ASSESSING THE PERFORMANCE OF AN EXPERIMENTAL GRADIOMETER ARRAY

Oluwaseyi J. Bolarinwa

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ASSESSING THE PERFORMANCE OF AN EXPERIMENTAL GRADIOMETER ARRAY

by

Oluwaseyi J. Bolarinwa

A Dissertation

Submitted in Partial Fulfillment of the Requirement for the Degree of Doctor of Philosophy

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This work culminates a journey towards obtaining a doctorate in Philosophy, however, I dare to say there is no Philosophy without people: the premises of this work lies on previous findings of excellent minds, and even this modest addition would not have come about without the help, tutelage, and encouragement from many individuals. First, I thank my Advisor and Professor, Dr. Charles Adam Langston for his in-depth insights, guidance, patience and kindness to me although my postgraduate studies at the Center for Earthquake Research and Information (CERI), University of Memphis. Over these years with him, he showed me that virtues like humility and exceptional intelligence can cohabit in a single being: thank you for communicating these dual virtues to me!

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PREFACE

This dissertation, Assessing the Utility of the Gradiometer Array in Characterizing Seismic Signals Within the Context of a Phased Array, is composed of three articles:

The first article, Calibrating the 2016 IRIS Wavefields Experiment Nodal Sensors for Amplitude Statics and Orientation Errors, co-authored by Charles A. Langston has been published by the Bulletin of the Seismological Society of America, doi:10.1785/0120200275.

The second article, Partitioning Local Seismogram Wavefields Using Continuous Wavelet Transform Methods for IRIS Wavefield Experiment Arrays, co-authored by Charles A. Langston has been submitted to the Geophysical Journal International.

The third article, Comparison of Regional Gradiometer and Phased Array performances from the 2016 IRIS Wavefields Experiment, co-authored by Charles A. Langston has been submitted to the Bulletin of the Seismological Society of America.
ABSTRACT

We examined the utility of an experimental gradiometer array within the context of a phased array using earthquakes and explosion data. These two arrays were fielded during the IRIS Community Wavefields Experiment that was conducted in northern Oklahoma during the summer of 2016. The central idea behind this assessment is to generate wave attribute estimates from the same event using gradiometry and phased array techniques, then appraise the precision of the gradiometer-derived attributes relative to those attribute estimates from the phased array method. The wave attributes estimated from gradiometry are derived from the wavefield and its spatial and temporal gradients; recording instrument and station site errors can introduce uncertainties into the recorded ground motion and its derivatives, and these uncertainties can propagate into the resulting wave attribute estimates from gradiometry. Therefore, it was necessary to calibrate the gradiometer sensors to mitigate the effects of these uncertainties before conducting gradiometry.

The instrument calibration exercise premised on the assumption that a teleseismic wavefield should be identical over the compact gradiometer elements, and that the deviations from this assumption in the teleseismic data are due to instrument and site errors. We used teleseismic P and S waves recorded by the experimental gradiometer to calibrate its sensors. We observed that the estimated correction factors were stable over frequency bands where the teleseismic signals are coherent and/or have high signal-to-noise ratio. We also saw notable improvement in the accuracy of the wave attributes estimates from a local earthquake after calibration.

Prior to gradiometry (and phased array) analysis, we applied two novel denoising and signal partitioning techniques to both gradiometer and phased array records to remove noise and
extract distinct energy packets in the continuous wavelet domain. Following denoising and signal partitioning, we observed improved signal coherence across array elements and we also saw a clearer P wave onset on the horizontal component records. Ultimately, we found that the denosing and signal partitioning exercises are not detrimental to phased array and gradiometry analyses.

To appraise the effectiveness of the gradiometer in characterizing seismic signals, we examined gradiometer and phased array records of three local earthquakes that happened during the 2016 Wavefields Experiment. We carried out gradiometry and phased array analyses on the body and surface wave data from these events and compared gradiometry results with their corresponding phased array equivalents. We found back azimuths estimates from gradiometry that disperse by as little as one degree from associated estimates from the phased array considering the same earthquake. Moreover, in some cases, we observed identical phase velocity estimates from using the two array methods to characterize an event. Overall, we saw a good correspondence between the wave attributes estimated from gradiometry and the phased array, which suggests that the gradiometer can be used to detect, locate, and characterize seismic events.
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wave record filtered between 1 and 4 Hz. (d) Top plot displays the ground displacement due to the fundamental mode at the gradiometer’s reference position. The middle and bottom panels respectively show the azimuth and slowness of the fundamental mode through time. (e) P wave record section filtered between 8.5 and 9 Hz. (f) P wave ground motion at reference position (top panel), wave azimuth and slowness through time are shown in the middle and bottom panels, respectively. The means of the wave attributes within the annotated windows in (b) (d) and (f) were discussed in the body of this article.

3.13 (a) Top panel shows the extracted mystery phase from 94 vertical-component gradiometer waveforms of event 3 (Table 3.1). The records have been scale band filtered between 1.02 and 1.16 s. The bottom panel consists of the estimate of the ground motion at the reference position of the gradiometer, and the mystery phase’s velocity and azimuth through time at the reference position. (b) Top panel displays the “high scale S and first higher mode” wave packet extracted from 97 vertical-component gradiometer records of event 1(Table 3.1). These records were filtered between scales of 1.25 and 1.48 s. The bottom panel display the “high scale S and first higher mode” ground motion at the gradiometer reference position, and the other two plots shows the phase velocity and azimuth of this wave packet at the reference position through time. The wave attributes within the annotated time windows in (a) and (b) were discussed in the body of this article.

3.14 Results from conducting 2D f-k analyses on raw and denoised H array radial records of event 2. The raw and denoised data were bandpass filtered between 0.3 and 1 Hz before beamforming. (a) Raw data record section is shown on the left panel, and the right panel is the f-k plot from analyzing the raw data. (b) Denoised data record section (left panel), and its corresponding f-k plot (right panel).

4.1 The 2016 IRIS Community Wavefields Experiment nodal and broad band sensors deployed near Enid, Oklahoma. The inset shows the location of the experiment over a background of the continental United States map. The circles in the main figure show the sensor distribution of the largest array of the deployment, with an aperture of 13 km. The Golay array elements expressed as pentagons are broadband instruments; the array was composed of 6 subarrays with 3 sensors each. The widest distance between a sensor pair in the Golay array is about 7 km. The Gradiometer was made up of 112 nodal instruments arranged into 7 concentric square rings. Each ring has 16 elements. We carved out the 3X3 km cross array from the largest array of the deployment to serve as a regional phased array. It is composed of 80 nodal seismometers.

4.2 A close-up view of the gradiometer array from Figure 4.1. The array was made up of 7 square subarrays in which each subarray is four times the area of the immediate smaller square. Each subarray is labelled clockwise from its north-west corner such that the first digit is the array code, the second the subarray code, and the last two digits define the location of each element in its respective subarray.

4.3 (a) The regional cross array, which is a subset of the largest array of the Wavefields Experiment and (b) the co-array plot that captures the spatial resolution of the cross array. It is a plot showing the distance and azimuth of every sensor pair of the cross array.
4.4 (a) Vertical component data of event B (Table 4.1) at station 4115 of the gradiometer array. The top panel shows the time-frequency representation of the data in which the magnitude of the wavelet coefficients is plotted as a function of scale and time. The presignal noise manifest as the high scale, low energy strip preceding the signal. The top plot to the left captures the mean noise amplitude between 0 and 60 seconds. The noise amplitude threshold that was computed from the empirical cumulative distribution function (ECDF) method is also shown on the same plot. The annotated time window from 0 to 60 seconds was used to compute the noise model employed to denoise the data. (b) The top panel shows the denoised data obtained from applying the soft thresholding procedure on the raw data in (a), while the second panel is the time domain representation of the denoised.

