Improved Image Uniformity Using Optimized Diverging-Wave Acquisition Sequence for High Frame Rate Pulse-Echo Ultrasound

Kashta Dozier-Muhammad

Follow this and additional works at: https://digitalcommons.memphis.edu/etd

Recommended Citation

This Thesis is brought to you for free and open access by University of Memphis Digital Commons. It has been accepted for inclusion in Electronic Theses and Dissertations by an authorized administrator of University of Memphis Digital Commons. For more information, please contact khggerty@memphis.edu.
IMPROVED IMAGE UNIFORMITY USING OPTIMIZED DIVERGING-WAVE ACQUISITION SEQUENCE FOR HIGH FRAME RATE PULSE-ECHO ULTRASOUND

by

Kashta Dozier-Muhammad

A Thesis
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

Major: Biomedical Engineering

The University of Memphis
May 2023
Dedication

I give praise to Allah for His Mercy and Blessings. I thank Him for The Most Honorable Elijah Muhammad and his student, The Honorable Minister Louis Farrakhan, and for their being the bearers of Light and Guidance, for giving to me any modicum of wisdom or strength and the impetus to complete this work.

I would like to dedicate this Thesis to all those who have supported me in every manner of ways throughout the duration of this process. I primarily devote this work to my mother and father, Brandi and Derrick Dozier-Muhammad, for instilling in me the value of education and secondly, my family and friends for encouraging me (especially Askia, Sakura, and Neeali). In addition, I dedicate any success in part to the Believers of the Savannah Study Group, Huntsville Study Group, Muhammad Mosque 15, and Muhammad Mosque 55 for being a guidance and community to me.

Last, but not least, I thank Alexandra Nelson and the Nelson Family for their inspiration and encouragement. This work’s inception is attributed to you.
Acknowledgements

I would like to express my appreciation and gratitude for my advisor, teacher, and mentor, Dr. Carl Herickhoff for his longstanding patience, commitment, and dedication to my success and the success of other young, and aspiring engineers like myself. Dr. Herickhoff has pushed me to excel in my studies and research within the field of Medical Ultrasound, and more broadly as a student. Above all, I am thankful for his kindness and that of Omar Yunis.

I would like to thank The University of Memphis for providing me the opportunity to pursue my education in Biomedical Engineering and to chart my course as a professional. I would also like to recognize my alma mater, Alabama A&M University, for cultivating from my novice the interest and drive to pursue science and engineering and my graduate education.

Lastly, this work was supported by the James E. West Graduate Fellowship for Minorities from the Acoustical Society of America. I would like to thank the Acoustical Society of America and Dr. James E. West for their dedication to the success of students and professionals in the Acoustical sciences.
Abstract

Ultrafast pulse-echo ultrasound imaging uses unfocused plane-wave transmit (PWT) or diverging-wave transmit (DWT) wavefronts and coherent compounding for image reconstruction. PWT imaging is more commonly utilized, but has a limited region of overlapping insonification. This work characterizes the tradeoffs between PWT and DWT, to determine an optimal DWT transmit scheme for given constraints on the imaging field-of-view (depth and width), frame rate, and resolution uniformity. Using Field II, the transmit energy field was analyzed for PWT and various active apertures and relative virtual source locations for DWT. This was followed by a Field II calculation and analysis of point-spread functions (PSFs) at many locations in the field for each PWT and DWT case, and several cases of PWT and DWT compounding. The amplitude and resolution of the PSFs, and the uniformity (variance) of each of these metrics over the field-of-view, was measured in each case. This framework was then implemented on a Verasonics Vantage-128 (V-128) research scanner for similar analysis on a wire-target phantom to determine objective guidelines for optimized DWT acquisition schemes based on the given speed of sound in the medium, transmit center frequency, aperture width, and desired field-of-view. Results suggest that a DWT scheme provides improved PSF amplitude and resolution uniformity over a broader field-of-view than PWT, with only a slight reduction in resolution with increasing depths.
Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2 Background</td>
<td>5</td>
</tr>
<tr>
<td>- Superposition principle of LTI system &amp; an ultrasound system’s Point Spread Function</td>
<td>5</td>
</tr>
<tr>
<td>- Conventional delay-and-sum beamforming</td>
<td>8</td>
</tr>
<tr>
<td>- Full-synthetic-aperture dataset</td>
<td>10</td>
</tr>
<tr>
<td>- Virtual Source beamforming</td>
<td>11</td>
</tr>
<tr>
<td>3 Research Strategy</td>
<td>16</td>
</tr>
<tr>
<td>- Preliminary Work &amp; Results</td>
<td>16</td>
</tr>
<tr>
<td>- Aim 1:</td>
<td>19</td>
</tr>
<tr>
<td>- Methods</td>
<td>19</td>
</tr>
<tr>
<td>- Results</td>
<td>21</td>
</tr>
<tr>
<td>- Aim 2:</td>
<td>22</td>
</tr>
<tr>
<td>- Methods</td>
<td>22</td>
</tr>
<tr>
<td>- Results</td>
<td>23</td>
</tr>
<tr>
<td>- Aim 3:</td>
<td>23</td>
</tr>
<tr>
<td>- Methods</td>
<td>23</td>
</tr>
<tr>
<td>- Results</td>
<td>26</td>
</tr>
<tr>
<td>4 Discussion</td>
<td>28</td>
</tr>
<tr>
<td>- Limitations and Potential Errors</td>
<td>32</td>
</tr>
<tr>
<td>5 Conclusion</td>
<td>34</td>
</tr>
<tr>
<td>- Significance</td>
<td>34</td>
</tr>
<tr>
<td>- Future Work</td>
<td>34</td>
</tr>
<tr>
<td>References</td>
<td>35</td>
</tr>
<tr>
<td>Appendices</td>
<td>38</td>
</tr>
<tr>
<td>Appendix A (Lateral Point-Spread Function)</td>
<td>38</td>
</tr>
<tr>
<td>Appendix B (Experimental Setup)</td>
<td>39</td>
</tr>
</tbody>
</table>
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Basic focus transmit (FT) scheme</td>
</tr>
<tr>
<td>2</td>
<td>Basic diagram of (L) PWT and (R) DWT schemes</td>
</tr>
<tr>
<td>3</td>
<td>Motion-compensation for high-contrast high-frame-rate echocardiography of the left ventricle at peak systole with 32 diverging-wave beams $[-25^\circ$ to $+25^\circ]$. Top row: apical 4-chamber; bottom row: parasternal long-axis</td>
</tr>
<tr>
<td>4</td>
<td>(L) point-to-point and (R) wave-field transmit scenes</td>
</tr>
<tr>
<td>5</td>
<td>Electronic receive focusing</td>
</tr>
<tr>
<td>6</td>
<td>Typical spatial coordinate system for a 1-D ultrasound transducer array</td>
</tr>
<tr>
<td>7</td>
<td>Electronic transmit focusing</td>
</tr>
<tr>
<td>8</td>
<td>Diagram of a zero-angle plane-wave</td>
</tr>
<tr>
<td>9</td>
<td>(L) PWT and (R) DWT travel paths</td>
</tr>
<tr>
<td>10</td>
<td>Example of single and compounded DWT sequence</td>
</tr>
<tr>
<td>11</td>
<td>FSA Tx angular response of a single-element in Field II</td>
</tr>
<tr>
<td>12</td>
<td>Regions of overlapping insonification for (L) PWT and (R) DWT, $\phi = [-20^\circ, 20^\circ]$</td>
</tr>
<tr>
<td>13</td>
<td>Designed imaging grid</td>
</tr>
<tr>
<td>14</td>
<td>Point Spread Function of $0^\circ$ DWT with $r_f = -3$ mm ($\approx 72^\circ$ sector)</td>
</tr>
<tr>
<td>15</td>
<td>$[-20^\circ, 20^\circ]$ compounded PWT and DWT PSFs</td>
</tr>
<tr>
<td>16</td>
<td>In silico image field comparison maps $\frac{\text{DWT}_{\text{PWT}}}{\text{PWT}} \times 100%$</td>
</tr>
<tr>
<td>17</td>
<td>PSFs generated in Verasonics using 5 DWTs $[-20^\circ, 20^\circ]$</td>
</tr>
<tr>
<td>18</td>
<td>CIRS MPMT phantom layout and selected experimental imaging field</td>
</tr>
<tr>
<td>19</td>
<td>Initial results for DWT using Top: Test 2 and Bottom: Test 1 setup</td>
</tr>
<tr>
<td>20</td>
<td>Phantom image and -3 dB resolution Top: DWT, Bottom: PWT</td>
</tr>
<tr>
<td>No.</td>
<td>Description</td>
</tr>
<tr>
<td>-----</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>21</td>
<td>Phantom study image field comparison maps for Test 1, $\frac{DWT - PWT}{PWT} \times 100%$</td>
</tr>
<tr>
<td>22</td>
<td>Phantom study image field comparison maps for Test 2, $\frac{DWT - PWT}{PWT} \times 100%$</td>
</tr>
<tr>
<td>23</td>
<td>Assessed uniformity statistics and imaging FOVs/feature targets</td>
</tr>
<tr>
<td>24</td>
<td>Normalized PSF with different Tx fractional bandwidths [7]</td>
</tr>
<tr>
<td>25</td>
<td>Experimental setup of phantom study</td>
</tr>
<tr>
<td>26</td>
<td>P4-2v probe and MPMT phantom detail</td>
</tr>
<tr>
<td>27</td>
<td>Test 1 configuration: P4-2v probe and MPMT phantom in contact using ultrasound gel buffer</td>
</tr>
<tr>
<td>28</td>
<td>Test 2 configuration: P4-2v probe and MPMT phantom in offset using water buffer</td>
</tr>
</tbody>
</table>
Introduction

