation in bulk average temperature for flow at different Reynolds numbers through an array of fixed solidity is presented, Figure 4.2 shows the variation in bulk average temperature for flow through arrays of different solidities for a Reynolds number of 40,000. Disregarding the anomaly of first plane and exit plane, it is seen that the average coolant temperature at the planes increases linearly for all the investigated solidities. The highest solidities correspond to the largest increases in bulk average temperature.

Figure 4.1. Bulk average temperature variation along planes of 35% solidity
It should be noted that because the stagger angle is fixed, the longitudinal and transverse spacing of pins in the array are a function of solidity, both decrease with increasing solidity. The flow Reynolds number is based on maximum average velocity at the minimum cross-sectional area. Therefore, although initially counterintuitive, the simulation inlet velocity for a specified Reynolds number is not same for all the arrays. Another important influence to note is that the distance between rows, and therefore the distance between planes, is not constant but rather decreases with increasing solidity. Longitudinal spacing, transverse spacing, inlet velocity and array transit time are presented in Table 4.1 for flow at a Reynolds number of 40,000 through the array geometries of interest.
The data in Table 4.1 clearly show the decrease in minimum flow cross-section with increasing solidity and therefore the decrease in inlet flow velocity (and therefore mass flow rate) with increasing solidity.

4.1.2 Heat transfer rate:

Figure 4.3 shows the dependence of heat transfer rate per unit height through an array of 45% solidity. It is noteworthy that although the rate of temperature rise is relatively small for a relatively high Reynolds number flow, (refer to section 4.1), the corresponding heat transfer rate is high. For example, in the middle of array (Region# 5), the heat transfer rate per unit height (627 W/m) for Re=40k is more than double of that (253 W/m) for Re=10k. In the previous section it is observed that temperature rise in case of Re=10k is higher than that of Re=40k. It is thus obvious that mass flow rate is responsible for high heat transfer rate at Re=40k. In fact, inlet velocity of Re=40k, and therefore the associated mass flow rate, is four times that of Re=10k. Therefore, although the coolant temperature decreases with increasing Reynolds number the rate of heat transfer increases with increasing Reynolds number. Again, the heat transfer rate of region#1 and region#9 is atypical because the geometric configurations of initial region and final region are different.
region are different from others. It is also noted that the heat transfer rate decreases as the flow proceeds towards the downstream regions because the temperature difference available to drive the heat transfer is decreasing as the coolant’s temperature increases as depicted in Figure 4.1.

![Graph showing heat transfer rate per unit height along the regions](image)

**Figure 4.3.** Variation of heat transfer rate in the pin array.

Figure 4.4 depicts the influence of solidity on the heat transfer rate per unit height from Region#2 to Region#9 for flow at a Reynolds number of 40,000. This depiction of the data reveals the complexity of the interacting influences on the heat transfer rate. For example, the initially high rate for highest solidity ($S=0.65$) decreases rapidly from row to row until at the last row where it is lower than the rate for lower solidity arrays. This suggests that the optimal number of rows in an array may depend on the solidity of the array. For example, the increase in heat
transfer rate between Regions 3 and 4 for solidities of 0.25 and 0.35 is somewhat puzzling. It may be due to development of the flow field in this region for the low solidity arrays. Although these variations in heat transfer rate per unit height within the array are interesting, in the final analysis it is the total rate of heat transfer for the entire row that is primary interest.

4.1.3 Nusselt Number:

Figure 4.5 shows dependence of Nusselt number on Reynolds number for flow through a conventional pin array of 25% solidity. Neglecting the values of Nusselt number for the first and last rows for reasons previously described, the Nusselt number does not change significantly with streamwise location for flows with a Reynolds number of 6,000 and 10,000. For flows with a Reynolds number of 20,000 and 40,000, the Nusselt number increases with increasing row

Figure 4.4. Variation of heat transfer rate for different solidity pin array at Re = 40,000.
number. It is believed that for the lower Reynolds number flows through the 25% solidity array, and for the other array geometries, the flow field inside the array quickly develops and does not change significantly as the fluid moves beyond the first few rows of the array. Therefore, the Nusselt number is approximately independent of streamwise position within the array. In contrast, the two higher Reynolds number flows in the lowest solidity array in the study continues to develop resulting in an increase in Nusselt number with increasing row number.

Figure 4.5. Nusselt number dependence on row number for 25% solidity pin array.

The influence of array solidity on Nusselt number variation within the array for flows with a Reynolds number of 40,000 is shown in Figure 4.6. For the lower solidity arrays the
Nusselt number increases with increasing row number whereas for the higher solidity arrays the Nusselt number is independent of row number to a good approximation.

Figure 4.6. Row-by-row distribution of Nusselt number at Re=40k for different solidity pin.

Figure 4.7 depicts the dependence of Nusselt number on solidity for the range of Reynolds numbers investigate in the present study. For all Reynolds numbers, the Nusselt number increases with increasing solidity. For all solidities, increasing the Reynolds number increases the Nusselt number. For a particular value of solidity, increasing the Reynolds number does not produce a proportionate increase of Nusselt number. For example, doubling the Reynolds number from 10,000 to 20,000 for an array of 35% solidity increases the Nusselt number from 72 to only 111.
The information presented in Figure 4.7 is reorganized to show the dependence of Nusselt number on Reynolds number for different solidities to produce Figure 4.8. This reorganization of information is presented in a log-log format to show the power law dependence of Nusselt number on Reynolds number frequently observed for forced convection heat transfer. The dependence of Nusselt number on Reynolds number for each array is well approximated by a straight line on the log-log plot suggesting that it is indeed well represented by a power law of the form:

$$\text{Nu} = a \text{Re}^b.$$
The curves in Figure 4.8 are approximately parallel suggesting that the value of the exponent b is approximately constant but the value of the coefficient a depends on array solidity. The values of the constants presented in Table 4.2 were determined using the same least-squares regression [14].

Table 4.2 Nu = f(Re) constants for conventional pin arrays

<table>
<thead>
<tr>
<th>Name of Parameter</th>
<th>Solidity of pin array</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S = 0.25</td>
</tr>
<tr>
<td>Value of a</td>
<td>0.2537</td>
</tr>
<tr>
<td>Value of b</td>
<td>0.6113</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9977</td>
</tr>
</tbody>
</table>
4.1.4 Resistance to flow:

The second most important parameter for the design of a cooling system is resistance to flow. An efficient cooling scheme demands more heat transfer with the consumption of less coolant. Although the pin array makes a significant contribution to increase the amount of heat transferred to the coolant, it also presents a significant resistance to flow through the cooling channel. Figure 4.9 shows how the static pressure difference across the channel required to produce a specific Reynolds number increase with increasing Reynolds number for each array in the present study. As is common for the display of resistance to flow data, (e.g., Moody Chart), the data is presented using logarithmic axes for both axes.