4.5 Signal partitioning in the continuous wavelet transform (CWT) domain. (a) The same denoised data from Figure 4.4(b) showing a polygon encompassing CWT coefficients of the fundamental mode surface wave on the top panel. In addition, the figure highlights prominent body and surface waves. The lower panel is the time representation of the scalogram above it. (b) A solo plot of the fundamental mode CWT coefficients on the scale time plane after the coefficients of other phases have been zeroed out is shown on the top panel. The panel below is derived from doing inverse CWT of the gated fundamental mode coefficients.

4.6 (a) Extracted seismic phases from the vertical component gradiometer records of Event B (Table 4.1). Top panel shows body wave phases from 98 gradiometer elements. A plot composed of 98 fundamental mode Rayleigh waveforms is shown on the lower panel. (b) Top panel shows plot of extracted body wave signals (using same technique expressed in Figures 4.5 and 4.6) from the cross array vertical component records of event A (Table 4.1). There are 79 waveforms on the plot. Bottom panel shows 97 body wave signals partitioned from the gradiometer vertical component data of event A. These two records will be examined later.

4.7 Phased array and gradiometry results from analyzing the vertical component body wave data of event A (Table 4.1; Figure 4.6b) within a scale band of 0.13-0.17 seconds. The gradiometer attributes have been shifted in time to remove wave propagation effect due to the spatial difference in the locations of the two arrays. The rectangular box that goes across the figure reveal the 0.5 second time window over which we estimated the mean attribute values from the two methods. The three dashed lines showed the time points at which the highlighted beams were computed. The circles with error bars are phases array results, and the red circles for the colored version of this article represent beams with sharp peaks, while those circles in black show diffused beams or beams with multiple peaks. The solid blue lines are the attributes derived from conducting gradiometry on the gradiometer subgeometry composed of squares 1 (smallest), 3 and 5. (a) Phased array reference station (station 3017) waveform filtered within a scale band of 0.13-0.17 seconds. (b) Horizontal phase velocity plot at every other 10 time points, and (c) associated signal back azimuth, also plotted at every other 10 time points.

4.8 Plots of gradiometry-derived mean wave attributes over a length of time (Table 4.2) against those attributes obtained from doing phased array analysis over the same length of time.
These plots are from analyzing body wave data of events A, B and C (Table 4.1). The proportionality line on each plot shows the ideal spots individual results should plot if the attributes from the two methods are identical. (a) BAZ estimates from analyzing vertical component P wave, and transverse and radial components S waves. (b) corresponding horizontal phase velocity estimates.

4.9 Filtered waveforms and 1-D f-k plot from analyzing (denoised and) extracted fundamental mode phase from event A vertical records on the linear array composed of all sensors along the east-west segment of the largest array in Figure 4.1. (a) Record section made up of 139 waveforms that have been filtered between 0.25-0.55 Hz. (b) 1-D f-k plot that reveal increase in slowness with frequency, capturing the dispersive property of the analyzed surface wave.

4.10 (a) Denoised and gated fundamental mode Rayleigh wave from the vertical component gradiometer data of event A. The plot is made up of 97 waveforms. (b) The ground displacement estimates at the reference position (geometric center of the gradiometer) of the gradiometer filtered between 0.25 and 0.35 Hz. (c) Wave horizontal phase velocity at the reference position. (d) Corresponding wave azimuth at the reference position. The results in (b), (c) and (d) were obtained from doing gradiometry on the 97 waveforms from (a) filtered between 0.25 and 0.35Hz. The time window bounded by the rectangular box shows the 9 second time window over which we estimated phase velocity and azimuth.

4.11 Plots showing absolute difference between wave properties obtained through gradiometry and corresponding wave properties derived from the beamforming method – see Figure 4.8 for actual values of the wave properties. (a) Phase Velocity difference. (b) BAZ difference. The circles, diamonds and squares in the plots shows that the associated gradiometry-derived attributes were computed from gradiometer subarray composed of 1 square, 2 squares and 3 squares of the gradiometer array, respectively (Table 4.2). The plots reveal that a gradiometer subarray composed of three gradiometer squares gives the most desirable attributes in the light of the beamforming results.

4.12 (a) Top panel shows raw (undenoised) radial component gradiometer data of event D composed of 95 waveforms, while the lower panel shows the same waveforms after denoising the records using the soft thresholding procedure. (b) Gradiometry result from analyzing the undenoised (raw) data. (c) Results from doing gradiometry on the denoised data. The bounding box shows the 6 second time window over which the mean wave attribute was computed. The results from the denoised records showed improved stability through time compared to the raw, noisy data.

4.13 (a) Waveform and results from conducting 1-D f-k analysis on cross array vertical component body wave data of the explosion. (a) Cross array records of the explosion filtered between 8 and 10 Hz. There are 78 waveforms in the record section. (b) The 1-D f-k result from analyzing the explosion data in (a).

4.14 Gradiometer and phased array vertical component data from the explosion (Table 4.1), and results from conducting gradiometry and phased array analysis on data from the two arrays.
(a) Denoised cross array body wave data composed of 78 waveforms. (b) Denoised body wave data from gradiometer subgeometry composed of squares 1 (smallest), 2 and 3. The plot contains 41 waveforms from the subgeometry. (c) The reference station waveform of the phased array filtered between a scale band of 0.1-0.13 seconds. (d) The symbol system in this panel and the panel below is the same as those of corresponding attribute plots on Figure 4.7 for the colored version of this article. The circles with error bars represent beamforming estimates of the horizontal phase velocity at every 10th time point in the time series, while the solid blue lines are the corresponding gradiometry-derived horizontal phase velocity estimates through time. (e) The associated BAZ to the horizontal phase velocity estimates in the panel above. The phase velocity and BAZ estimates from gradiometry have been shifted in time to remove wave propagation effect resulting from the spatial difference in the locations of the two arrays. The rectangular box that runs through Figure 4.14 (c), (d) and (e) shows the time window over which mean phase velocity and azimuth estimates were computed for the two array processing techniques. The wave properties in this figure were estimated with a scale band of 0.1-0.13 seconds.
Chapter 1 Introduction

The Incorporated Research Institutions for Seismology (IRIS) embarked on a unique Large-N experiment in the vicinity of Enid, Oklahoma in the summer of 2016. One of the objectives of the experiment was to assess the relative utilities of a geodetic array and a phased array that were both fielded during the experiment (Sweet et al., 2018). The geodetic array, also called the gradiometer, was composed of seven concentric rings that spreads over 800 square meters. The data from the experiment offers an unprecedented opportunity to compare the attributes of a seismic event as derived from gradiometry with those attributes obtained from analyzing a 3X3 km phased array record of the same event. A gradiometer can analyze signal wavelength up to 10 times its aperture (Langston, 2007c); this compact property could make the gradiometer preferred in, for instance, geophysical campaigns where phased array-size space is a constraint.

Gradiometry uses a wavefield and its time and spatial derivatives to estimate wave attributes (Langston, 2007c, 2007a), and since the spatial derivative is a function of relative amplitude over space, attendant wave amplitude errors will propagate into resulting spatial gradient estimates, which will ultimately affect the accuracy of the end products: the wave attributes. These amplitude errors could stem from subtle variation in instrument characteristics, differences in sensor site responses and instrument misorientation (Bungum et al., 1971; Cranswick et al., 1985; Ekstrom et al., 2006; Tasič and Runovc, 2013; Langston, 2018). The second chapter of this dissertation dwelled on results and procedures we employed to calibrate the Wavefields Experiment gradiometer in a bid to mitigate amplitude errors prior to doing gradiometry analysis on the calibrated datasets.
The wave ground motion and its spatial gradient at a designated reference location are usually estimated as model parameters of a system of equations, and the solutions of the inverse problem can be negatively impacted by data noise. Moreover, gradiometry models a wavefield as a generalized single propagating wave (Langston, 2007c, 2007a), but heterogeneities within the earth can permit a number of arrivals per time at a recording station. These concurrent arrivals are aberrations to the fundamental premise of gradiometry so that ensuing wave attributes reveal gaps and spikes at these critical times where multiple waves arrive at the gradiometer. The third chapter of this dissertation addressed this problem as we used the CWT denoising and signal partitioning technique (Langston and Mousavi, 2019; Langston, 2021) to remove noise and extract body and surface wave phases in the wavelet domain before conducting gradiometry and phased array analyses on several seismic events.