Conventional diagnostic ultrasound imaging utilizes a sequence of acoustic pulse-echo acquisitions, each composed of a transmit beam focused to a particular depth, to reconstruct the target region of interest (ROI). In the past, ultrasound transducers consisted of a single plate of piezoelectric material (e.g. lead zirconate titanate (PZT)), and the transmit and receive focus beams were mechanically translated or steered [8, 9]. However, modern probes utilize transducer arrays composed of many isolated PZT elements, and the ultrasound beams are electronically steered by applying temporal delay profiles across the array of elements [3, 7, 8, 10, 11]. Figure 1 shows a schematic diagram of focusing on transmit (Tx) using the electronic steering method.

Subsequent advances in signal processing techniques and data management capabilities have led to rapid advancement of ultrasound imaging techniques and technology [7]. Acquisition schemes using unfocused wavefronts, such as plane-waves and diverging-waves, to insonify the field can reduce the number of transmit events required to produce a quality image. Plane-wave transmit (PWT)-based imaging uses planar wavefronts at multiple angles, allowing for rapid coherent compounding and synthetic transmit focusing [4]. PWT, however, is restricted by its limited region of overlapping insonification (Figure 2 displays a basic schematic of the PWT approach and its expected transmit energy fields). Diverging-wave transmit (DWT)-based imaging has also been shown to increase frame rates by compounding beams with broad curvature using virtual sources [6, 12, 13]. In theory, these compounded sector beams exhibit greater coverage compared to compounded planar beams (as can be seen in Figure 2).

DWT can be applied in areas of clinical significance, where it can outperform traditional phased-array techniques and recover image cross-sections of broad areas within the
body. This can be especially useful for imaging at many of the body’s acoustic windows, as is done when imaging the cardiac muscle beyond the rib cage. In such applications, DWT can demonstrate frame rates 800-1000 frames per second or more, as compared to the roughly 30 frames per second achieved in focused transmit (FT) sequences; this can reduce the effects of cardiac motion artifacts and, in many instances, allow for high-sensitivity tissue tracking [1,5,6,14,15]. For example, in cardiac motion-compensation, Doppler shifted received signals from the moving myocardial wall tissue are correlated at neighboring time points over a series of acquisitions during the cardiac cycle. The signals are then phase adjusted according to the distance traveled between acquisitions to improve the coherent beamform summation and remove motion artifacts (smearing) [1,14]. In this way, the imaging frame rate must be more than twice the maximum tissue velocity expected, in accordance with the Nyquist sampling criterion [7,16]. In healthy individuals, myocardial wall tissue can reach velocities of 20 cm/s, and decreases with age and in the presence of some cardiomyopathy [1,17,18]. According to [1] the frame rate required by the ultrasound system can be found preemptively by the expression

\[
PRF = \frac{v_{min} N f_0}{c}
\]

where \(PRF\) is pulse-repetition-frequency (PRF), \(v_{min}\) is the Nyquist velocity, \(N\) is the number of used frames for compounding, \(f_0\) is the transmit frequency, and \(c\) is the speed of sound.
With $v_{min} = 40 \text{ cm/s}$, $N = 5$, $f_0 = 2.5 \text{MHz}$, and an assumed sound speed of 1540 m/s, the required PRF would be approximately 3250Hz, and the image frame rate would be $PRF/N = 650\text{fps}$. Therefore, it can be seen that the greater frame rates achieved in DWT sequences provide greater imaging advantage in the presence of motion. Figure 3 shows an example of motion-compensation performed when imaging the heart with a high frame rate DWT sequence.

![Figure 3: Motion-compensation for high-contrast high-frame-rate echocardiography of the left ventricle at peak systole with 32 diverging-wave beams $[-25^\circ \text{ to } +25^\circ]$. Top row: apical 4-chamber; bottom row: parasternal long-axis][1]

Similarly, unfocused Tx schemes can be applied to cardiovascular flow imaging to reduce aliasing and velocity estimation error [13,19]. DWT also has utility in transcranial and extracranial Doppler ultrasonography, where tailoring of the transmit wavefront direction, geometry, and spread can improve flow estimations, image uniformity and contrast compared to conventional focused Doppler [19–22].

In our preliminary work, we conducted a parametric study to investigate the potential for optimal acquisition sequences, based on the observed transmit energy fields for a number of transmit schemes (i.e. PWT, DWT, FT). Simulations were performed in Field II using a transducer array geometry identical to the Verasonics P4-2v cardiac probe. Experimental parameters were chosen for their potential effect on the transmit energy field’s geometry, uniformity/distribution, and strength, and the parameters were varied to produce distinct imaging cases. These parameters included the size and number of active sub-apertures, the axial coordinate of the virtual source, the transmission angle, and the arrangement and number of...
compounded image frames. Each imaging case was studied by applying the appropriate temporal delay profiles and observing the resultant transmit energy fields. From these data, we first confirmed the angular response of a single-element as compared to Selfridge’s cosine-adjusted formulation [23]; from this, we inferred a compounded transducer field-of-view based on the region of uniform overlapping insonification from multiple transmits.

To quantitatively determine an optimal DWT imaging scheme with respect to these identified constraints and parameters, we extended our study to pulse-echo (transmit-receive beamforming) analyses. An imaging system’s point-spread function (PSF) is a clear indication of its imaging capability [7,9]; therefore, measurements of the PSF amplitude geometry, resolution, and uniformity throughout the field can be used to objectively compare the simulated PWT and DWT acquisition sequences. The framework developed in silico was then used to conduct a PSF imaging study on a wire-target phantom using a V-128 research scanner. Image quality assessments of the two transmit methods were compared by investigating a CIRS Multi-Purpose, Multi-Tissue Phantom and estimating the image resolution, contrast, and optimal FOV (width and depth). These outputs were used to study differences in image quality between DWT and PWT within our chosen FOV and ultimately help determine objective guidelines for an optimized DWT acquisition scheme based on the given speed of sound in the medium, transmit center frequency, aperture width, and desired FOV.
Background

Superposition principle of LTI system & an ultrasound system’s Point Spread Function:

An ultrasound imaging system can be modeled as a linear time-invariant (LTI) system, hereon defined as \( \text{LTI} \), with the fundamental properties:

1. linearity: \( ax \to \text{LTI} \to ay \), where scaling the input, \( x \), by a constant, \( a \), only scales the output, \( y \), by the constant.