![Figure 4.9. Static pressure difference vs Reynolds number for different array solidities.](image-url)
Figure 4.10 describes the pressure loss characteristics of five different solidity conventional pin arrays in the range of Reynolds number 6,000 to 40,000. An increase in Reynolds number reduces the friction factor for each array. The increase in friction factor between 55% and 65% solidities is significantly greater than the incremental increase of 10% for all of the other solidities. Again, it appears that a power law relationship can be used to represent the influence of friction factor on Reynolds number.

![Friction factor vs Reynolds number](image)

Figure 4.10. Friction factor vs Reynolds number for different conventional pin arrays.

**4.2 Results of static pressure difference boundary condition:**

Although the specified Reynolds number simulations produce data with the most common independent variable in the literature, these simulations do not well represent the design process in which the pressure difference available to drive the flow through the cooling passage is determined by other choices in the gas turbine system design process. To gain a better
understanding of the influence of cooling pin array geometry with more typical constraints, two sets of simulations were conducted using a specified static pressure difference across the cooling channel. Simulation of flow at a Reynolds number of 40,000 through an array of 45% solidity predicts a static pressure difference across the channel of 1017 Pa and flow at a Reynolds number of 6,000 through an array of 65% solidity predicts a static pressure difference of 122 Pa. These two static pressure differences were chosen for the “design case” simulations because it was thought that they would produce thermal performance in the same parameter space as the specified Reynolds number simulations.

Conducting simulations for a specified static pressure difference was not straightforward because ANSYS Fluent permitted specification of the total pressure difference but not the static pressure difference. A trial-and-error process in which the total pressure difference was adjusted until the desired static pressure difference was produced before the heat transfer performance data could be obtained. Appendix C presents plots showing the variation of bulk average temperature with array plane number.

4.2.1 Heat transfer rate:

A plot of heat transfer rates in the regions bounded by the imaginary planes for a static pressure drop of 1017 Pa is shown in Figure 4.11. Values for the first and last regions are omitted because they are geometric anomalies as explained previously. The lowest solidity (25%) pin array in the investigated range shows highest heat transfer rate and the highest solidity array the lowest heat transfer rate. In case of the highest solidity among the considered pin arrays, coolant flow rate is decreased due to reduction in flow velocity and flow area. The area normal to the flow is reduced in 65% solidity pin array because transverse spacing is the lowest among those of other solidity pin arrays. Although the rate of coolant temperature rise is highest for 65%
solidsity, the gain in coolant temperature difference cannot compensate for the loss in mass flow rate. As the solidity is increased, a decreasing pattern is found indicating reduced capacity of coolant to extract heat in the downstream regions. It is interesting to note that the monotonic decrease in heat transfer rate with each 10% increase in solidity and that by far the largest decrease in heat transfer rate is from 55% to 65% solidity.

The total heat transfer rate for each solidity pin array for which a static pressure difference of 1017 Pa is imposed is displayed in Figure 4.12. The curve with (+) symbol shows the trend of decreasing heat transfer rate per unit height with increasing solidity. In contrast, the curve with the solid dots shows the trend of increasing heat transfer rate per unit volume with
increasing solidity. The heat transfer per unit volume accounts for the decrease in array volume as the longitudinal and streamwise pin spacing decreases with increases in solidity for a fixed stagger angle. The volume available in the blade trailing edge is very limited suggesting that although the higher solidity 8-row arrays have a lower total heat transfer they may well be a better choice when used in a channel constrained in size by the blade’s shape. The slight decrease in total heat transfer rate per unit volume from 55% solidity to 65% solidity suggests that the disadvantage of diminished flow rate due to heightened resistance outweighs the advantage of a more compact array.

Figure 4.12. Heat parameters vs solidity for static pressure difference of 1017 Pa.
4.2.2 Nusselt number:

The variation of Nusselt number with pin row number is shown in Figure 4.13. The dependence on row number is clearly different for different array solidity. The dependence of Nu on row number increases with decreasing solidity. At the highest Reynolds number Nu is almost independent of row number suggesting that the flow is almost fully developed after passing through the first three rows. In contrast, the Nu increases significantly with increasing row number suggesting that the flow is still developing as it exits the array.

![Nusselt Number along the rows](image)

Figure 4.13. Nusselt number along the rows for static pressure difference of 1017 Pa.

This average Nusselt number for each array is shown in Figure 4.14 for two static pressure differences of 122 Pa and 1017 Pa. Obviously, a relatively high static pressure
difference across the array of same solidity pin array generates more flow through the pins. For each solidity, the Nusselt number corresponding to 1017 Pa static pressure difference is higher than that for a static pressure difference of 122 Pa. A decreasing tendency is found to occur in row average Nusselt number as the solidity of pin array increases in the investigated range. This decrease is more pronounced for the higher static pressure difference.

4.2.3 Resistance to flow:

The resistance to flow characterized by the friction factor is plotted vs conventional pin array solidity in Figure 4.15. As expected, friction factor increases with increasing solidity. The strong increase at the higher solidities, doubling with an increase in array solidity from 55% to
65% supports the suggested reason for the decrease in total heat transfer rate per unit volume observed in Figure 4.12.

![Friction Factor vs Solidity](image)

**Figure 4.15.** Friction factor vs solidity of pin array for two static pressure differences.

### 4.3 Flow field:

Contour plots for flow through conventional circular pin arrays of three solidities (25%, 45%, 65%) at a Reynolds number of 20,000 are shown in Figure 4.17. The geometry is not drawn to scale but the color map of velocity in m/s is the same for the three contour plots. The relative spacing of different solidity arrays is shown in Figure 2.7. The pin diameter is the same for all three images in Figure 4.17. If they were all drawn to the same scale, the 65% image would be less than half of the size of the 25% image. Each is presented at the scale that maximizes the information that can be extracted from viewing them in their present medium.
As previously discussed, although all three simulations in Figure 4.17 are of flow at a Reynolds number of 20,000, the inlet velocity is not the same for all three simulations because the characteristic velocity used in computing the Reynolds number depends on the array solidity. In all three images the flow seems to be fully developed after passing through the first two rows of the array. The size of the wake aft of each pin seems to decrease with increasing solidity. The maximum velocity occurs adjacent to the pins in the minimum cross-sectional area. An increase in solidity influences the high velocity red regions. As the diagonal passage is reduced for a relatively high solidity pin array, the coolant flow is more tightly squeezed in this region at the highest solidity pin array among the investigated conventional pin arrays.