The central objective of this work is to evaluate the effectiveness of the gradiometer array in characterizing seismic waves, within the purview of a phased array. The wave attributes derived from gradiometry are themselves time series that shows signal azimuth and slowness as functions of time, and in a quest to duly assess the usefulness of the gradiometer we appraised its performance against the phased array method of Langston (2021) that also yields attributes that are time dependent. In Chapter 4, we estimated wave azimuths and phase velocities of three local earthquakes using gradiometry and phased arrays, and we compared the results from the two arrays. We also investigated the effect of seismic noise on gradiometry result as we conducted gradiometry analysis on raw and denoised data from a low signal-to-noise ratio (SNR) record of an event and compared wave attribute estimates from the two datasets. For the rest of this chapter, the underpinning theories of gradiometry and the phased array techniques employed in this study will be introduced to facilitate proper understanding of later chapters.
Wave gradiometry is an array analysis technique that employs the wavefield alongside its spatial and temporal gradients to compute wave attributes. For a 2-D array, gradiometry can be used to estimate wave horizontal slowness and wave azimuth – variations in geometric spreading and radiation pattern are additional attributes that can be computed using gradiometry (Langston, 2007c, 2007a). The technique models the displacement ground motion in the Cartesian coordinate system as a generalized single propagating wave as expressed in equation (1.1) (Langston, 2007c).

\[
\begin{align*}
    u(t, x, y) &= G(x, y) f \left( t - p_x(x - x_0) - p_y(y - y_0) \right) \\
    \text{(1.1)}
\end{align*}
\]

Where the amplitude variations with distance and azimuth from the source are encoded in \( G(x, y) \). \( p_x \) and \( p_y \) are the space-dependent slownesses in \( x \) and \( y \) directions, respectively, while \( x_0 \) is the reference position.

Following Langston (2007c), applying the chain rule to equation (1.1) with respect to the spatial variables gives

\[
\begin{align*}
    \frac{\partial u(t, x, y)}{\partial x} &= A_x(x) u(t, x, y) + B_x(x) \frac{\partial u(t, x, y)}{\partial t} \\
    \text{(1.2a)} \\
    \frac{\partial u(t, x, y)}{\partial y} &= A_y(y) u(t, x, y) + B_y(y) \frac{\partial u(t, x, y)}{\partial t} \\
    \text{(1.2b)}
\end{align*}
\]

Where,

\[
A_x(x) = \frac{1}{G(x, y)} \frac{\partial G(x, y)}{\partial x} \quad \text{(1.3a)}
\]
Langston (2018) showed that the most desirable estimates of the time series in equations (1.2a and b) can be obtained by doing a Taylor series expansion of the wavefield record at each station about a central reference position of the gradiometer array and then solving for the wave displacement and its derivatives at the reference position as follows:

\[
\begin{align*}
A_y(y) &= \frac{1}{G(x, y)} \frac{\partial G(x, y)}{\partial y} \\
B_x(x) &= -\left( p_x(x) + \frac{\partial p_x}{\partial x} (x - x_0) \right) \\
B_y(y) &= -\left( p_y(y) + \frac{\partial p_y}{\partial y} (y - y_0) \right)
\end{align*}
\]

\[1.3b \]

\[1.4a \]

\[1.4b \]

\[1.5 \]

The reference position displacement, \(u(x_0, y_0)\) and its first and higher-order derivatives can be computed by casting equation (1.5) as an inverse problem as in equation (1.6):

\[
\begin{bmatrix}
1 & (x_1 - x_0) & (y_1 - y_0) & \frac{1}{2}(x_1 - x_0)^2 & \frac{1}{2}(y_1 - y_0)^2 & (x_1 - x_0)(y_1 - y_0) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & (x_n - x_0) & (y_n - y_0) & \frac{1}{2}(x_n - x_0)^2 & \frac{1}{2}(y_n - y_0)^2 & (x_n - x_0)(y_n - y_0)
\end{bmatrix}
\begin{bmatrix}
\frac{\partial u}{\partial x} |_{x_0} \\
\frac{\partial u}{\partial y} |_{y_0} \\
\vdots \\
\frac{\partial^2 u}{\partial x^2} |_{x_0} \\
\frac{\partial^2 u}{\partial x \partial y} |_{x_0} \\
\frac{\partial^2 u}{\partial y^2} |_{x_0}
\end{bmatrix}
= \begin{bmatrix}
u(x_0, y_0) \\
\frac{\partial u}{\partial x} |_{x_0} \\
\frac{\partial u}{\partial y} |_{y_0} \\
\vdots \\
\frac{\partial^2 u}{\partial x^2} |_{x_0} \\
\frac{\partial^2 u}{\partial x \partial y} |_{x_0} \\
\frac{\partial^2 u}{\partial y^2} |_{x_0}
\end{bmatrix}
\]

\[1.6 \]
\[ \mathbf{G} \quad \mathbf{m} \quad = \quad \mathbf{d}. \]

The least square solution to equation (1.6) is given by

\[ \mathbf{m} = (\mathbf{G}^\mathsf{T}\mathbf{G})^{-1}\mathbf{G}^\mathsf{T}\mathbf{d} \quad (1.7). \]

Having estimated the three desired time series, the coefficients \( A_x, A_y, B_x, \) and \( B_y \) are estimated using the time domain analytical signal procedure (Langston, 2007b). Then, through coordinate transformation from the Cartesian to the cylindrical coordinate system, the horizontal slowness, \( p \), and signal azimuth, \( \theta \), can be computed as expressed in equations (1.8) and (1.9) (Langston, 2007c).

\[ p = \sqrt{p_x^2 + p_y^2} \quad (1.8) \]

\[ \theta = \tan^{-1}\left(\frac{B_x(x)}{B_y(y)}\right) = \tan^{-1}\left(\frac{p_x}{p_y}\right) \quad (1.9). \]

The phased array technique adopted in this work takes advantage of the reality that a time shift process in time amounts to an equal time shift of the wavelet coefficients in the continuous wavelet transform (CWT) domain (Langston, 2021). The CWT is defined by Grossmann et al. (1989) as

\[ W(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t - \tau}{a}\right)dt \quad (1.10). \]

The independent variables \( a \) and \( \tau \) in equation (1.10) are the scale and time lag, respectively, while the asterisk denotes the complex conjugate of the function. \( f(t) \) is the signal and \( \psi(t) \) is a scaled basis function. The basis function is complex and is also called
The “mother wavelet.” \( W(a, \tau) \) is the wavelet coefficient and it is also complex and can be expressed in the Fourier domain as

\[
\hat{W}(a, \omega) = \sqrt{a} \hat{f}(\omega) \hat{\psi}^*(a\omega)
\] (1.11).

Since the CWT is a linear process, its inverse transform is expressed as follows:

\[
f(t) = \frac{1}{c} \int_0^\infty \int_{-\infty}^{+\infty} \frac{1}{\sqrt{a}} W(a, \tau) \psi\left(\frac{t - \tau}{a}\right) \frac{da d\tau}{a^2}
\] (1.12),

in which \( c \) is given by

\[
c = \int_0^{+\infty} \frac{\hat{\psi}^*(\omega) \hat{\psi}(\omega)}{\omega} d\omega
\] (1.13).

The CWT produces a map of amplitude as a function of scale and time lag for the signal.