2. time-invariance: \( x(t + T) \to \text{LTI} \to X(t) \), for \( T \in \mathbb{R} \geq 0 \), where the behavior of the system is invariant to the time of operation on a temporally stable input: \( x(t + T) = x(t) \), after any time increment from \( t \) to \( (t + T) \).

A key feature of an LTI system is it’s impulse response, \( h(t) \), to an input delta function, \( \delta(t) \), given by \( \delta(t) \to \text{LTI} \to h(t) \), which fully characterizes its operation. The impulse response then allows one to determine any output of the LTI system by simply convolving the input with \( h(t) \): \( x(t) \to \text{LTI} \to y(t) = h(t) \ast x(t) \). In addition, the system’s transfer function, \( H(jw) \), is defined as the Fourier transform of the impulse response. Taking advantage of the convolution property for Fourier pairs, \( h(t) \ast x(t) \equiv H(jw)X(t) \), where \( H(jw) \) acts in the frequency domain as a filter on the input signal, \( X(jw) \) \([2,16]\).

This theory can be applied in the context of a pulse-echo ultrasound sequence, in which acoustic waves are sent from the transmit (Tx) aperture into the medium of propagation and the reflected waves are recorded by the receive (Rx) aperture. For the Tx case, a delta voltage excitation sent to each individual element of the transducer array produces a pulsed acoustic wave (i.e., the transducer’s impulse response) that is propagated spatially from the Tx aperture to a particular depth within a homogeneous medium with constant speed of sound, \( c \), and density, \( \rho_0 \). Components of the ultrasound probe (i.e. acoustic backing, impedance matching layers, and lens) ensure optimal energy transfer from the transducer to the medium \([8,24,25]\). The measured complex pressure field at an observation point within this medium can be found by considering the acoustic spatial impulse response for this system. The total response at a given point in the medium can be determined by applying Huygen’s principle of superposition such that the wavefront emitted with the entire aperture can be broken down
into the summation of the individual contributions from each of the spherical waves generated by the point resonators–array elements–making up the aperture. The spherical wave generated at each point resonator is given by

\[ p_s(r_1, t) = k_p \frac{\delta(t - \frac{|r_2 - r_1|}{c})}{|r_2 - r_1|} \]  

(2)

where \( r_1 \) is the point within the medium, \( r_2 \) is the point on the aperture, \( t \) is the instantaneous time of intersection with the wavefront, and \( k_p \) is a constant \[2\]. The Huygen’s principle can thus be expressed as the Rayleigh integral,

\[ p(r_1, t) = \frac{\rho_0}{2\pi} \int_S \frac{\partial}{\partial t} v_n(t - \frac{|r_1 - r_2|}{c}) \frac{dS}{|r_1 - r_2|} \]  

(3)

where \( v_n \) is the uniform particle velocity (initially zero everywhere except the aperture) and \( S \) represents the aperture surface. Equation 3 represents the summation of the pressure fields generated by each point on the aperture and can be rewritten as the convolution of the excitation velocity and the spatial impulse response, such that:

\[ p(r_1, t) = \rho_0 \frac{\partial v_n(t)}{\partial t} * h(r_1, t), \]  

(4)

where

\[ h(r_1, t) = \frac{1}{2\pi} \int_S \frac{\delta(t - \frac{|r_1 - r_2|}{c})}{|r_1 - r_2|} \frac{dS}{}, \]  

(5)

[2,9]. Thus, the total response is calculated by summing the impulse responses for the geometry-dependent point-to-point observations and convolving them with the time-varying input pressure field, which resembles the first-order linear wave equation [24, 26]. The point-to-point and the collective resonator wave-field scenes are shown in Figure 4.

The Rx case can be described by the same process. A point scatterer within the medium insonified by the Tx wave field also emits a spherical wave defined by Equation 2. Similar to the point resonators on the transmit aperture, the received wavefront can be decomposed into the contributions from each of the array elements making up the Rx aperture. The sensed transient pressure on each element is then converted into a temporal-voltage trace represent-
ing the complex radio frequency (RF) signal. The total reflected pressure incident on the aperture can again be described by Equation 3 as the convolution of the summed responses on each array element and the reflected input pressure field from the point scatterer. The received pressure field for the pulse-echo sequence can be defined as

\[ p(r_1, r_2, t) = \rho_0 \frac{\partial v_n(t)}{\partial t} * h(r_1, t) * h(r_2, t), \]  

(6)

where \( h(r_1, t) \) is the Tx impulse response and \( h(r_2, t) \) is the Rx impulse response. Assuming that the transducer and medium behave identically on Tx and Rx for the travel paths \( |r_2 - r_1| \) and \( |r_1 - r_2| \), \( h(r_1, t) = h(r_2, t) \) and the pulse-echo response for any point in the imaging field can be determined by simply knowing its one-way impulse response \([2,3]\).

For an aperture with uniform \( \delta(x) \) excitation, across all of its array elements along an axis \( x \), an aperture function may be approximated as a rect function, \( \Pi(x) \). The definition of the transfer function can then be used to transform the one-way pressure field of the system to its frequency domain equivalent. As stated earlier, the transfer function is the Fourier transform of the impulse response, and in this case is defined by a sinc function, \( \text{sinc}(u) \), and is called the imaging system’s lateral point-spread function (PSF) in the far field (when the observation distance \( >> \) the extent of the aperture) \([3]\). Properties of the Fourier transform allow the pulse-echo pressure field in Equation 6 to be rewritten in the frequency domain as

\[ P(w_2, w_2, \omega) = j\omega P_0(\omega)\text{sinc}^2(w_1, w_2) \]  

(7)
where $w_{1,2}$ are used as spatial frequency coordinates, $\omega$ is used as the temporal frequency, $P_0(\omega)$ is the transform of the initial excitation pressure, and $j$ is the imaginary unit. Convolution in the spatial domain is exchanged for multiplication in the frequency domain and the imaging output can be quickly calculated based on the aperture functions. Figure 24 in Appendix A shows the characteristic $\text{sinc}^2$ PSF for an idealized ultrasound system [2, 3, 7]. Like the impulse response, the PSF is dependent on the point scatterer’s position, the properties of the medium, and the transducer and its excitation scheme.

**Conventional delay-and-sum beamforming:** In order to effectively determine the pulse-echo pressure field using Huygen’s principle, the complex RF signals received from the reflected spherical wave must be summed accounting for their respective geometric travel paths, or rather, their arrival times. As can be seen in Figure 5, the received pulse from a point scatterer arrives first at the nearest element and last at the furthest element. To compensate, temporal delay profiles are applied across the channels connected to the array of elements (assuming 1-to-1 channel-element arrangement), aligning the voltage traces with respect to their unique travel times before coherent summation of the complex RF signals [11].

![Figure 5: Electronic receive focusing](image)

In the majority of applications, we tend to define our coordinate system with respect to the transducer, by placing the origin at the geometric center of the transducer array, and taking the dimension across the transducer face as the $x$-axis (azimuthal) and the dimension forward the transducer and into the field as the $z$-axis (axis). The dimension along the array element height is the $y$-axis (elevation) and is omitted from our calculations, as the elevational focus is narrow and often fixed (see Figure 6). Using the azimuthal and axial point locations, the element channel time-delays can be easily calculated using the Euclidean distance formula
and the speed of sound, $c$,

$$\Delta t_e = \frac{1}{c} \sqrt{(x_e - x_p)^2 + (z_e - z_p)^2} - t_c$$

(8)

where $(x_e, z_e)$ is the coordinate of the element center, $(x_p, z_p)$ is the position of the point scatterer in the field, and the subscript $e \in 1, 2, \ldots N$ may be used to distinguish the $e^{th}$ transducer element. The last term, $t_c$, is the reference time and is defined by

$$t_c = \frac{1}{c} \sqrt{(x_c - x_p)^2 + (z_c - z_p)^2}$$

(9)

where $(x_c, z_c)$ is the reference center point of the transducer aperture at $(0, 0)$. In this manner, Rx focusing may be used to hone in on point scatterers throughout the entire imaging FOV and realize their PSFs.