Figure 4.16. Streamwise velocity component for Re = 20,000 for conventional pin array of (a) Solidity 25% (b) Solidity 45% (c) Solidity 65%
5. Novel Pin Array

A novel pin shape was proposed in [7] to increase the rate of heat transfer to the coolant flowing through the pin array in the cooling channel inside the trailing edge of a gas turbine blade. The current study is an extension of that work. The novel pin shape is best thought of as a slotted pin in which the two halves of the conventional circular pin are separated by a channel aligned with the mean flow direction. Figure 5.1 shows how the new pin geometry compares to the conventional geometry for an array of 35% solidity. Cooling flow is from left to right. For the present study, the novel geometry is formed by splitting the conventional pin and displacing each semi-circular half one-half of the desired slot width in the transverse direction. Forming the slotted-pin array geometry in this fashion results in a slotted-pin array with the same solidity and stagger angle as the circular-pin array from which it is formed. Although this is an approximation, using the circular pin diameter as the characteristic length for the dimensionless parameters of interest facilitates comparison of cooling performance of the slotted-pin array to its parent circular pin array.

Figure 5.1. 35% solidity conventional and novel pin geometry in 2D plane

The parameter space for comparing the slotted-pin array performance to the circular pin performance includes the same range of Reynolds numbers as the parameter space for the circular pin study (Re= 6k, 10k, 20k, 40k), and the same range of array solidities (S= 0.25, 0.35,
0.45, 0.55, 0.65), augmented by a new geometric feature: the slot width as a percentage/fraction of the pin diameter (5%, 10%, 15%, 20%, 25%). A slot with of 25% was chosen because at that width the array starts to look more like lines of semi-circular pins of alternating transverse orientation than like lines of slotted circular pins. Not surprisingly the associated flow field is becoming less and less similar to that of the narrower slot pins. With this selection of the studies parameter space there are five slotted pin arrays for each array solidity. This enables a performance comparison of each slotted-pin array to the parent circular pin array and to the other slotted-pin arrays to determine if there is an optimal slot width. Forming the array geometries revealed that for the highest array solidity of 65% the pins are so close together that arrays with the two highest slot-width pins are not possible because they would require adjacent pins to occupy the same space which is of course impossible. Therefore, the parameter study of slotted-pin array performance required simulation of the flow through a cooling channel with twenty-three different arrays.

The same methodology described in chapter 3 for computing quantities of interest for the conventional pin arrays is used for novel pin arrays. For example, the planes are placed in the center of rows of the slotted-pin arrays to facilitate calculation of bulk average fluid temperature, heat transfer coefficients of regions and rows, and Nusselt numbers. The following subsections summarize the performance of the novel slotted-pin arrays.

5.1 Results of velocity inlet boundary condition

As was true for the conventional circular-pin arrays, investigation of the performance of the slotted pin arrays began with specification of inlet velocity to produce a desired flow Reynolds numbers. The area open to flow at the pin centers is the same for the slotted-pin arrays as for parent circular-pin arrays so the inlet velocity for the two sets of simulations is identical.
5.1.1 Bulk average temperature of coolant

The variation in coolant temperature for the slotted pin array of solidity 35% and slot width equal to 0.20D is shown in the Figure 5.2. Appendix D contains similar plots for other slot width arrays. The coolant temperature dependence on row number and Reynolds number is very similar to the pattern exhibited by the circular-pin arrays. The temperature increases with increasing row number and for any row the temperature increases with decreasing Reynolds number. It is noteworthy that comparing the temperature for a specific Reynolds number at a row reveals that the coolant temperature is consistently higher for the slotted-pin arrays than for the parent circular pin array. This suggests that the hope for increase in cooling performance may be realized by the slotted-pin geometry. This increase may be due to the addition of exposed surface for heat transfer provided by both walls of the slots, or it may be due to changes in the flow field due the flow through the slot.
In Figure 5.3, coolant temperature is plotted for flow with a Reynolds number of 40,000 through arrays of varying solidity containing pins with a slot width of 0.15D. Again, the trend of curves for the different solidity slot pin arrays is identical to that found for conventional solid pin arrays. The current plot can be described using the same explanation given in section 4.1.1 for the corresponding plot. The difference is, for each solidity pin geometry, coolant temperature at exit of slot pin array is higher than it is for the parent conventional solid pin array. Therefore, both plots of coolant temperature confirm the fact that coolant temperature rise increases with the inclusion of a slot in pin geometry. For all the investigated solidities of novel pin shape and Reynolds number, the coolant temperature at the exit of array is higher than its conventional counterpart.

Figure 5.2. Bulk average temperature variation along planes of 35% solidity and SW/D = 0.20 slot pin array.
Figure 5.4 shows the dependence of coolant exit temperature on pin slot width for flow through a 55% solidity array at a fixed Reynolds number. For every array of slotted-pins the exit temperature is higher than the conventional pin array. For every slot width the highest exit temperature corresponds to the lowest Reynolds number. The maximum exit temperature for all Reynolds numbers occurs for a slot width of 10% for 55% solidity novel pin arrays. Because the flow rate is fixed for a specified Reynolds number, the 10% slot width array also provides the most cooling for each Reynolds number, this array may not result in the highest rate of heat transfer because that quantity also depends on the mass flow rate with the maximum exit temperatures. Appendix D can be seen to check for the data of other novel pin array solidities.
5.1.2 Heat transfer rate

The dependence of heat transfer rate per unit height on slot width for flows at a Reynolds number of 10,000 through arrays of different solidities is shown in Figure 5.5. As discussed earlier, an array of 65% solidity is not geometrically possible for slot widths of 0.20D and 0.25D. From the figure, it is observed that heat transfer rate per unit height of the conventional solid pin array is much smaller than that of any slotted-pin array. Although the 65% solidity array has the highest rate of heat transfer per unit height, that rate falls precipitously as slot width is further increased for that array. For array solidities of 35% and 45% the rate of heat transfer per unit
height has a local maximum for a slot width of 15%. In contrast, the rate of heat transfer per unit height for flow through the 25% solidity array continues to increase with increasing slot width for all slot widths simulated.