Equation (1.10) can be extended to express the CWT of the \( k \)th array channel \( f_k(t) \) as

\[
W_k(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f_k(t) \psi^\star\left(\frac{t - \tau}{a}\right) dt
\] (1.14).

\( W_k \) is a collection of wavelet coefficients which are the time series that are the constituents of the scalogram. The scalogram is a scale-time plot of the transformed time-series record.

Beams of individual wavelet scale time series can be realized by shifting each scale using a plane-wave model across all array stations, and then stacking the shifted scalograms for individual array element. A narrow-scale beam is expressed as

\[
W_{\text{Beam}}(a, \tau) = \sum_k W_k(a, \tau + p_x x_k + p_y x_y)
\] (1.15).

This beam is obtained by adding up the shifted wavelet series for a given scale over all array elements after removing plane wave moveout across the array by means of a supposed horizontal slowness vector.

The broadband analog of equation (1.15) for a specified scale band is given by
\[ \bar{W}_{\text{Beam}}(\tau) = \sum_j \sum_k W_k(a_j, \tau + p_x x_k + p_y y) \]  

These array beams are continuous functions of time; therefore, they can give a continuous estimate of slowness over time (Langston, 2021).

Reference


Cranswick, E., R. Wetmiller, and J. Boatwright, 1985, HIGH-FREQUENCY OBSERVATIONS AND SOURCE PARAMETERS OF MICROEARTHQUAKES RECORDED AT HARD-ROCK SITES.


Chapter 2  Calibrating the 2016 IRIS Wavefields Experiment Nodal Sensors for Amplitude Statics and Orientation Errors

2.1 Introduction

Among other objectives, the 2016 Incorporated Research Institutions in Seismology (IRIS) Community Wavefield Experiment carried out near Enid, Oklahoma, presented a novel opportunity to appraise the utility of an experimental gradiometer array compared to a traditional phased array that was also fielded during the experiment (Sweet et al., 2018). The accuracy of wave characteristics obtained from gradiometry analysis relies primarily on the correct computation of spatial gradients from a noise-free wavefield, and the spatial gradient in itself is a function of amplitude differences between array elements (Langston, 2018). However, instrument error and site heterogeneity can introduce significant errors into recorded wavefield amplitudes (Cranswick et al., 1985; Ekström and Busby, 2008; Ekström et al., 2006; Langston, 2007a). This study presents results and methods used to calibrate the compact, high-frequency, three-component sensors of the wavefield experiment’s small-aperture arrays to mitigate attendant site heterogeneity and instrument errors.

Station amplitude statics describe nonuniform sensor and digitizer gains combined with the effects of local site resonance and instrument installation condition on wave amplitude (Langston, 2018). The response of sensors nowadays is precise to a few percent (Gomberg et al., 1988); this coupled with human error in reporting instrument responses can significantly impact relative amplitudes of array seismometers (Ekström et al., 2006, Langston, 2018). Moreover, nonstandard digitizer gains across array elements can pose a sensitivity problem to recorded data (Hutt and Ringler, 2011). Amplitude statics resulting from geological heterogeneities across sensor sites can also introduce notable amplification to array records (Bungum et al., 1971,
Due to local and shallow site response, a recording site was observed to record more than twice as much wave amplitude as adjacent stations for an event that occurred 24 km away (Mueller et al., 1982). A 100% amplification at a site will significantly affect spatial gradient computations involving the spurious site records.

The precisions of horizontal component orientations within the instrument case are typically given to at least 0.1°, but the accuracy of published field orientation azimuths is not well known and difficult to ascertain (Ekström and Busby, 2008). GSN seismic instruments have been misoriented by as much as 10° (Larson, 2000; Larson and Ekström, 2002). The resultant effect of instrument misorientation and amplitude statics can considerably degrade wave attribute estimates (in form of wave azimuth and slowness) obtain from gradiometry analysis on array data (Poppeliers, 2010; Baker and Langston, 2016; Langston, 2018).

The main rationale behind this study is to implement a method to quality-check the high-frequency nodal data acquired during the 2016 IRIS community experiment before conducting gradiometry analyses on the array records. To achieve this goal, we extend the array calibration method of Langston (2018) to high-frequency nodal instruments based on the premise that a common wavefield should be recorded over a small-aperture array using teleseismic observations. In particular, we seek to ascertain if nodal array elements can be calibrated so that we can do ensuing spatial gradient computations to 1% accuracy or less. Unlike the earlier study that used zero-lag correlations of broadband array records to find relative amplitudes and in situ sensor misorientation angles, we attempt to correct for wave propagation in the high-frequency nodal records by using the maximum of the correlations of array records to compute instrument correction parameters. This step is necessary to enhance signal coherence across array waveforms which ultimately aids the precision of the estimated correction parameters. We
analyzed teleseismic records to obtain amplitude correction factors (ACF) and orientation correction factors (OCF) that can be used to calibrate local and regional event data before gradiometry analysis. Then, we applied the ACF and OCF results to data from a M4.4 earthquake that occurred 102 km away from the gradiometer array center. Ultimately, we compared the gradiometry analysis results we obtained from the calibrated dataset with those from the uncalibrated array records to evaluate the performance of the array calibration technique on high-frequency nodal instrument data.

Another motive for this work is to quantify relative misorientations of seismic array sensors. The horizontal components of the 2016 array sensors were aligned by means of magnetic compasses (Sweet et al., 2018). This method of orienting seismometers is cheap, but could be prone to large errors: inaccuracies emanating from sensor sites within proximity of automobiles, incorrect declination, etc. (Ringler et al., 2013). Thus, beyond its implementation in gradiometry, the OCF results also have simple but important implications for any three-component field deployment. This work aims to mitigate the attendant influence of instrument and site errors on the 2016 IRIS community experiment’s gradiometer records.

2.2 IRIS Wavefields Experiment

A community wavefield experiment was conducted in the summer of 2016 near Enid, Oklahoma by IRIS personnel and university participants including postdocs and graduate students. The experiment consisted of fielding three seismic arrays over a period of 5 months. The largest of the arrays was a 13-km aperture configuration consisting of 254 three-component, 5 Hz nodal sensors installed from June 20 to July 20, 2016 – this array will be subsequently referred to as the “H array” (Figure 2.1). An unconventional regional array, the “Golay” array,
was made up of 18 broadband Guralp CMG-3T sensors; its widest sensor-sensor distance is about 7 km. This broadband array was decommissioned 5 months after its installation on November 10, 2016 (Sweet et al., 2018).

The third array deployed during the experiment was a 1-km aperture gradiometer composed of 112 three-component, 5 Hz nodal seismometers. It consisted of 7 concentric square subarrays, with each subarray being four times the area of the immediate smaller square. Each ring of the gradiometer had 16 sensors (Figures 2.1 and 2.2). The gradiometer was deployed over the same time period as the H-array.

Besides the gradiometer, two gradiometer-style, cross-shape, subarrays were embedded around the junctions of the two longitudinal segments of the H-array and their east-west counterpart (Figure 2.1). For the purpose of this current study, the cross subarray around the west junction of the H-array will be designated as the D-array, while its sister cross subarray to the east will be referenced as the E-array. Moreover, these two subarrays will be jointly named cross-arrays going forward. The E-array is comprised of 28 seismometers, while the D-array had 22 sensors. The nodes of these subarrays were sited 33 m apart, and they share the same deployment history with the H-array – in fact, these two subarrays are subsets of the H-array (Sweet et al., 2018). In all, the Oklahoma seismic arrays are made up of 370 nodal and 18 broadband seismometers, and the records from these sensors are archived under IRIS array code “YW (2016-2016)” at the IRIS Data Management Center (DMC). In addition, preceding installation, the wavefield experiment station sites were located to centimeter precision with a real-time kinematic GPS unit; each node was buried with its top being about 3-5 cm beneath the surface.
Figure 2.1: Spatial distribution of array elements for the 2016 IRIS Community Wavefield Experiment. Top-right inset shows the location of the IRIS experiment (the filled circle) on the map of continental United States. The filled circles in the main figure are positions of the three-component nodal seismometers made up of 254 elements – also called H-array - and the array of nested square symbols is the gradiometer that comprises of 112 three-component 5Hz nodal seismometers. The spatial distribution of the Golay array is shown by the distributed triangles; the array is made up of 18 3-component broadband sensors. The two subarrays circled at the junctions of the H-array are gradiometer-styled arrays that were also considered in this study; the subarray to the west is named D-array, while the one to the east is the E-array.
Figure 2.2: The gradiometer showing its nested square subarrays and the station codes of its 112 sensors. The first station code digit indicates the gradiometer array, while the second digit specifies the subarrays with the innermost subarray designated as 1 and the outermost subarray as 7. The last two digits of the station code describe the clockwise numbering of each subarray element beginning from the north-west corner.
And the horizontal components of the nodal sensors were oriented onsite using magnetic compasses (Sweet et al., 2018).