In order to optimize the sensitivity and resolution of the imaging system, the magnitude and geometry of the PSF amplitude may be improved by applying focusing delays on Tx as well. Sensitivity is measured by the contrast between the desired signal and the surrounding background, and the resolution is typically measured as the “full-width at half maximum” (FWHM), or the diameter of the PSF’s main lobe at half the maximum amplitude (-6 dB) (see Figure 24). Focusing on Tx will increase the magnitude of the main lobe by maximizing the energy incident on the point scatterer, and will decrease the FWHM (improving resolu-
tion) by reducing off-axis insonation of the surrounding medium. To achieve this, the outgoing spherical waves generated by the resonating elements making up the Tx aperture use the same delay profiles, $\Delta t_e$, as those on the Rx aperture to steer and focus the ultrasound beam directly to the point of interest [10]. Figure 7 shows the reciprocal focusing on Tx. Focusing on Tx and Rx only requires one delay calculation and this process can then be repeated at different scatterer locations to bring various features into focus throughout the FOV.

**Full-synthetic-aperture dataset:** The innovation of the FSA approach takes advantage of the superposition principle by fully characterizing the impulse responses for each Tx-element-to-Rx-element pair, for the entire FOV. The FSA scanning sequence is composed of a serial single element unfocused transmit of acoustic energy into the medium followed by an unfocused reception of the echoed signal on all receive elements; this is repeated for all transmit elements on the Tx aperture [27, 28]. The output can be constructed as a multi-dimensional matrix containing the signal traces for each Tx-element-to-Rx-element pairing across the Tx and Rx apertures, with size $N_{e,Tx} \times N_{e,Rx} \times N_s$, where $N_{e,Tx}$ and $N_{e,Rx}$ represent the number of elements composing the Tx and Rx apertures, respectively, and $N_s$ is the discrete number of samples in the received signal traces. Often times, the Tx and Rx apertures are identical, such that the data at each sample along $N_s$ is square ($N_e \times N_e$), and the size of the dataset is $N_e^2 \times N_s$.

The application of post-scan temporal delay profiles can then be used to synthetically beamform the unfocused raw data. As was shown previously, temporal delay profiles can facilitate focusing and steering of the ultrasound beam on Tx and Rx. For a conventional point focus on both Tx and Rx the temporal delay profile is described by Equation [8]. In order to ap-

![Figure 7: Electronic transmit focusing](image-url)
ply this regime to the FSA dataset, the signal corresponding to the \( n \times m \) (\( n, m \in \mathbb{N} \) and \( n \neq m \)) Tx-Rx pair must be delayed according to it’s total pulse-echo travel time \([27, 28]\).

Each trip can be described similar to Equation 8 and the reference time is now twice that of a single travel direction. The total travel time-delay is

\[
\Delta t_{e, tot} = t_{e, Tx} + t_{e, Rx} - 2t_c
\]

where \( t_{e, Tx} = t_{e, Rx} = t_e \) when considering the \( n \times n \) pair and the total delay becomes

\[
\Delta t_{e, tot} = 2(t_e - t_c) = 2 \times \Delta t_e
\]

After aligning the individual traces using \( \Delta t_{e, tot} \), the beamformed signal is acquired by summing along the Tx and Rx aperutres. That is

\[
S(t) = \sum_{n=1}^{N_e, Tx} \sum_{m=1}^{N_e, Rx} s_c^e(t - \Delta t_{e, tot})
\]

where \( s_c^e(t - \Delta t_{e, tot}) \) refers to the time-adjusted Tx-Rx signal trace and superscript \( \mathbb{C} \) denotes \( s_e \) as a complex signal.

Using the FSA approach and unique temporal delay profiles provides robust control over the image resolution and contrast by applying different focusing regimes, altering the shape and magnitude of the lateral PSF amplitude. Finally, imaging techniques can be easily compared, as the output of the FSA approach is dependent only on the scatterering field (echogenicity and location of features), the properties of the medium, and the transducer and its excitation scheme.

**Virtual source beamforming:** Unfocused wavefronts used to insonify the field (i.e., PWT and DWT) can reduce the number of transmit events typically required to produce an image using FT. Implementing the FSA approach, a series of unfocused beams can be retrospectively compounded to create a synthetic focus. In PWT schemes, a plane-wave is transmitted into the medium by providing excitation to all the elements on the Tx aperture without application of any delay profiles.
As shown in Figure 8, the plane-wave intersects a point scatterer within the medium at \((x, z)\) with travel time, \(t_e = \frac{z}{c}\), where \(x\) and \(z\) denote the azimuthal and axial coordinates of the point scatterer and \(c\) is the speed of sound. Notably, all of the elements’ wavefronts share the same Tx travel time, \(t_e\), as the reference travel time, \(t_c\), and therefore, \(\Delta t_{e,Tx} = t_e - t_c = 0\).

Angularly decomposing a FT scheme resembles an ensemble of numerous angled plane-waves, each tangential to the original FT wavefront. PWT compounding is achieved by combining multiple variable-angle plane-wave acquisitions to steer the off-axis components of the zero-angle wavefront, and approach a FT regime. In practice, a plane-wave angled \(\alpha : [0^\circ \leq \alpha \leq 90^\circ]\) off the Tx aperture azimuth is transmitted into the medium with a temporal delay profile

\[
\Delta t_e = \frac{1}{c}(x\sin(-\alpha))
\]  

The Tx temporal delay is found as \(\Delta t_{e,PWT} = t_e - t_c\), where \(\Delta t_{e,PWT} = 0\), iff \(\alpha = 0^\circ\) [4].

The beam intersects the point scatterer at \((x, z) = (x_p, z_p)\) after a travel time based on the path from the angled aperture to \((x, z)\), such that

\[
t_e = \frac{1}{c}(r\cos(\alpha - \Theta))
\]  

where

\[
r = \sqrt{x_p^2 + z_p^2}
\]  

and

\[
\Theta = \tan\left(\frac{x_p}{z_p}\right)
\]

as can be seen in Figure [9].

On Rx, the temporal delay profile, \(\Delta t_{e,Rx}\), is found according to Equation [8] and re-
mains focused. The total time delay $\Delta t_{e,\text{tot}}$ is thus

$$\Delta t_{e,\text{tot}} = \Delta t_{e,PWT} + \Delta t_{e,Rx} \quad (17)$$

The beamformed signal after each $\alpha$-angled PWT is again found by summing the time-adjusted signals along the $N_{e,Tx}$ and $N_{e,Rx}$ dimensions of the FSA dataset and coherent compounding is achieved by summing the beamformed, complex-valued, pixel data from various $\alpha$-angled PWT acquisitions. This retrospective compounding recomposes an approximation of the angularly sampled FT wavefront, and thus our above scheme approaches a conventional FT, focused receive regime—given a large number of distinct and appropriately spaced PWT angles are used.

Similar to the PWT sequence, a DWT sequence is composed of steered diverging-waves that are retrospectively compounded to recreate a wavefront with a synthetic Tx focus. The curved DWT wavefronts can be described as spherical waves originating from a virtual point source located behind the Tx aperture at $(x_v, z_v) : [x_v \in \mathbb{R}, -\infty < z_v < 0]$. The instant the virtual source wavefront intersects the Tx aperture only one element is excited, followed by its neighbors and so on. Thus, a DWT temporal delay profile can be defined by the virtual circular wavefront travel time to each Tx element. Considering first a virtual source directly
behind the origin at \((0, r_f)\) : \(r_f < 0\). The diverging-wave temporal delay is given by

\[
\Delta t_{e,v} = t_{e,v} - \min(t_{e,v})
\]

(18)

where

\[
t_{e,v} = \frac{1}{c} \sqrt{(x_e - x_v)^2 + (z_e - z_v)^2}
\]

(19)

and the reference time is now taken as the minimum travel time \([1, 5, 6, 14]\).