The heat transfer rate per unit volume is of great interest because it accounts for array volume changes with changing solidity. The dependence of total heat transfer rate per unit volume on Reynolds number for flow through the 55% solidity containing pins of different slot widths is shown in Figure 5.6. Again, it is clear that the cooling performance of the conventional solid circular pin array is significantly lower than the performance of all the slotted-pin arrays. It
is also clear that for each array the heat transfer rate increases with increasing Reynolds number. The performance of the 5% slot width, 10% slot-width, and 15% slot widths is closely grouped for flows at all Reynolds numbers investigated with the 15% slot width array resulting in the highest heat transfer rate per unit volume for flow at each Reynolds number. Increasing the slot width to 20% results in a decrease in cooling performance and further increasing the slot width to 25% results in an even more significant decrease in cooling performance.

Figure 5.6. Heat transfer rate per unit volume vs Reynolds number for different slot widths.
5.1.3 Nusselt number

For each slot width, Nusselt number dependence on array solidity for flow at each Reynolds number is very similar to that shown in Figure 4.7 for the solid-pin array so a figure to show that dependence is not presented in this section. A plot of Nusselt number vs ratio of slot width and diameter is shown in Figure 5.7 shows the dependence of Nusselt number on slot width for flow at a specific Reynolds number through an array of 45% solidity. Again, the slotted-pin arrays outperform the conventional solid-pin array. The largest Nusselt number corresponds to flow at the largest Reynolds number. There is a relatively weak dependence of Nusselt number on slot-width with the maximum occurring for a slot width of 15% for each Reynolds number.

Figure 5.7. Nusselt number vs slot width/diameter for different Reynolds number.
For each slot width, Nusselt number dependence on array solidity for flow at each Reynolds number is very similar to that shown in Figure 4.7 for the solid-pin array so a figure to show that dependence is not presented in this section. A plot of Nusselt number vs ratio of slot width and diameter is shown in Figure 5.7 shows the dependence of Nusselt number on slot width for flow at a specific Reynolds number through an array of 45% solidity. Again, the slotted-pin arrays outperform the conventional solid-pin array. The largest Nusselt number corresponds to flow at the largest Reynolds number. There is a relatively weak dependence of Nusselt number on slot-width with the maximum occurring for a slot width of 15% for each Reynolds number.

Figure 5.8. Nusselt number vs Reynolds number for different slot pin array.
5.1.4 Resistance to flow:

The influence of slot width on friction factor for flow at a specified Reynolds number through an array of 55% solidity is depicted in Figure 5.9. As seen from Figure 5.9, friction factor decreases with each increase in Reynolds number. At Reynolds numbers of 6,000 and 10,000, the narrowest slot width pin array produces more resistance to flow than conventional solid circular pin array. For other slot widths the friction factor is smaller for the slotted-pin arrays than for the conventional pin array. Although the resistance to flow decreases with increasing slot width for all slotted-pin arrays it should be recalled that the heat transfer performance experienced a local maximum for the intermediate slot widths.

![Friction Factor vs Slotwidth/Diameter](image)

Figure 5.9. Friction factor vs slot width/diameter for different Reynolds number
To further explore the influence of geometry on friction factor, Figure 5.10 shows the dependence of friction factor on pin slot width for flow at a Reynolds number of 10% through arrays of different solidities. Again, the narrowest slot width arrays show an increase in resistance to flow as compared to the solid pin array for solidities of 55% and 65% but the other slot widths for all solidities show a decrease in resistance to flow. This figure clearly shows that the 55% solidity array presents more resistance to flow than the lower solidity arrays and that the 65% solidity array shows a much larger resistance to flow than the 55% solidity array.

Figure 5.10. Friction factor vs slot width/diameter for different solidity pin arrays.
A plot of friction factor vs Reynolds number is shown in Figure 5.11 for 45% solidity pin array. In log-log scale, straight lines are found for all the investigated pin geometries. From the figure, it is seen that friction factor decreases with increase in Reynolds number for each pin configuration. The friction factor of any slot width pin array is lower than that of conventional counterpart except for 0.05D slot width at 6,000 and 10,000 Reynolds number. Again, the largest slot width array presents the least resistance to flow.

![Friction Factor vs Reynolds Number](image)

**Figure 5.11. Friction factor vs Reynolds number for different slot pin array**
5.2 Results of static pressure difference boundary condition

As was done for the conventional circular pin arrays, simulations were conducted for static pressure differences of 122 Pa and 1017 Pa to better understand the competing influences on total heat transfer rate for a fixed static pressure difference constraint imposed by the design of the gas turbine system.

5.2.1 Bulk average temperature of coolant

Figure 5.12 shows the dependence of bulk average temperature on longitudinal position in 45% solidity arrays of pins of different slot width for flows due to an imposed static pressure difference of 122 Pa across the channel. For all planes except the first, which is upstream of the first row of pins, the coolant temperature is higher for all slotted-pin arrays than for the conventional pin array. A slot width of 0.10D results in the highest exit temperature but the temperatures for 0.05D, 0.10D, and 0.15D arrays are all closely grouped at each pin row. Flow for an imposed pressure difference of 1017 Pa results in a very similar set of curves.
Figure 5.13 shows the influence of pin slot width and array solidity on the bulk average temperature of the coolant as it leaves the channel. The exit temperature for all slotted pin arrays is higher than the exit temperature for the conventional array of circular pins. The exit temperature is weakly dependent on slot width for lower solidity arrays. For array solidities of 55% and 65% there is a local maximum in exit temperature for a slot width of 0.05D.

Figure 5.13. Coolant temperature distribution for different slot width pin arrays of 45% solidity.
Figure 5.13. Coolant temperature at exit of array for different solidity slot pin arrays.

### 5.2.2 Heat transfer rate

Figure 5.14 shows the dependence of heat transfer rate per unit height on array solidity and pin slot width for a static pressure difference across the channel of 1017 Pa. As can be seen from the figure, for a fixed static pressure difference the heat transfer rate per unit height decreases with an increase in array solidity for all solidities modeled. Again, the array of circular pins is outperformed by all the slotted pin arrays. Among the slotted pin configurations, arrays...
with a pin slot width of 0.25D has the highest rate of heat transfer per unit height for array solidities of 25%, 35% and 45% solidity cases. The array of pins with a slot width of 0.15D has the highest heat transfer rate per unit height for array solidities of 55% and 65%.