2.3 Data

The Oklahoma array nodal sensors recorded 25 teleseisms with magnitudes greater or equal to 5.5 within the period they were deployed. We obtained event magnitudes and other hypocentral parameters from the National Earthquake Information Center (NEIC). Many of these event records could not be used for array calibration due to low SNR resulting from their large epicentral distances from the Oklahoma arrays and the high frequency nature of the sensors. However, a suitable magnitude 5.7 earthquake happened 2232 km away from the gradiometer’s center in the vicinity of Pinotepa de Don Luis, Mexico while the array was deployed (Table 1). The gradiometer records of this event exhibited a mean SNR (ratio of rms amplitude of the signal window to the rms amplitude of the pre-event noise window) of 21 on nodal vertical components. The next closest teleseism to the Oklahoma arrays was a magnitude 6.3 earthquake that occurred 4415 km away, close to Rosa Zarate, Ecuador (Table 1). Ecuador event vertical records from the gradiometer have a mean SNR of only 4.5. By reason of these statistics, we chose the high-SNR Mexico event over other archived teleseisms to calibrate the Oklahoma small-aperture arrays. Henceforth, we will refer to the Pinotepa de Don Luis, Mexico earthquake as event 1 and call the Rosa Zarate, Ecuador earthquake event 2.

Table 2.1: Parameters of teleseismic events used for array calibration. Event 1 occurred in Mexico, while Event two happened in Ecuador. Date format: yyyy-mm-dd.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date/time</th>
<th>Latitude (deg)</th>
<th>Longitude (deg)</th>
<th>Depth (km)</th>
<th>M\textsubscript{w}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2016-07-11T02:11:04.800Z</td>
<td>0.5812</td>
<td>-79.638</td>
<td>21</td>
<td>6.3</td>
</tr>
</tbody>
</table>
The high-frequency nature of the nodal seismometer is inherent in its response curve (Figure 2.3) with a corner frequency of 5 Hz. But the flat segment of the response curve is beset by high-frequency noise acquired at measuring stations (Figure 2.3) so that the most favorable frequency band for the calibration analysis is within 0.1 and 2 Hz. This is the frequency range where the signal spectrum clearly rides above the pre-event noise spectrum. In addition, we will subsequently show that the optimum signal coherence that is most desirable for the instrument calibration exercise exist within this frequency range.

![Figure 2.3](a) Response curve for the nodal instrument at station 4101 and (b) associated signal and pre-event noise amplitude spectra plots of event 1 recorded at the station. SNR is favorable for instrument calibration between 0.1 and 2 Hz.

We removed the same nominal response of the nodal system (obtained from IRIS) from the data to extract the ground motion at each station. This allowed waveform comparison across array records before embarking on array calibration. Five of the nodal array sensors returned no data through the course of the deployment (Sweet et al., 2018), and we took out a number of stations with transients on one or more components in this study. The entire nodal instruments recorded data at a sampling rate of 250 samples/second.
We filtered high SNR teleseismic records of events 1 and 2 within several frequency bands between 0.1 and 2 Hz. As will be later emphasized, this is the frequency range where SNR and signal coherence is optimal for calibration. We applied a 10% (5% on each side) cosine taper on the time series before applying a single-pole, zero-phase Butterworth filter within frequency bands of 0.1-0.5, 0.25-0.5 and 0.5-2 Hz to these datasets.

Following calibration, we will use the instrument calibration parameters obtained by analyzing event 1 to calibrate a local event located 102 km west of the gradiometer array (Table 2). We will later use this local event to assess the effect of array calibration on gradiometry results.

Table 2.2: Parameters of the local event used to appraise calibration result. Date format: yyyy-mm-dd.

<table>
<thead>
<tr>
<th>Date/time</th>
<th>Latitude (deg)</th>
<th>Longitude (deg)</th>
<th>Depth (km)</th>
<th>MwT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2016-07-09T02:04:27.400Z</td>
<td>36.4638</td>
<td>-98.7584</td>
<td>7.242</td>
<td>4.4</td>
</tr>
</tbody>
</table>

2.4 Array Calibration Method

The analysis initially followed the theory and procedures detailed in Langston (2018) for calibrating small-aperture array elements for amplitude statics and instrument orientation errors. The basis of this method is to employ inversion techniques to extract calibration parameters from a system of equations involving zero-lag correlations of ground motions records of an array. Its end goal is to compute calibration parameters that can be used to correct small-aperture array data for instrument and site errors. However, unlike Langston (2018) who calibrated a broadband small-aperture array, we attempted to calibrate untested high-frequency nodal seismometers in a bid to later use accurate estimates of array record amplitudes for gradiometry studies.

Our calibration exercise will predicate on four assumptions:
1. The horizontal components are misoriented but orthogonal, while the vertical component is correctly oriented.

2. The instrument properties – in form of gain, sensitivity and orientation – are constant through time.

3. Each component has a distinct but constant amplitude multiplier.

4. Teleseismic wavefield is matching in phase and amplitude on all the instruments.

The first assumption stems from the nature of the inverse problem that relates the calibration parameters to the inner products of array element signals. This supposition ultimately permits the vertical and horizontal components inversions to be carried out separately.

The input ground motion, \( u_3(t) \), along the vertical axis from a teleseismic event can be related to recorded vertical displacement, \( z_i(t) \) at the \( i \)th station by the following equation:

\[
z_i(t) = c_i u_3(t)
\]  \hspace{1cm} (2.1)

Where the amplification (or deamplification) resulting from amplitude static sources is encoded in the amplitude multiplier, \( c_i \). The individual amplitude multiplier at each array station can be estimated by applying the least squares technique to solve a system of equations involving the amplitude multipliers and corresponding amplitude ratio between station records; sensor amplitudes are corrected in relation to the optimum array average in the least squares sense.

Having evaluated \( c_i \), equation (2.2) expresses how the recorded vertical waveform, \( z_i(t) \), can be corrected for amplitude static errors.

\[
\hat{u}_3(t) = \frac{1}{c_i} z_i(t) = \gamma_i z_i(t)
\]  \hspace{1cm} (2.2)

The input ground motion, \( u_3(t) \), has been replaced with its estimate (\( \hat{u}_3(t) \)) in equation (2.2) because the input ground motion cannot be found independently (Langston, 2018). By
"input ground motion", we refer to the ground motion devoid of instrument and site response errors. Thus, the "input ground motion" is the actual ground motion. \( \gamma_i \) in equation (2.2) is the vertical component ACF.