Just as the plane-wave beam can be steered by applying a delay profile given by Equation [13] that produces an angular tilt to the zero-angle PWT, an identical delay profile can be used to alter the zero-angle diverging-wave beam to produce a series of \(\alpha\)-angled DWTs. The DWT delay profile thus becomes

\[
\Delta t_{e,v} = (t_{e,v} - \min(t_{e,v})) + \frac{1}{c} (x_e \sin(-\alpha))
\]

(20)

The DWT wavefront may also be described using the total travel time to a point \((x_p, z_p)\) in the medium as

\[
t_e = \frac{1}{c} \sqrt{(x_p - x_{v,1})^2 + (z_p - z_{v,1})^2} - r_f
\]

(21)

where the point \((x_p, z_p)\) takes the place of the element coordinates in Equation [16] and the virtual source coordinate \((x_{v,1}, z_{v,1})\) is defined as

\[
x_{v,1} = r_f \sin \alpha
\]

(22)

and

\[
z_{v,1} = r_f \cos \alpha
\]

(23)

The travel path for a DWT is shown in Figure [9].

DWT compounding combines multiple variable-angle transmits to reproduce the off-axis components of a FT regime. In a DWT sequence, this is accomplished by changing the position of the virtual source coordinate, as can be seen in Figure [10]. Notably, the curvature of the wavefront in Equation [20] and [21] is entirely dependent on the length of \(r_f\) with respect
to the Tx aperture and \((x_p, z_p)\). Thus, a plane-wave can be taken as originating from a virtual source at \((x_v, -\infty)\) in order to produce a diverging-wave with infinite curvature (flat).

The DWT temporal delay is found as \(\Delta t_{e,DWT} = t_e - t_c\). On Rx, the temporal delay profile, \(\Delta t_{e,Rx}\), is again found according to Equation \(8\) and remains focused. The total time delay \(\Delta t_{e,tot}\) is thus

\[
\Delta t_{e,tot} = \Delta t_{e,DWT} + \Delta t_{e,Rx}
\]

The beamformed signal in each case is again found by summing the time-adjusted signals along the \(N_{e,Tx}\) and \(N_{e,Rx}\) dimensions of the FSA dataset, and compounding is achieved by summing the beamformed pixel data from various \(\alpha\)-angled DWT schemes.

![Diagram](image)

Figure 10: Example of single and compounded DWT sequence [6]
Research Strategy

Preliminary Work & Results: In our preliminary work, a parametric study was conducted to investigate the potential for optimal acquisition sequences based on the observed transmit energy fields for a number of transmit schemes (i.e. PWT, DWT, FT). Simulations were performed in Field II using a transducer array geometry identical to the Verasonics P4-2v cardiac probe. Table 1 gives design values for the simulated P4-2v probe. Using these data, we first confirmed the Tx angular response of a single-element in Field II as compared to the cosine-adjusted formulation developed by Selfridge in [23]. Results agreed with the reference material with the angular response, $\Phi \approx 52.5^\circ$ at magnitude -6 dB level (see Figure [11]).

Table 1: Field II transducer parameters

<table>
<thead>
<tr>
<th>P4-2v</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. elements</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Transducer pitch</td>
<td>300 $\mu$m</td>
<td></td>
</tr>
<tr>
<td>Aperture width</td>
<td>19.15 mm</td>
<td></td>
</tr>
<tr>
<td>Center frequency</td>
<td>2.7 MHz</td>
<td></td>
</tr>
<tr>
<td>Frequency bandwidth</td>
<td>74 %</td>
<td></td>
</tr>
</tbody>
</table>

Then, experimental parameters were chosen for their potential effect on the transmit energy field’s magnitude, geometry, and uniformity/distribution. These parameters included the size and number of active sub-apertures, the axial coordinate of the virtual source, the transmission angle, and the arrangement and number of compounded image frames. The size of the active sub-aperture—expressed as a fraction of the full Tx aperture, $\delta A$, was given by

$$\delta A = \frac{N_{sub}}{N_e}$$

(25)

where $N_{sub}$ is the number of elements making up the sub-aperture and $N_e$ is the total number of Tx elements. The number of possible distinct sub-apertures used in any compound sequence is many, but finite, where sup-apertures may have varying degrees of overlap, be entirely contiguous, or be sparsely populated across part of or the whole Tx aperture. In order to first examine the effect of the aperture size on the scan, the number and location of sub-apertures remained constant throughout experimentation (i.e. 1, centered, respectively). The
extent of the virtual source, \( r_f \), was set at three particular values: \(-7\), \(-10\), and \(-15\times\) the transducer pitch, which was defined by the expression, \( p = (w + k) \), where \( w \) is the element width and \( k \) is the kerf or inter-element spacing. The transmission angle, \( \phi \), was defined as the beam steering angle or the degree that the delay profile was projected off of the original Tx aperture azimuth. A sparse range of transmission angles, \( \phi = [-20^\circ, -10^\circ, 0^\circ, 10^\circ, 20^\circ] \), were used for compounding the PWT and DWT sequences. Single image frames for each value of \( \phi \) were acquired and later compounded to form higher resolution images. The number of compounded image frames used to produce a higher resolution PWT or DWT image was labeled \( N_\phi \), which ranged \([2–5]\), in accordance with the number of distinct transmission angles of a specific aperture length. The sparsity of the acquisition sequence, \( \delta\phi \), was given by

\[
\delta\phi = \frac{2\phi_{\text{max}}}{N_\phi}
\]

where \( \delta\phi \) is assumed regular and symmetric across the transducer center z-axis. Because \( \phi_{\text{max}} \in \phi \), the possible range of \( \delta\phi \) was confined to \([10^\circ, 20^\circ]\).

Figure 11: FSA Tx angular response of a single-element in Field II

The above parameters were used to create the different PWT and DWT temporal-delay profiles, and compound image sequences. Each imaging case was studied by applying the appropriate temporal-delay profiles and observing the resultant transmit energy fields. Initial results showed that the best field coverage was achieved with \( N_{sub} = N_e, N_\phi = \max(N_\phi) \), and \( \delta\phi = \min(\delta\phi) \) for both PWT and DWT.

The PWT region of uniform overlapping insonification was restricted by \( \phi_{\text{max}} \), on account of the intersection between \( \phi_{\text{max}} \) and \(-\phi_{\text{max}}\) as can be seen in Figure 12. The maximum
The extent of the PWT uniform imaging field was found by the following

\[ z_{\text{max}} = \frac{D/2}{\tan(\alpha)} \]  

(27)

where \( D \) is the width of the aperture, found by

\[ D = N_e \times p - k \]  

(28)

With \( \phi_{\text{max}} = 20^\circ \), \( z_{\text{max}} = 26.3 \text{ mm} \).

![Figure 12: Regions of overlapping insonification for (L) PWT and (R) DWT, \( \phi = [-20^\circ, 20^\circ] \)](image)

The DWT case, however, was not affected by a limited extent. Instead, a measurement of the maximum -6 dB angular falloff was used to assess the region of uniform overlapping insonification for the different values of \( r_f \). As stated earlier, \( r_f \) was set to \(-7p\), \(-10p\), and \(-15p\). The magnitude of \( r_f \) determines the angular width of the propagated sector beam by

\[ \beta = 2 \times \arctan \left( \frac{D/2}{r_f} \right) \]  

(29)

where \( \beta \) is twice the arc tangent of the aperture and \( r_f \). In this way, a PWT is equivalent to a DWT with \( r_f \) at infinity.

According to [23] and our simulations the angular response for the elements of the P4-2v was found to be \( \pm 52.5^\circ \) off the transducer center z-axis. Thus to ensure an accurate simulation of DWT and optimal spread from the P4-2v, \( r_f \) was chosen such that the -6 dB
angular falloff beyond $\pm D/2$ approached the angular response. The best fit was found at $r_f = -10p$ for a $-6$ dB angular falloff of $60^\circ$ (see Figure 12). The transducer angular response and inferred optimum FOV from our transmit energy field analyses would then be used to construct an imaging field for FSA pulse-echo analyses using PWT and DWT sequences.