![Graph showing heat transfer rate per unit height vs Solidity for different pin geometries.](image)

**Figure 5.14.** Heat transfer rate per unit height vs solidity for different pin geometries.

The dependence of heat transfer rate per unit volume on array solidity and pin slot width for flow due to a specified static pressure difference of 1017 Pa is shown Figure 5.15. This figure of merit takes in account the variation in array volume with array solidity and it is important because the space for the cooling channel is highly constrained by the geometry of the blade’s trailing edge. All the slotted pin arrays outperform the conventional circular pin array of the
same solidity. Except for the array of largest slot width, (SW=0.25D), the rate of heat transfer per unit volume has a local maximum for an array solidity of 55%.

![Heat transfer rate per unit volume vs Solidity](image)

**Figure 5.15. Heat transfer rate per unit volume vs solidity for different pin geometries.**

### 5.2.3 Nusselt number

The average Nusselt number for an array is computed as the arithmetic average of the Nusselt number computed for row 2 to 7. This average Nusselt number is reported in Figure 5.16 as a function of array solidity and pin slot width for flow due to an imposed static pressure difference of 1017 Pa. For this figure of merit all the slotted pin arrays outperform the conventional circular pin array. The array of 45% solidity produces the highest Nusselt number.
except for the narrowest slot width of 0.05D for which the highest Nusselt number corresponds to an array of the lowest solidity, 25%. The pin array of 0.20D slot width and 45% solidity produces the highest Nusselt number of all the pin arrays under consideration.

![Nusselt number vs Slotwidth/pin diameter](image)

**Figure 5.16.** Row average Nusselt number vs slot width/pin diameter for different pin arrays.

### 5.2.4 Flow resistance

A plot of friction factor vs solidity is shown in Figure 5.17 as a measure of flow resistance presented by pin arrays of different solidity and different slot widths for flow due to an imposed static pressure difference of 1017 Pa. across the arrays. As was observed in 5.14, except for the narrowest slot width of 0.05D in the highest solidity pin array, the resistance to flow of
the slotted pin arrays is lower than the conventional circular pin array. The resistance to flow of the larger slot width arrays, SW= 0.02D and SW= 0.25D do not exhibit a monotonic trend of friction factor with increasing solidity. For the other pin slot width arrays an increase in friction factor is observed for an increase in array solidity.

Friction Factor vs Solidity
Static Pressure Drop = 1017 Pa

![Friction Factor vs Solidity Graph]

Figure 5.17. Friction factor vs solidity for different pin geometries.

5.3 Flow field

The three images in Figure 5.18 show the velocity field for flow at a Reynolds number of 20,000 through an array of 35% solidity in which the slotted pins have slot widths of 0.05D, 0.15D, and 0.25D. The color map for stream wise velocity applies to all the three cases.
As was observed for flow through an array of circular pins, after passing through the first two rows the flow seems to be fully developed and the maximum velocity occurs near the planes of minimum cross-sectional area. Not surprisingly, the flow velocity through the slot increases with increasing slot width. Comparison to Figure 4.17 showing the flow field through an array of circular pins, the jet exiting the slot disrupts the wake behind each pin significantly altering the flow around the circumference of the pin or, depending on the viewer’s perspective, the flow field between the pins in the next row.

Figure 5.18. Streamwise velocity component for Re = 20,000 in 35% solidity novel pin array of (a) SW/D = 0.05 (b) SW/D = 0.15 (c) SW/D = 0.25
6. Summary and Conclusions

6.1 Parameter Space of Investigation

Computational simulations were conducted to investigate the dependence of the cooling performance of flow through a channel in the trailing edge of a gas turbine blade on the geometry of a pin array placed in the channel to increase the rate of heat transfer to the coolant. The geometric parameter space of the investigation includes two sets of arrays with solidities of 25%, 35%, 45%, 55%, and 65%, all with the same pin diameter and stagger angle. Simulations conducted with the first set of arrays, in which the pins are of the conventional circular cross section, were conducted to investigate the influence of array solidity on cooling performance. Simulations conducted with the second set of arrays, in which the pins are of a novel slotted cross section, were conducted to determine if the slotted-pin array provides better cooling performance than the “parent” solid circular pin array. This second set of arrays is composed of subsets in which the pins have slot widths of 5%, 10%, 15%, 20% and 25% of the pin diameter to gain an understanding of the influence of slot width on cooling performance. Simulations with specified flow Reynolds numbers of 6k, 10k, 20k, and 40k were conducted to characterize the cooling performance of each array as a function of Reynolds number as is typically reported in the literature. Two sets of “design case” simulations in which the static pressure difference across the channel is fixed were conducted to gain an understanding of the array design trade-offs when the pressure difference available to drive flow through the channel is imposed by other choices in the design of the gas turbine system.

It should be noted that the characteristic length used to compute the Reynolds number is the pin diameter which is a fixed value for all arrays in the present investigation. The characteristic velocity is the maximum bulk average velocity in the array, which corresponds to
the minimum area open to flow in the array, which depends on the transverse spacing of the array’s pins, which in turn depends on the solidity of the array and the stagger angle of the pins in the array. The stagger angle is the same for all arrays in the present study. The value of the characteristic velocity used to compute the Reynolds number is therefore a function of array solidity. Therefore, although this may at first seem counterintuitive, for a fixed Reynolds number, the flow rate through arrays of different solidities will be at different flow rates.

6.2 Nusselt Number, Circular Pin Arrays

Figure 6.1 shows the dependence of array Nusselt Number on flow Reynolds number for each conventional circular pin array in the parameter space of the present study. It also shows the dependence of Nusselt number on array geometry for flows due to specified static pressure differences of 122 Pa and 1017 Pa.