The horizontal component inversion is more involved as the ACF and corresponding OCF are coupled into a non-linear system of equations. To obtain the model parameters, going by the coordinate system in Figure 2.4, the horizontal component displacements are modelled by equations (3a) and (3b):

\[
e_i(t) = a_i [u_1(t) \cos \delta_i - u_2(t) \sin \delta_i] \quad (2.3a)
\]

\[
n_i(t) = b_i [u_1(t) \sin \delta_i + u_2(t) \cos \delta_i] \quad (2.3b)
\]

Where \( e_i(t) \) is the recorded east-west displacement and \( n_i(t) \) is the observed north-south displacement at the \( i \)th station. \( u_1(t) \) and \( u_2(t) \) are the ground displacements in the true geographical coordinate system as illustrated in Figure 2.4. \( a_i \) and \( b_i \) are the amplitude multipliers that encodes the effects of amplitude statics on the recorded east-west and north-south components of the \( i \)th station. \( \delta_i \) is the deviation of the horizontal components of station \( i \) from the orientation of the horizontal components of the reference station.

Equations (2.3a) and (2.3b) can be solved for ground displacements in the true geographical coordinate systems to get

\[
u_1(t) = \frac{1}{a_i} e_i(t) \cos \delta_i + \frac{1}{b_i} n_i(t) \sin \delta_i \quad (2.4a)
\]

\[
u_2(t) = -\frac{1}{a_i} e_i(t) \sin \delta_i + \frac{1}{b_i} n_i(t) \cos \delta_i \quad (2.4b)
\]

Replacing \( \frac{1}{a_i} \) with \( \alpha_i \) and \( \frac{1}{b_i} \) with \( \beta_i \) in equations (4a) and (4b) gives

\[
u_1(t) = \alpha_i e_i(t) \cos \delta_i + \beta_i n_i(t) \sin \delta_i \quad (2.4a)
\]

\[
u_2(t) = -\alpha_i e_i(t) \sin \delta_i + \beta_i n_i(t) \cos \delta_i \quad (2.4b)
\]
Figure 2.4: Coordinate system showing angular deviation ($\delta_i$) of potentially misoriented horizontal components at the $i$th station (the primed system) relative to the true north and true east (the unprimed system).

Where $\alpha_i$ is the east-west component ACF, $\beta_i$ is the north-south component ACF, and $\delta_i$ is the corresponding OCF, all for the $i$th station. In order to compute these ACF and OCF factors, it is necessary to assume that one of the horizontal components (in this case, the east-west component) of a particular station was immune from instrument and site errors. This station will be henceforth called the reference station, with an ACF of one and an OCF of zero. All the horizontal components will be calibrated in relation to the reference station.
The second assumption has been critically examined by Langston (2018) who looked into the calibration parameters as a function of time using teleseisms that happened over a period of two years. The correction factors observed over the two-year period were notably stable and precise regardless of the reference station used for the horizontal components’ calibration. The third assumption inherently supposes that the instrument correction factors are frequency independent. It also assumes implicitly that each sensor has an identical pole-zero response function. We will test this supposition later by band-pass filtering and discuss the result.

The fourth assumption above was generally valid for Langston (2018) since broadband sensors were calibrated using teleseisms whose horizontal wavelengths were about two orders of magnitude greater than the array dimension. Thus, all elements over the array effectively measured identical teleseismic ground motions in both amplitude and phase. The threshold requirement to faithfully meet this criterion is that an array aperture should be at most one-tenth the horizontal wavelength of the waveform that will be used to calibrate the array sensors (Langston, 2018). This fourth assumption does not quite hold for the high-frequency nodal sensors and we will further examine it in the next section. Given that signal coherence often has a negative correlation with frequency (e.g., Toksöz et al., 1991; Zerva and Zervas, 2002) it is necessary to examine the coherence of the high-frequency instrument records used for this study before using the dataset to calibrate the nodal arrays.

2.4.1 Wavefield Coherence

For this analysis, we computed coherence in the time domain according to equation (2.5), after its continuous form by Langston (2006). For a given dataset, coherence range from -1 to +1 such that an outcome of +1 reflects perfect semblance between two waveforms (Langston, 2006).
The proposition that the teleseismic input wavefield recorded by all instruments match in phase and amplitude may not be necessarily true for the high-frequency instrument records as signal incoherence could prevent phase alignment. As an example, zero-lag correlations of the east-west components of H-array records for event 1 reveal coherence as low as zero between the reference waveform and approximately 3 and 6 km distant station records (Figure 2.5). The pronounced decorrelation with distance on the horizontal components data is likely due to instrument misorientation. Even vertical component data coherence degrades by as much as 37% along the 5-km north-south segment to the east. In addition, signal similarity between the smaller aperture gradiometer waveforms on the east-west component diminished by 13% for one of the remote stations from the reference station (Figure 2.6). The loss of signal coherence across the arrays precludes the desired criterion of phase uniformity.

$$C_{xy} = \max \left( \sum_{k=-N}^{N} x[k] y[k + \tau] \right) \left( \sum_{k=-N}^{N} (x[k])^2 \sum_{k=-N}^{N} (y[k])^2 \right)^{1/2}$$ (2.5)

Minimum horizontal wavelength for teleseismic S waves filtered between 0.5 and 2 Hz that travel at a horizontal phase velocity of 5.5 km/s is about 3 km; this signal wavelength is less than the 10 km threshold wavelength required for a wave to be suitable for the gradiometer sensor calibration. More so, the most remote station to the south of the central station of the H-array receives S waves on the east-west component close to 1 second ahead of the central station, and the east-west component waveform at this station degrades by approximately 82% in coherence compared to the central station waveform. For these reasons, we applied a time-shift (determined by maximum cross-correlation) to all station records relative to corresponding component waveform at a reference station to remove the effect of wave propagation on the data before embarking on the calibration exercise (Figures 2.5 and 2.6).
Figure 2.5: (a) Zero-lag and (b) maximum correlation coefficient plots of H-array records of event 1 bandpass filtered between 0.25 and 0.5 Hz. Waveforms were time-shifted relative to station 1064, which is at the center of the H-array. The Z component coherence was obtained using the P wave time window, while the horizontal component coherences were computed from the S wave time window.

Moreover, the horizontal component inversion seeks to solve a nonlinear system of equations via the iterative least squares technique. The known quantities of this nonlinear problem are correlations of the reference waveform with those of other stations. Being a nonlinear problem, significant signal attenuation at stations far away from the reference station can degrade waveform amplitude which can potentially undermine the accuracy of the estimated model parameters. For this reason, we calibrated the horizontal components of the arrays “piecemeal” by grouping nearby stations into subarrays. Due to the segmented calibration procedure, we will call the very first reference station the “master station” onward.
Using stations away from the master station as subsequent reference stations, we corrected all uncalibrated station sensors within 200m of a reference station relative to the reference station (Figure 2.6). If there are no uncalibrated stations within the stated distance, we selected the uncalibrated station closest to the current reference station and calibrate it relative to the reference station. It should be noted that we incorporated all calibrated station (that are not prior reference stations) records within 200 m of each reference station into the system of equations used to calibrate station sensors; however, we did not update the correction factors of the already calibrated stations in subsequent inversions where we used them. The 200 m radius is less than 10% of the wavelength of a 2 Hz teleseismic S wave traveling at a horizontal phase velocity of 5.5 km/s across the gradiometer. In contrast to the small-aperture arrays, the H-array sensors are
mostly 100 meters apart, therefore, we used a moving window of 400 m to calibrate the H-array horizontal components.

2.4.2 Effect of Noise in the Inversion

We computed the correction factors used to calibrate the small-aperture arrays by inverse techniques, which can be significantly influenced by noise (Aster et al., 2013). In a bid to mitigate the effect of noise on inversion results, we carried out a synthetic test to appraise the threshold noise level allowed in the data to obtain accurate correction factors. Specifically, the synthetic test sought to evaluate the amount of noise one can tolerate to have the percentage difference between true and computed ACF less than 1%.