**Aim 1:** To develop and optimize simulated PWT and DWT acquisition sequences and objectively compare the amplitude geometry, resolution, and uniformity of their PSFs throughout the imaging field.

**Methods:** The framework and results of our preliminary work were leveraged to develop pulse-echo (Tx-Rx beamforming) simulations of the PWT and DWT acquisition sequences in the Field II MATLAB environment.

The simulated transducer was based on the geometry and frequency characteristics of the Verasonics P4-2v cardiac array probe (specifications for the P4-2v probe are outlined in Table 1). The virtual imaging field was designed according to the initial results for the transducer angular response and inferred optimum FOV from our transmit energy field analyses detailed in the Preliminary Work & Results section. Point scatterers were populated throughout the ROI defined on $[x, 0, 20]$ mm by $[z, 5, 35]$ mm with regular azimuthal and axial spacing, $\delta_x = 5$ mm and $\delta_z = 5$ mm, respectively. The simulations carried out within this imaging field analyzed the PSFs of each point scatterer in an isolated window spanning $[x_p + (-2, 2)]$ mm laterally by $[z_p + (-1, 1)]$ mm axially–as an 8 mm$^2$ rectangular area about a central point scatterer at $(x_p, z_p)$, for the different imaging conditions. The pixel size within each of the PSF observation windows was chosen as $\delta_x = 0.05$ mm and $\delta_z = 0.05$ mm. Figure 13 depicts the chosen imaging field with defined measurements.

After defining the properties of the medium of propagation, the array transducer, and the imaging field, the Field II function, `calc_scatt_all(...)` was used to calculate the FSA dataset, returning the received relative scattered pressure response for the defined point scatterer for...
all possible Tx-Rx pairings on the 64-element array. Then, using Equations \[13, 14, 17, 18, 20,\] and \[21,\] the characteristic Tx and Rx delay profiles were calculated according to experimental parameters from the Tx energy field analyses (i.e. \(\delta A, N_{\text{sub}}, N_e, r_f, \phi, N_\phi,\) and \(\delta \phi\)).

\(r_f = -10p\) for all DWT simulations. These temporal delay profiles were applied to the FSA dataset to synthetically implement the PWT and DWT wavefronts at each Tx angle, along with the parallel receive focusing wavefronts to every point in the imaging field.

Image frames of the PSFs were obtained by interpolating the Tx-Rx traces to the points within the windowed ROIs. The beamformed sum was then calculated using Equation \[12,\] displaying the magnitude of the complex RF traces in pixel brightness intensities. The image data was log-compressed and displayed in decibels relative to the maximum relative pressure in each PSF frame (see Figure \[14].\) This framework was used to generate PSFs of all point scatterers in the imaging field for all experimental parameter values. Finally, complex pixel-data from the variable angle PSF frames were coherently summed in order to achieve retrospective focusing (compounding) for each Tx scheme. Figure \[15\] shows a compounded PSF image frame from a point scatterer at \((0, 20)\)mm for PWT and DWT. Afterwards, quality metrics including the PSF amplitude geometry, resolution, and image uniformity were used to compare the performance of each transmit scheme.
The PSF amplitude geometry was assessed by calculating the PSF’s main lobe amplitude maximum and the difference between the location of the main lobe amplitude maximum and the ground truth location of the point scatterer in the field (location error). The PSF resolution was assessed by calculating the PSF lateral and axial diameter of the PSF’s main lobe in mm at -3 dB relative to the maximum amplitude of the PSF’s main lobe (lateral and axial resolution). Image uniformity was then calculated by estimating the variance of 1) the lateral resolution, $\sigma_{lat,res}^2$, 2) the axial resolution, $\sigma_{ax,res}^2$, 3) the amplitude maxima, $\sigma_{amp}^2$, and 4) the location error, $\sigma_{loc}^2$, throughout the imaging field.

**Results:** Figure 16 shows the image field comparisons with regard to the (a) axial and (b) lateral resolution, (c) amplitude maxima, (d) location error, and (e) output transmit energy. All field comparison maps were generated by determining the listed metrics for each PSF and using the expression

$$\frac{DWT - PWT}{PWT} \times 100\%$$

(30)

to quantify the percent difference between the values for compounded DWT and compounded PWT image frames throughout the FOV. Maxima within each map are as follows: 120% increase in axial resolution at (20, 20) mm, 40% increase in lateral resolution ≤ 10 mm laterally and ≥ 30 mm depth, 900% increase in amplitude maxima > 15 mm laterally and ≤ 10 mm depth, 350% increase in location error at (20, 20) mm and (15, 35) mm, and > 1000% increase in transmit energy ≥ 15 mm and ≤ 20 mm. In addition, the variance in these quality metrics over the FOV for DWT and PWT are given in Table 2. A comparison of the variances according to the expression

$$\frac{DWT}{PWT} \times 100\%$$

(31)

is placed across the last row of Table 2.
Figure 16: *In silico* image field comparison maps $\frac{DWT}{PWT} \times 100\%$

Table 2: *In silico* variance of quality metrics over FOV in Field II

<table>
<thead>
<tr>
<th>lat res</th>
<th>ax res</th>
<th>max amp</th>
<th>max loc</th>
<th>Tx eng</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT</td>
<td>0.092</td>
<td>$4.3 \times 10^3$</td>
<td>0.006</td>
<td>0.31</td>
</tr>
<tr>
<td>PWT</td>
<td>0.1</td>
<td>$7.0 \times 10^3$</td>
<td>0.11</td>
<td>0.92</td>
</tr>
<tr>
<td>$\frac{\sigma_{\text{DWT}}^2}{\sigma_{\text{PWT}}^2}$</td>
<td>88</td>
<td>570</td>
<td>61</td>
<td>6.1</td>
</tr>
</tbody>
</table>

**Aim 2:**—To implement the optimized PWT and DWT acquisition sequences on a Verasonics Vantage-128 (V-128) research scanner.

**Methods:** Building upon the results of Aim 1, an optimized framework for PWT and DWT acquisition sequences developed *in silico* was adapted for real-time acquisitions using the V-128 research scanner in a phantom imaging study. The V-128 research scanner is equipped
with a simulation package that is accessed within the MATLAB software environment and is used to build and test executable imaging scripts.

As both are designed using object-oriented programming formats, the Field II framework was readily translated to the Verasonics simulation framework. Scan parameters were set to match the P4-2v probe and the PWT and DWT experimental parameters from Aim 1. To test the Verasonics framework, images were acquired from a simple simulated phantom with point scatters at \((-10, 35)\) mm, \((-10, 15)\) mm, \((0, 25)\) mm, \((10, 15)\) mm, \((10, 35)\) mm and attenuation = \(-0.7\) dB/cm/MHz. Tx sequences included angles \([-20^\circ\, 20^\circ]\) with \(10^\circ\) angular spacing and a receive focus was placed at \((0, 25)\) mm. The channel data was pixel-beamformed, envelope-detected, and log-compressed to pixel intensity values. Image frames of simulated point scatterers within the Verasonics environment were compared to those generated within the Field II environment, and thus established a qualitative baseline. Once proven, the Verasonics PWT and DWT acquisition sequences were to be deployed on the research scanner in a phantom imaging study.

**Results:** Verasonics scripts were developed to conduct an image study on a wire-target phantom and assess the feasibility of the optimized acquisition sequences. Figure 17 shows an image generated using the Verasonics simulation tool. These methods served as a platform for implementing custom acquisition sequences on the V-128 system.