For any Reynolds number within the range of the present study, the array of highest solidity produces the largest Nusselt number, with Nusselt number monotonically decreasing with decreasing solidity. The nearly straight-line relationship between Nusselt number and Reynolds number on log-log axes confirms that the relationship between these dimensionless parameters is well modeled by the typical power law of the form $\text{Nu} = a\text{Re}^b$ where $a$ and $b$ are constants. The line for each solidity has approximately the same slope as the lines for the other solidities suggesting that the value of the coefficient is a function of solidity but a single value for the exponent will well represent the relationship for all solidities studied. For the “design cases”, (static pressure differences of 122 Pa and 1017 Pa), the Nusselt has a modest dependence on solidity with its value increasing with decreasing solidity. As expected, the larger static pressure difference produces a larger mass flow rate and therefore a larger Nusselt number for each array. Because the curves for both static pressure differences are of almost identical shape,
it is reasonable to assume that interpolation between them should produce a reasonably good prediction of Nusselt number for intermediate static pressure differences. Therefore, within the parameter space investigated, the information in Figure 6.1 can be used to predict the Nusselt number for flow driven by a specified static pressure difference across the cooling channel.

![Nusselt Number vs Reynolds Number](image.png)

**Figure 6.1.** Nusselt number vs Reynolds number for both velocity inlet and static pressure difference boundary conditions

### 6.3 Nusselt Number, Slotted-Pin Arrays

To compare the performance of arrays of pins with slot withs of 10% and 15% to the performance of their parent arrays presented above in Figure 6.1, the relationship between
Nusselt number and Reynolds for the slotted-pin arrays is presented in Figure 6.2 using the same axes as were used in Figure 6.1. Similar plots for the other slot widths are included in Appendix D. It is clear that both of these slotted-pin arrays outperform their parent conventional circular pin arrays throughout the parameter space investigated. Although the Nusselt number increases monotonically with either Reynolds number or solidity, the flows due to a fixed static pressure difference show only a weak dependence of Nusselt number on solidity and that dependence is not monotonic. The curves for different solidities become more tightly grouped with increasing slot width.
Figure 6.2. Nusselt number vs Reynolds number for novel pin array of
(a) SW/D = 0.10 (b) SW/D = 0.15
6.4 Friction factor:

Figure 6.3 shows the dependence of friction factor on Reynolds number for the conventional circular pin arrays of the present study. Superimposed on these curves are curves showing the variation of friction factor with array solidity for the two “design cases” for which a fixed static pressure difference is imposed across the cooling channel.

For a specific solidity, the friction factor decreases in an almost linear fashion with Reynolds number in a log-log plot. Higher solidity arrays present more resistance to the flow and therefore are characterized by a larger friction factor for a specified Reynolds number. For the “design cases”, the friction factor decreases with decreasing solidity because the flow rate, and therefore the Reynolds number increases with decreasing solidity.
A similar plot of friction factor vs Reynolds number using logarithmic axes is shown in Figure 6.4 for flow through a novel pin array of slot width equal to 10% of the pin diameter. To facilitate comparison with the parent conventional circular pin arrays, the axes in Figure 6.4 are identical to those in Figure 6.3. Like the conventional pin array, the friction factor of novel pin array decreases with Reynolds number. For a fixed slot width and Reynolds number, the friction factor increases with solidity. Comparison of the data in Figures 6.3 and 6.4 reveals that the slotted-pin arrays present less resistance to flow than their parent circular pin array and they are therefore characterized by a lower friction factor. This conclusion holds true for all slotted-pin arrays in this investigation’s parameter space.

Figure 6.4. Friction factor vs Reynolds number for novel pin array of SW/D = 0.10
6.5 Relative Cooling Performance Slotted and Circular Pin Arrays

The current investigation compares the thermal performance of novel pin arrays with that of conventional circular pin arrays. Augmentation of heat transfer is the most significant feature of an effective cooling design and various parameters have been used to characterize it. From Figure 5.6, it is found that heat transfer rate per unit volume of the novel slotted pin arrays is higher than their parent circular pin array. Referring to Figures 5.7 and 5.8, the Nusselt number for flow through a slotted-pin array is higher than the Nusselt number for flow through the parent conventional pin arrays throughout the parameter space investigated. For flow due to a fixed static pressure difference across the cooling passage, Figure 5.16 shows that the novel pin arrays provide better performance than their conventional pin parent arrays in terms of heat transfer rate per unit volume. A conclusion can be drawn from Figure 5.16 that within the parameter space investigated, an array of slotted-pins of any slot width and any solidity results in a higher array Nusselt number than its parent conventional circular pin array. All these data show that the novel slotted pin arrays will remove more heat than the parent conventional circular pin array for any flow in the parameter space investigated and are a strong candidate for replacing the conventional circular pin array for cooling the trailing edge of gas turbine blade.

6.6 Future Research Recommendations

The current study investigated the influence of array solidity on the performance of the cooling passage in the trailing edge of a gas turbine blade for arrays of pins of a conventional circular cross section and for arrays of pins of a novel slotted cross section. The slotted-pin arrays of any width in the study consistently outperformed their “parent” conventional pin arrays. Opportunities for extending the study are identified below.
• The improvement in cooling performance for the slotted-pin arrays is substantial. It cannot be explained entirely by the increased exposed surface for heat transfer provided by the slot. Review of the flow fields in the slotted-pin arrays shows that the jet exiting the slot significantly alters the wake behind the pin. The presence of a slot therefore significantly alters the flow field around the pin and perhaps it significantly alters the heat transfer to the circumferential surface of the pin. Computational simulations and/or experiments should be conducted to gain a better understanding of how the presence of the slot alters the rate of heat transfer to an otherwise circular cylinder and ultimately to the dependence of the rate of heat transfer on the slot width.

• All arrays investigated in the present study have the same diameter, the same stagger angle, and the same number of rows. The geometric parameter space of the investigation can be expanded by considering the influence of each of these geometric parameters on the array’s cooling performance.

• 2D simulations of the entire blade trailing edge passage/array would permit investigation of variations in geometry and flow rate in the passage/array.
  o For a typical passage, the cross section of the passage changes with position along the passage.
  o For a typical passage, the flow rate changes with position along the passage because some coolant flows through the side of the channel to exit from the blade’s trailing edge.
  o Although a uniform array is the simplest to analyze and build, variation of any of the geometric parameters on position within the array may improve cooling performance.
• 3D simulations and experiments would permit the investigation of all of the above-mentioned influences on the passage’s cooling performance.

• The size and geometry of a typical blade trailing edge cooling passage is highly constrained by the exterior shape of the blade which is designed to maximize the blade’s performance as a component of the gas turbine system. All research that leads to a better understanding of how to optimize the blade trailing edge passage’s cooling performance for the imposed geometric constraints will contribute to increasing the efficiency of a gas turbine system.