For the synthetic test, we designated station 4101, which is close to the gradiometer center, as the reference station. We denoised thirty-minute-long three-component teleseismic records of event 1 at 4101 after Langston and Mousavi (2019) and extracted a five-minute time window containing P waves from the vertical component data, while we obtained an equal length time window that included S waves from the horizontal components (Figure 2.7). We preferred the S wave window data for the horizontal components because of the enhanced amplitude of the S wave relative to the P wave on the horizontal waveforms (Figure 2.7). Following phase extraction, we perturbed the resulting windowed records with uniformly distributed random amplitude multipliers (inverse of ACF) and OCF values (according to equations 2.1, 2.3a and 2.3b) to generate synthetic seismograms for the other gradiometer stations. Then, we [linearly] added scaled amplitudes of pre-event noise at each station to corresponding synthetic seismograms. Thus, for each set of scaled amplitudes of pre-event noise added to the synthetic waveforms, we were able to generate a SNR distribution for the gradiometer stations.
Figure 2.7: Reference station seismograms used to generate synthetic seismograms for other gradiometer stations. The three seismograms, obtained from station 4101 records of event 1, have been denoised before extracting the time windows outlined in red.
We calibrated the resulting noise-corrupted signals to obtain computed ACF and OCF. After calibration, we compared the computed correction factors at each station for individual scaled amplitudes of pre-event noise with their true counterparts. Our subsequent analyses used the optimal SNR distribution – that just kept the percentage difference between true and computed correction factors less than 1% - as a quality control measure for the real dataset before inverting teleseismic data for correction factors. It should be noted that we computed SNR for each waveform as the ratio of the rms amplitude of the signal window to the rms amplitude of the pre-event noise window.

The synthetic tests results revealed that to compute calibration parameters to less than 1% error accuracy, the mean of the SNR distributions of the vertical, north-south and east-west components of the gradiometer must be 6, 11 and 11, respectively – the associated orientation residuals to these statistics are mostly less than 0.2° (Figure 2.8). We realized these optimal statistics between 0.5 and 2 Hz; predictably, the SNR of the gradiometer data is maximum within this same frequency band. The skewness of the horizontal components residuals from zero-mean has to do with the tradeoff of the horizontal component ACF with the OCF in the inversion: the inversion is more sensitive to the OCF. Given the listed thresholds, we did not include stations having at least one teleseismic signals with SNR smaller than two standard deviations of the mean SNRs in the calibration exercise. In addition, we discarded event 1 waveforms at some stations because of local transients observed on one or more station components.
2.5 Array Calibration Results

2.5.1 Entire Array

We made attempts to calibrate the entire H-array for instrument and site errors and, to a certain degree, the H-array calibration fairly improved signal relative amplitudes, particularly for the vertical components (Figure 2.9). We observed the correction factors that yielded the optimal boost in signal coherence for the vertical component records are between 0.25 and 0.5 Hz.

Figure 2.8: Residuals showing optimal difference between the actual and computed ACF and OCF from the noise-corrupted synthetic records for the vertical, north-south and east-west components of the gradiometer stations. These results were obtained between 0.5 and 2 Hz.
Figure 2.9: (a) Uncorrected and (b) corrected H-array waveforms filtered between 0.25 and 0.5 Hz. Two hundred and four waveforms from event 1 (Table 1) have been plotted over each other in each plot. \( u_z \), \( u_1 \) and \( u_2 \) are respectively vertical, east-west and north-south components of the associated nodal instruments. The vertical component records show some relative amplitude improvement after inversion.
Maximum correlation coefficients for H-array vertical component prior to calibration are all greater than 0.9, while a number of the corresponding horizontal component records showed little or no coherence with the master station’s horizontal component waveforms (Figure 2.10). Following inversion, the vertical component result shows a fair improvement in relative amplitudes (Figure 2.9). We saw the most prominent boost in relative amplitudes at stations 1009, 1074 and 1085. These three stations exhibited pronounced amplitude dissimilarities from other station vertical records before inversion. On the other hand, horizontal component waveforms showed at best a minute change in signal coherence after calibration (Figure 2.9). For example, the maximum correlation of the most remote station’s north-south waveform with that of the master station only change by 0.0056 after calibration.

The vertical component ACF for the H-array generally vary by 10.7% (maximum deviation) with a precision of 3.6% (standard deviation), we did not include the three outlier stations in this statistics (Figure 2.11). Associated horizontal components’ ACF largely spread up to 25.1% and have a precision of 7.8%. The H-array sensor OCF vary in general by as much as 18.6° and are precise to within 7°.
Figure 2.10: (a) Maximum correlation coefficients before and (b) after calibration for the H-array horizontal components records from event 1 (Table 1). All correlations were made relative to station 1064 (the master station) records. There is little to no improvement in signal coherence after calibration on the two horizontal component records.
Figure 2.11: H-array ACF and OCF computed between 0.25 and 0.5 Hz from event 1 waveforms. The three outlier ACF of the vertical component corresponds to the three waveforms with distinct amplitude contrast in Figure 2.9.

2.5.2 Gradiometers

On all accounts, the instrument calibration exercise improved waveform coherence on all component records across the small-aperture arrays (Figures 2.12 and 2.13). We obtained the results that show the most improvement in signal coherence between 0.25 and 0.5 Hz – with the exception of the horizontal component inversion for the D-array that best enhances signal coherence between 0.1 and 0.5 Hz. We calibrated each station of the gradiometer, D-array and E-array relative to reference stations 4101, 2017 and 3017, respectively.
Figure 2.12: (a) Uncorrected and (b) corrected gradiometer seismograms filtered between 0.25 and 0.5 Hz. Ninety-nine waveforms from event 1 (Table 1) have been plotted over each other in each plot. $u_z$, $u_1$ and $u_2$ are respectively vertical, east-west and north-south components of the associated nodal instruments. There are observable relative amplitude improvements after inversion, most especially on the vertical component. The $Z$-component teleseismic signal at station 4607 exhibits selective attenuation that are not observed on local event record at the station.
Figure 2.13: (a) Uncorrected and (b) corrected E-array seismograms filtered between 0.25 and 0.5 Hz. Twenty-four waveforms from event 1 (Table 1) have been plotted over each other in each plot. $u_z$, $u_1$, and $u_2$ are as designated in Figures 2.9 and 2.12. There are also noticeable relative amplitude improvements after inversion, particularly on the vertical component.
The horizontal component inversions typically converged after four iterations, or less (Figure 2.14), whereas the vertical component inversions are intrinsically linear (void of data and null spaces) so that we precisely determined individual vertical component ACF. Maximum correlation coefficients for gradiometer vertical component before calibration were higher than 0.99, while the associated horizontal components’ correlation coefficients are greater than 0.91. After inversion, correlation coefficients of the gradiometer’s horizontal components are better than 0.98 (Figure 2.15). The coherence of the least correlated horizontal component waveform – which is unsurprisingly distant from the reference station – increased from 0.93 to 0.99 following inversion. In situ estimates of ACF for the gradiometer vertical component typically vary by 7.8% (maximum deviation) with a one standard deviation precision of 2.3% (Figure 2.16 and Appendix A). Horizontal component ACF for the same array generally disperse by 13.7% and are precise to within 4.2%, and its OCF distribution mostly spread by as much as 8.2° with 3° precision. In general, improvement in the three-component relative amplitudes is clearly evident after inversion (Figures 2.12 and 2.13).