**Aim 3:**—To assess performance of the optimized PWT and DWT acquisition sequences on a Multi-Purpose, Multi-Tissue phantom by estimating the image resolution, contrast, optimal FOV (width and depth), and validate objective guidelines for optimal DWT acquisition sequences based on the given speed of sound in the medium, transmit center frequency, aperture width, and desired FOV.
Methods: The platform developed for implementing the optimized PWT and DWT acquisition sequences on the V-128 system was used to investigate a CIRS Model 040GSE Multi-Purpose, Multi-Tissue (MPMT) Ultrasound Phantom, courtesy of the Medical Ultrasound Imaging & Instrumentation Innovation Lab at The University of Memphis. The MPMT phantom is a 7 in × 5 in × 8 in hydrogel polymer-based phantom designed for a variety of image quality assessments (e.g., contrast, uniformity, depth of penetration, etc.). The MPMT phantom includes targets of varying diameter and spacing such as nylon wire targets of 100 µm diameter and several gray-scale, anechoic, and elasticity targets of various diameters and orientations. In addition, the phantom medium is divided into two layers, parallel to the lateral-depth plane, each with different signal attenuation: the low attenuation zone of 0.7 dB/cm/mHz and the high attenuation zone of 0.95 dB/cm/mHz, describing the reduction in received signal intensity with increasing depth and frequency. All data and figures from this study represent acquisitions from the low attenuation zone in order to resemble the attenuation by body tissue (0.5 dB/cm/mHz to 0.7 dB/cm/mHz) [25]. Figures 26 and 25 in Appendix B display the experimental setup for the CIRS MPMT phantom study.

A selection of 100 µm diameter wire-targets including those of the Near-Field Group, Vertical Group, and Horizontal Group were used to define the phantom imaging field (see Figure 18). The FOV spanned an image sector approximately 140° angularly and extending...
60 mm in depth with a pixel density of \(\approx 7\) samples/\(\lambda\) or \(\approx 12\) samples/mm both laterally and axially. Image frames of the MPMT phantom were acquired using the optimized Verasonics framework and the P4-2v probe.

![Image](image.png)

**Figure 19:** Initial results for DWT using Top: Test 2 and Bottom: Test 1 setup

Seen in Figure [19] initial results of the phantom study included image distortions over the Near-Field Group for the DWT acquisition, possibly due to reverberation clutter [29, 30]. This prompted the need for two tests to be performed. Test 1 consisted of the setup shown in Figure [27] from Appendix B, with the probe in contact with the surface of the phantom using ultrasound gel. Test 2 consisted of the setup shown in Figure [28] in Appendix B, with the probe offset from the phantom, in contact with the surface a pool of water approximately 5 mm level above the phantom—the water serving as an alternative acoustic buffer. The axial coordinates of all the points were later readjusted to reflect this initial offset. Figure [19] shows the DWT image recovered using the setup in Test 1 and Test 2. All proceeding processes and
Calculations were performed on data from each test, with Test 1 sacrificing the data at point [1] (compare to Figure 18) due to distortions in both DWT and PWT acquisitions.

Frames containing various features from the MPMT phantom were segmented by finding the precise location of the local maximum within an image window 1.5 mm laterally × 0.75 mm axially. The segmented image frames were minimally processed within MATLAB using algorithms developed in Aim 1. The image data was dB log-compressed and image quality metrics including the PSF image lateral and axial resolution, local amplitude maximum (contrast), maxima location error were calculated using modified scripts from Aim 1. Figure 20 shows the initial images recovered from Test 2 of the phantom study and the measured lateral and axial (-3 dB) resolution for compounded DWT and PWT acquisitions. Lastly, uniformity of the phantom study field with respect to each of the image quality metrics was used to compare the performance of each transmit scheme. From these data inferences on an optimal FOV for each acquisition was derived.

Results: The Verasonics PWT and DWT acquisition sequences were used to investigate the MPMT phantom. Image quality metrics including the image resolution, contrast, location er-

---

Figure 20: Phantom image and -3 dB resolution Top: DWT, Bottom: PWT
ror, and the uniformity of these metrics were used for comparison of the different transmit schemes. Figures 21 and 22 show the image field comparisons with regard to the (a) axial and (b) lateral resolution, (c) amplitude maxima, and (d) maxima location error for Test 1 and Test 2, respectively. All field comparison maps were generated by determining the listed metrics for each PSF and using Equation 30 to quantify the percent difference between the values for compounded DWT and compounded PWT image frames throughout the FOV. Maxima within each map are as follows: ≥ 10% increase in axial resolution at (−11.5, 4) mm and (−30, 40) mm, > 40% increase in lateral resolution beyond 40 mm depth and ≤ ±10 mm laterally, > 100% increase in amplitude maxima beyond 10 mm in the negative lateral direction, and > 50% and > 120% increase in location error at < 10 mm depth and (−20, 40) mm, respectively. In addition, the variance in these quality metrics over the FOV for DWT and PWT are given in Table 3. A comparison of the variances between DWT and PWT for Test 1 and Test 2 according to Equation 31 are placed across rows 3 and 6 of Table 3, respectively.

<table>
<thead>
<tr>
<th></th>
<th>lat res</th>
<th>ax res</th>
<th>max amp</th>
<th>max loc</th>
</tr>
</thead>
<tbody>
<tr>
<td>test 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DWT</td>
<td>0.27</td>
<td>0.019</td>
<td>1.5</td>
<td>0.17</td>
</tr>
<tr>
<td>PWT</td>
<td>0.2</td>
<td>0.013</td>
<td>7.2</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>130</td>
<td>140</td>
<td>20</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DWT</td>
<td>0.2</td>
<td>1.5×10⁻⁴</td>
<td>6.1</td>
<td>0.099</td>
</tr>
<tr>
<td>PWT</td>
<td>0.17</td>
<td>5.2×10⁻⁴</td>
<td>29</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>28</td>
<td>21</td>
<td>110</td>
</tr>
</tbody>
</table>

Table 3: Phantom study variance of quality metrics over FOV using V-128
Figure 21: Phantom study image field comparison maps for Test 1, $\frac{DWT - PWT}{PWT} \times 100\%$

**Discussion**

The experimental approach in Aims 1-3 provided a robust framework for the development of custom algorithms for analyses of pulse-echo PSFs. Observations of the field comparison maps in Figure 16 provide inference on the regions of best performance for DWT and,
alternatively for PWT. Figure 16(a) shows little difference in the axial resolution, except for a few outliers at (10, 10) mm, (20, 15) mm, and (20, 20) mm—with the prior two indicating the improvement in DWT over PWT and the latter vice versa. In contrast, the lateral resolution in 16(b) increases significantly with depth. However, DWT outperforms PWT in regions closer
and beyond the extent of the transducer width and remains nearly equivalent to PWT in the Near-Field within the extent of the transducer. Referring to the data in Table 2, there is a 12% improvement in the uniformity of the field with regard to lateral resolution. Figure 16(c) and (e) appear similar, as expected, indicating that a DWT sequence propagates more acoustic energy beyond the extent of the transducer (especially in at shallower depths). Columns 3-5 of Table 2 represent that the uniformity of the contrast and precision over the FOV improves (a decrease variance of maxima) using DWTs.

In Aim 2, the PWT and DWT acquisition and PSF analysis framework developed using Field II was transitioned to the Versasonics simulation environment and an eventual phantom study on a chosen group of feature targets of identical diameter, specified spacing, and type 100 – µm diameter nylon wire. Observations of the field comparison maps in Figures 21 and 22 provide inference on the regions of best performance for the phantom study. As was shown for the Field II results, Figure 22(a) shows little difference in the axial resolution. The lateral resolution in 22(b), however, increases with depth. The improvement in DWT compared to PWT is again reduced within the extent of the transducer width and outperformed by PWT at all depths except in the Near-Field or areas beyond the extent of the transducer. Figure 22(c) resembles the simulated results, indicating that a DWT sequence propagates more acoustic energy and achieves greater amplitude maxima beyond the extent of the transducer (especially in at shallower depths). 22(d) shows that some degree of location error persists in the areas of optimal transmit energy improvement, but is otherwise similar to the location error distribution for a PWT sequence.