This brief list of topics recommended for future research makes it clear that a large amount of research lies between our current understanding of the fluid flow and heat transfer processes occurring in the cooling passage of the trailing edge of a gas turbine blade before the cooling performance of that passage can be fully optimized.
7. References


Determination of longitudinal spacing \( (S_L) \) and transverse spacing \( (S_T) \):

Information regarding model geometry:

1. Diameter of solid pin, \( D = 2.012 \text{ cm} \).
2. Solidity of pin array, \( S = 35\% = 0.35 \)
3. Array angle, \( \alpha = 38.66 \) degree.

From the figure:

\[
\tan \alpha = \frac{1}{2} \frac{S_T}{S_L} \Rightarrow S_T = 2 \times S_L \tan \alpha = (2\tan \alpha)S_L
\]

(1)

According to the definition of solidity,

\[
S = \frac{\pi D^2}{4} \left( \frac{1}{S_T S_L} \right) = \frac{\pi D^2}{4} \left( \frac{1}{(2\tan \alpha) S_L^2} \right); \text{[using (1)]}
\]

\[
\Rightarrow S_L = \sqrt{\frac{\pi D^2}{4} \left( \frac{1}{(2\tan \alpha) S_L} \right)} = \sqrt{\frac{\pi \times 0.02012^2}{4} \left( \frac{1}{(2 \times \tan 38.7) \times 0.35} \right)} = 0.0238 \text{ m}
\]

\[
\Rightarrow S_L = 2.38 \text{ cm}
\]

From (1), \( S_T = (2\tan \alpha)S_L = (2 \times \tan 38.66) \times 2.38 \text{ cm} = 3.81 \text{ cm} \)

Therefore, when solidity is 35% and stagger angle is 38.66 degree, the longitudinal spacing \( (S_L) \) and transverse spacing \( (S_T) \) are 2.38 cm and 3.81 cm, respectively.
9. Appendix – B  Inlet conditions for various solidity pin array

Diameter of pin, \( D = 2.012 \) cm

Height of pin, \( H = 0.95D = 1.91 \) cm

Hydraulic diameter,

\[
D_h = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}} = \frac{4HS_T}{2} = \frac{HS_T}{H + \frac{ST}{2}}
\]

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<th>( V_{\text{max}} )</th>
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<td>( \frac{\pi D^2}{4} \left( \frac{1}{ST} \right) ) ( \frac{\rho V_{\text{max}} D}{\mu} ) ( \frac{\text{Re}<em>D \mu}{\rho D} ) ( \frac{(ST - D)}{ST} ) ( \frac{\rho V</em>{\text{in}} D_h}{\mu} ) ( \times V_{\text{max}} )</td>
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<td>4.36</td>
<td>1.47</td>
<td>1700</td>
<td>6.31</td>
<td></td>
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<tr>
<td>10,000</td>
<td>7.26</td>
<td>2.46</td>
<td>2840</td>
<td>5.92</td>
<td></td>
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<tr>
<td>20,000</td>
<td>14.52</td>
<td>4.91</td>
<td>5680</td>
<td>5.43</td>
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<tr>
<td>40,000</td>
<td>29.04</td>
<td>9.82</td>
<td>11360</td>
<td>4.98</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>S_L = 0.0175 m, S_T = 0.0280 m, D_h = 0.0162 m</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>6,000</td>
<td>4.36</td>
<td>1.23</td>
<td>1360</td>
<td>6.49</td>
<td></td>
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<tr>
<td>10,000</td>
<td>7.26</td>
<td>2.04</td>
<td>2270</td>
<td>6.09</td>
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</tr>
<tr>
<td>20,000</td>
<td>14.52</td>
<td>4.09</td>
<td>4530</td>
<td>5.59</td>
<td></td>
</tr>
<tr>
<td>40,000</td>
<td>29.04</td>
<td>8.17</td>
<td>9060</td>
<td>5.12</td>
<td></td>
</tr>
</tbody>
</table>
10. Appendix – C  Plots of conventional pin array

Coolant temperature along the planes
Solidity = 25%

Coolant temperature at the planes
Solidity = 45%
Coolant temperature along the planes
Solidity = 55%

- Re=6k
- Re=10k
- Re=20k
- Re=40k

Coolant temperature along the planes
Solidity = 65%

- Re=6k
- Re=10k
- Re=20k
- Re=40k
Coolant temperature along the planes
Static Pressure Drop = 122 Pa

Coolant temperature along the planes
Static Pressure Drop = 1017 Pa
Bulk Avg. Temperature along the planes

Re = 6,000

S = 0.25  S = 0.35  S = 0.45  S = 0.55  S = 0.65

Bulk Avg. Temperature along the planes

Re = 10,000

S = 0.25  S = 0.35  S = 0.45  S = 0.55  S = 0.65
Bulk Avg. Temperature along the planes
Re = 20,000

- S = 0.25
- S = 0.35
- S = 0.45
- S = 0.55
- S = 0.65
Heat transfer rate per unit height along the regions

**Solidity = 25%**

- Re=6k
- Re=10k
- Re=20k
- Re=40k

Heat transfer rate per unit height along the regions

**Solidity = 35%**

- Re=6k
- Re=10k
- Re=20k
- Re=40k
Heat transfer rate per unit height along the regions

**Solidity = 55%**

- Re=6k
- Re=10k
- Re=20k
- Re=40k

**Solidity = 65%**

- Re=6k
- Re=10k
- Re=20k
- Re=40k
Heat transfer rate per unit height along the rows

Re = 6,000

Heat transfer rate per unit height along the rows
Re = 10,000
Heat transfer rate per unit height along the rows
Re = 20,000

- Static Pressure Drop = 122 Pa

Region #
Heat Transfer Rate/height, Qdot/H (W/m)
Nu along the rows
Solidity = 35%

- Re=6k
- Re=10k
- Re=20k
- Re=40k

Row #

Nu along the rows
Solidity = 45%

- Re=6k
- Re=10k
- Re=20k
- Re=40k

Row #
Nu along the rows
Solidity = 55%

Nu along the rows
Solidity = 65%
Nu along the rows
Re = 6,000

Nu along the rows
Re = 10,000
Nu along the rows
Re = 20,000

Static Pressure Drop = 122 Pa

Row #
Heat parameters vs Solidity
Static Pressure Drop = 122 Pa

- Heat Transfer Rate Per Unit Height
- Heat transfer rate per unit volume
11. Appendix – D  Plots of novel pin array