According to Test 2, the variance of the axial resolution $\sigma_{DWT}^2 < \sigma_{PWT}^2$, but data from Test 1 may disqualified this metric. Column 3 of Table 3 indicates that the uniformity of the amplitude over the FOV improves by $\approx 20\%$ using DWTs as compared to PWT. Finally, differences in Column 4 for Test 1 and 2 are indicative of the differences in their experimental setups (i.e. Test 1 uses 12 points and Test 3 uses an additional point in the Near-Field for analysis). Figure 21(d) shows that the outlier skews the calculation of the variance in Table 3. Additionally, distortions present in the Near-Field region for the PWT acquisition also offset the value of the location error metric for point 2 in Test 1, thus further impacting the compari-
Figure 23: Assessed uniformity statistics and imaging FOVs/feature targets

(a) In silico variance of image quality metrics in Field II

<table>
<thead>
<tr>
<th>lat res</th>
<th>ax res</th>
<th>max amp</th>
<th>max loc</th>
<th>Tx eng</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT</td>
<td>0.092</td>
<td>0.007</td>
<td>4.3×10³</td>
<td>0.006</td>
</tr>
<tr>
<td>PWT</td>
<td>0.1</td>
<td>0.001</td>
<td>7.0×10³</td>
<td>0.11</td>
</tr>
<tr>
<td>(%)</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>(V_{\max}/V_{\text{PWT}})</td>
<td>88</td>
<td>570</td>
<td>61</td>
<td>6.1</td>
</tr>
</tbody>
</table>

(b) Simulation Point Grid

(c) Phantom variance of image quality metrics using V-128

<table>
<thead>
<tr>
<th>lat res</th>
<th>ax res</th>
<th>max amp</th>
<th>max loc</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT</td>
<td>0.27</td>
<td>0.019</td>
<td>1.5</td>
</tr>
<tr>
<td>PWT</td>
<td>0.2</td>
<td>0.013</td>
<td>7.2</td>
</tr>
<tr>
<td>(%)</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>(V_{\max}/V_{\text{PWT}})</td>
<td>130</td>
<td>140</td>
<td>20</td>
</tr>
<tr>
<td>DWT</td>
<td>0.2</td>
<td>1.5×10⁻⁴</td>
<td>6.1</td>
</tr>
<tr>
<td>PWT</td>
<td>0.17</td>
<td>5.2×10⁻⁴</td>
<td>29</td>
</tr>
<tr>
<td>(%)</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>(V_{\max}/V_{\text{PWT}})</td>
<td>120</td>
<td>28</td>
<td>21</td>
</tr>
</tbody>
</table>

(d) MPMT Imaging Phantom

This work was composed of the initial development and experimentation of transferable PWT and DWT acquisition sequences using 5 angles ranging \([-20^\circ, 20^\circ]\) within imaging fields greater than 40 mm laterally and [40, 60] mm in depth. The results of these studies provide objective initial conclusions on the optimal application of specific transmit sequences given particular imaging parameters and constraints. Transducer properties, such as the transmit center frequency and aperture width, are known \textit{a priori} and the speed of sound in the...
medium is often assumed uniform and constant. Knowledge of the relationship between the
desired imaging FOV and the values of the input parameters (i.e. \( \delta A \), \( N_{sub} \), \( N_e \), \( r_f \), \( \phi \), \( N_\phi \), and \( \delta \phi \)) provided in this study give adequate starting points for further investigation toward im-
proving DWT acquisition sequences for a variety of imaging applications.

**Limitations and Sources of Error:** Traditional medical ultrasound, and all the temporal de-
lay equations derived in earlier sections, rely on the Euclidean distance formula and an ideal
medium with uniform speed of sound. Yet, within the initial reconstruction algorithm, shift
parameters were estimated using approximations of the normalized number of samples for
each travel path. This manner of interpolation makes an inherent compromise on the accu-
curacy and precision of the FSA beamforming approach. To alleviate interpolation errors, the
reconstruction algorithm was revisited and tuned to use the raw travel paths as inputs. Linear
interpolation using neighboring samples along the complex RF trace was also performed to
achieve the best results. Additionally, the uncertainty of the image quality metrics outlined
above are proportional to the resolution of the ROI pixel grid. A decrease in the pixel size
from 0.1 mm to 0.05 mm reduced propagation of estimation errors, at the cost of marginal
increase in computational burden.

Whereas the Field II delay-and-sum algorithm relies on pixel interpolation using sam-
ples in millimeters for each Tx-Rx travel path of the FSA dataset, the V-128 system is de-
signed to operate in acoustic wavelengths and instead returns receive channel data with re-
spect to a transmit event using all 64 elements of the array. Hence, challenges were encoun-
tered transitioning the FSA beamforming approach in Field II to real-time scanning on the
V-128. The approach used for the PWT and DWT acquisition sequences and PSF analyses
were readjusted for the Verasonics system, while maintaining the degrees of freedom for the
parameters of interest.

The phantom study was conducted using two different experimental setups with the
intention of circumventing the image distortions realized in initial results for Aim 3. Although
the images, quality metrics, and uniformity assessments acquired from Test 2 data were of
better quality compared to those from Test 1, the transducer offset from the surface of the
phantom may have led to image reconstruction errors. Because traditional medical ultrasound
relies on the assumption of an ideal medium, the transition of the propagated ultrasound beam from water \( (c \approx 1480 \text{ m/s}) \) to tissue \( (c \approx 1540 \text{ m/s}) \) over a distance \( \Delta z \leq 5 \text{ mm} \) not only would have reduced the transmitted acoustic power, but also alter arrival times. These errors in the time-of-flight might have contributed to greater PSF diameter (degraded resolution) and reduced amplitude maxima upon coherent summation and compounding. Nevertheless, additional data is required to validate the results of Aim 3, especially in the Near-Field.

Figure 23(a) and (c) again show the calculated variances in Tables 2 and 3 alongside their respective imaging fields in Figure 23(b) and (d). Looking at (b) and (d), the Simulation Point Grid was designed to study image uniformity in areas where each Tx scheme was expected to perform well. On the other hand, the MPMT Phantom was pre-configured and the choice of a subset of wire-targets was therefore limited to what is shown in (d). Obviously, there were fewer targets in the phantom study compared to in silico. However, the distribution of these gives rise to the inconsistent statistics seen between Tables 2 and 3. In the MPMT phantom, after the near-field group, there are no laterally spaced targets in the FOV until reaching a depth of 40mm. This is already beyond the tested depth of the in silico study and resultant maps in ?? showed PWT’s increasing advantage in resolution with increasing depth and on-axis compared to DWT. Evidently, there is a \( \geq 20\% \) decrease in DWT’s uniformity of field with regard to lateral resolution compared to PWT. Although the DWT shows greater uniformity of field in transmit energy and PSF amplitude, additional phantom studies, in which targets are appropriately distributed for optimal FOV for PWT and DWT are required. Ideally, custom phantoms capable of representing a near equivalence to simulated imaging fields, or vice versa, would provide better data for the comparison between imaging schemes.
Conclusion

Significance: The relationship between the desired imaging FOV and the values of the input parameters considered was investigated by performing experiments using simulated data and innovating this approach for real-time scanning. The procedures and results realized in this work may provide a precedence for future studies and contribute toward validating objective guidelines for optimal DWT acquisition sequences. A variety of challenging diagnostic imaging applications including cardiac cycle tracking and motion compensation, high frame rate cardiovascular flow imaging of extracranial blood vessels, and high sensitively transcranial Doppler will ultimately benefit from such advances in ultrasound knowledge, innovation, and instrumentation.

Future Work: Additional studies using similar and improved experimental frameworks will be used to investigate the relationships between field parameters and a broader range of values for the imaging parameters used in this work. The inclusion of more transmit angles, virtual source locations, and apodizations (aperture weighting schemes) will be used to identify the best acquisition sequences and further establish the relationship between the variables. Finally, k-space analysis of the pulse-echo responses may assist objective comparison of PWT and DWT sequences.
References


Figure 24: Normalized PSF with different Tx fractional bandwidths [7]
Appendix B (Experimental Setup)

Figure 25: Experimental setup of phantom study
Figure 26: P4-2v probe and MPMT phantom detail
Figure 27: Test 1 configuration: P4-2v probe and MPMT phantom in contact using ultrasound gel buffer
Figure 28: Test 2 configuration: P4-2v probe and MPMT phantom in offset using water buffer