Coolant temperature along the planes
Solidity = 25%, SW/D = 0.05

Coolant temperature along the planes
Solidity = 25%, SW/D = 0.10
Coolant temperature along the planes
Solidity = 25%, SW/D = 0.15

Coolant temperature along the planes
Solidity = 25%, SW/D = 0.20
Bulk avg. temperature, K

Coolant temperature along the planes
Solidity = 25%, SW/D = 0.25

Re=6k  Re=10k  Re=20k  Re=40k

Bulk Avg Temperature along the planes
Solidity = 25%, Static Pressure Drop = 122 Pa

SW/D=0  SW/D=0.05  SW/D=0.10
SW/D=0.15  SW/D=0.20  SW/D=0.25
Bulk Avg Temperature along the planes
Solidity = 35%, Static Pressure Drop = 122 Pa

- SW/D=0
- SW/D=0.05
- SW/D=0.10
- SW/D=0.15
- SW/D=0.20
- SW/D=0.25

Bulk Avg Temperature along the planes
Solidity = 55%, Static Pressure Drop = 122 Pa

- SW/D=0
- SW/D=0.05
- SW/D=0.10
- SW/D=0.15
- SW/D=0.20
- SW/D=0.25
Bulk Avg Temperature along the planes
Solidity = 65%, Static Pressure Drop = 122 Pa

- SW/D=0
- SW/D=0.05
- SW/D=0.10
- SW/D=0.15

Bulk average temperature, K

Coolant temperature vs Slotwidth/diameter
Static Pressure Drop = 1017 Pa

- S = 0.25
- S = 0.35
- S = 0.45
- S = 0.55
- S = 0.65

Bulk average temperature, K
The graphs show the coolant temperature along the planes for different $S$ values with $Re = 40,000$, $SW/D = 0.05$ and $SW/D = 0.10$.
Coolant temperature along the planes
Re = 40,000, SW/D = 0.20

Coolant temperature along the planes
Re = 40,000, SW/D = 0.25
Bulk Average Temperature vs Slotwidth/Diameter
Solidity = 25%, Re = 6k ~ 40k

Bulk Average Temperature vs Slotwidth/Diameter
Solidity = 35%, Re = 6k ~ 40k
**Bulk Average Temperature vs Slotwidth/Diameter**

**Solidity = 45%, Re = 6k ~ 40k**

- **Re=6k**
- **Re=10k**
- **Re=20k**
- **Re=40k**

**Solidity = 65%, Re = 6k ~ 40k**

- **Re=6k**
- **Re=10k**
- **Re=20k**
- **Re=40k**
Heat transfer rate per unit height vs Slotwidth/Diameter
Re = 6k, Solidity = 25% ~ 65%

Heat transfer rate per unit height vs Slotwidth/Diameter
Re = 20k, Solidity = 25% ~ 65%
Heat transfer rate per unit height vs Slotwidth/Diameter
Re = 40k, Solidity = 25% ~ 65%

Heat transfer rate per unit height vs Solidity
Static Pressure Drop = 122 Pa
Heat transfer rate per unit volume vs Reynolds number
Solidity = 25%, SW/D = 0.00 ~ 0.25

Heat transfer rate per unit volume vs Reynolds number
Solidity = 35%, SW/D = 0.00 ~ 0.25
Heat transfer rate per unit volume vs Reynolds number
Solidity = 45%, SW/D = 0.00 ~ 0.25

Heat transfer rate per unit volume vs Reynolds number
Solidity = 65%, SW/D = 0.00 ~ 0.25
Heat transfer rate per unit volume vs Solidity
Static Pressure Drop = 122 Pa, SW/D = 0.00 ~ 0.25
Nusselt Number vs Slotwidth/Diameter
Solidity = 25%, Re = 6k ~ 40k

Nusselt Number vs Slotwidth/Diameter
Solidity = 35%, Re = 6k ~ 40k
Nusselt Number vs Slotwidth/Diameter

Solidity = 55%, Re = 6k ~ 40k

Solidity = 65%, Re = 6k ~ 40k
Nusselt Number vs Reynolds Number
Solidity = 25%, SW/D = 0.00 ~ 0.25

Nusselt Number vs Reynolds Number
Solidity = 35%, SW/D = 0.00 ~ 0.25
Nusselt Number vs Reynolds Number
Solidity = 45%, SW/D = 0.00 ~ 0.25

Nusselt Number vs Reynolds Number
Solidity = 65%, SW/D = 0.00 ~ 0.25
Nusselt number vs Slot Width Ratio
Static Pressure Drop = 122 Pa
Friction Factor vs Slotwidth/Diameter
Solidity = 25%, Re = 6k ~ 40k

Friction Factor vs Slotwidth/Diameter
Solidity = 35%, Re = 6k ~ 40k
Friction Factor vs Slotwidth/Diameter
Solidity = 45%, Re = 6k ~ 40k

Friction Factor vs Slotwidth/Diameter
Solidity = 65%, Re = 6k ~ 40k
Friction Factor vs Slotwidth/Diameter
Re = 6k, Solidity = 25% ~ 65%

Friction Factor vs Slotwidth/Diameter
Re = 20k, Solidity = 25% ~ 65%
Friction Factor vs Slotwidth/Diameter

Re = 40k, Solidity = 25% ~ 65%

Friction Factor vs Solidity

Static Pressure Drop = 122 Pa
Friction Factor vs Slotwidth/Diameter
Re = 40k, Solidity = 25% ~ 65%

Friction Factor vs Solidity
Static Pressure Drop = 122 Pa
Friction Factor vs Reynolds Number
Solidity = 25%, SW/D = 0.00 ~ 0.25

- Solidity = 35%, SW/D = 0.00 ~ 0.25
Nusselt Number vs Reynolds Number
SW/D = 0.05, Solidity = 25% ~ 65%

Nusselt Number vs Reynolds Number
SW/D = 0.20, Solidity = 25% ~ 65%
Nusselt Number vs Reynolds Number

$SW/D = 0.25$, Solidity = 25% ~ 65%

- $S=0.55$
- $S=0.45$
- $S=0.35$
- $S=0.25$
- SPD=122Pa
- SPD=1017Pa
Friction factor vs Reynolds Number
SW/D = 0.20, Solidity = 25% ~ 65%

Friction factor vs Reynolds Number
SW/D = 0.25, Solidity = 25% ~ 65%