The inversion process also enhanced the relative amplitude characteristics of the two cross-array records. Before inversion, maximum correlation coefficients for the array’s vertical components are better than 0.997, and those of corresponding horizontal components are greater than 0.96. Following inversion, the coherences of the horizontal component waveforms are generally higher than 0.99 for the two cross-arrays. The ACF of the cross-arrays’ vertical components typically vary up to 5.1% (maximum deviation) with a precision of 2.4% (standard deviation) (Figure 2.17 and Appendix A), and their horizontal components’ ACF largely spread up to 9.3% and have a precision of 3.6%. OCF for these subarrays vary in general by as much as 15.2° and are precise to within 6.5°.
Figure 2.14: Representative plot of the least square residual over ten iterations for the horizontal components’ inversion using event 1 (Table 1) records from the gradiometer. The inversion converged following three iterations, with the residuals exhibiting about an order of magnitude boost in the fit of relative amplitudes between the horizontal components of the gradiometer elements.
Figure 2.15: (a) Maximum correlation coefficients before and (b) after calibration for the gradiometer horizontal components records from event 1 (Table 1). All correlations were made relative to station 4101 (the master station) records. There is a clear boost in signal coherence after calibration on the two horizontal component records.
Figure 2.16: Gradiometer ACF and OCF computed between 0.25 and 0.5 Hz from event 1 waveforms. The outlying ACF on the vertical component corresponds to station 4607 of the gradiometer; event 1 record amplitude on this station’s vertical component is about half of those of adjoining stations.
Figure 2.17: (a) E-array and (b) D-array ACF and OCF distributions. All correction factors were obtained between 0.25 and 0.5 Hz, except the horizontal component ACF and OCF of D-array that were computed between 0.1 and 0.5 Hz.
It is tough to evaluate the uncertainties in the correction factor estimates for all three small-aperture arrays because of the paucity of high SNR teleseismic event records during their deployment. However, the observed improvement in relative amplitude behavior of the waveforms across the three arrays lends credence to the fidelity of the computed correction factors.

2.6 Discussion

For this study, we mostly computed the optimal correction factors between 0.25 and 0.5 Hz; the correction factors we obtained over the frequency band of 0.5 to 2 Hz also exhibited satisfactory precision with reference to the optimal parameters (e.g., Figure 2.18). The ACF we evaluated from 0.25 to 0.5 Hz are in general within 1% of those we computed between 0.5 and 2 Hz for the three small-aperture arrays. And differences in OCF that we obtained at these two frequency bands are mostly less than 0.4° for the three array setups. The coherence across the three arrays is best in the frequency band from 0.25 to 0.5 Hz, while, as earlier disclosed, the SNR of the uncalibrated data set is maximum between 0.5 and 2 Hz. Therefore, it appears high SNR and/or excellent signal coherence aided the precision of the estimated correction factors across different frequency bands.

Also, we made attempts to calibrate the entire 13-km aperture nodal array sensors for amplitude statics and instrument orientation errors. Analyses of signal similarity across the array reveal minimal coherence at most remote stations from the reference station, especially on the horizontal component records. For instance, we observed coherence value as low as 0.32 (maximum correlation coefficient) on the horizontal components of the H-array at a distant station (Figure 2.5).
Figure 2.18: Comparing correction factors computed between 0.25 and 0.5 Hz with those obtained between 0.5 and 2 Hz for gradiometer elements. (a) The left column shows ACF and OCF plots at the listed frequency bands. (b) The column to the right shows the corresponding variability between station correction factors computed at the two frequency bands.
The loss of coherence across this larger array is also evident in the systematic increase in the ACF values with distance from the master station on all three components (Figure 2.11). The limited signal coherence at distance along the array precluded useful waveform relative amplitude improvement after calibration.

Given the systematic increase in H-array horizontal component ACF values away from the master station, it is likely that each time a new reference station is chosen, the inversion will change the orientation and amplitude factors of all stations involved in the inversion. We tracked these changes for each station as a measure of error in the H-array calibration. The maximum absolute deviation of OCF from zero are as large as 19°, while the maximum absolute deviation of the horizontal components ACF from 1 are as big as 9. We also tracked these changes for the horizontal components of the gradiometer. The gradiometer’s maximum absolute deviation of OCF from zero are mostly less than 0.5°, and the maximum absolute deviation of its horizontal components ACF from 1 are generally less than 0.02. We calibrated some stations only once; therefore, we could not include their error estimates in the given bounds. It is necessary to emphasize that the very first correction factors we obtained for a station are the designated correction factors for that station.

The results we have reported so far are from our analyses of teleseismic records of event 1. In an attempt to evaluate the correctness of the correction factors generated from event 1 records, we computed correction factors for the gradiometer elements using waveforms from event 2 between 0.5 and 2 Hz frequency band. The SNR of gradiometer horizontal component records of event 2 are all lower than the data quality control thresholds so that we can only compare the vertical component ACF we computed from event 2 with those of event 1. Percentage residuals between the vertical component ACF of events 1 and 2 are mostly less than
2% (Figure 2.19). These observed deviations are likely a result of the lower SNR of event 2 records, with mean and standard deviation of 4.5 and 0.2, respectively.

![Image](image-url)

**Figure 2.19:** (a) Comparing gradiometer vertical component ACF obtained from event 1 waveforms with those estimated from event 2 records. All correction factors shown on this plot were computed between 0.5 and 2 Hz. (b) The associated residual plot.

Another interesting, but peculiar, observation was the outlying vertical component ACF estimate corresponding to station 4607 of the gradiometer (Figure 2.12). This result is interesting in the sense that the vertical component record amplitude of event 1 at station 4607 is about half the amplitudes of those records at neighboring stations for the same event, and the inversion process markedly improved its amplitude relative to other stations. In a similar fashion, the vertical component record of event 2 at the same station also exhibited comparable amplitude disparity when compared to adjoining station records. However, the vertical component waveform of a local event (Table 2) recorded at the same station is rather similar in amplitude to its neighboring stations’ waveforms. This selective attenuation of teleseismic signals at station 4607 is difficult to explain.

The distributions of correction factors we obtained are more dispersed compared to previous studies. As an example, the OCF determined for the E-array vary as much as 14.7°, which is more than twice the maximum OCF variability determined by Donner et al. (2017) who
optimized horizontal component cross-correlation functions to evaluate instrument OCF. The vertical component ACF precision (one standard deviation) from our results is mostly less than 2.4% for the three small-aperture arrays. This statistic is low in comparison to a precision of 0.25% reported by Langston (2018) when he applied a similar inversion technique to compute ACF for broadband instruments. The horizontal component ACF precision from our study for all three arrays is, in general, less than 4.2%; this precision is rather low with respect to the horizontal component precision of 0.61% obtained by Langston (2018). An earlier study by Pavlis and Vernon (1994) used background noise measured by two or more effectively collocated sensors for instrument calibration – where the calibration of one of the sensors is known. Unlike this study, the pre-event noise in our work does not correlate between stations.

The results of Donner et al. (2017) and Langston (2018) were obtained from analyzing high SNR (the waveforms used by Langston (2018) have SNR in the hundreds) broadband data containing frequencies as low as 0.018 and 0.005 Hz, respectively. These are about an order of magnitude (or more) smaller than the lowest frequency we considered in this study. These lower frequencies – coupled with high SNR - virtually guarantee higher data coherence since horizontal wavelengths were 100 times larger than the array dimension. Using data with horizontal wavelengths of the order of the array dimension also guarantees that velocity heterogeneity will likely be an important factor in the degradation of wave coherence.

The purpose of calibrating the Oklahoma gradiometers for instrument and site errors was to use the calibrated dataset for gradiometry studies. We did gradiometry analysis of a local earthquake (Table 2) recorded by the gradiometer. We performed the analysis on both calibrated and uncalibrated waveforms of the local event to evaluate the effect of array calibration on the computed wave attributes. In all, data from 98 stations met the data quality control criteria
emphasized in the methodology section. We denoised individual station data after Langston and Mousavi (2019) and we rotated the horizontal components into the radial and transverse directions of the source-station azimuth. We transformed the resulting 3-component data into the continuous wavelet domain where we extracted prominent body wave phases following Langston and Mousavi (2019), inverse transformed back to the time domain and analyzed by means of gradiometry technique (e.g., Figure 2.20).

We used the means and standard deviations of wave attributes we estimated within designated time windows to postulate gaussian statistical models for the computed wave parameters. Of all the 3-components records analyzed, the extracted S wave phase on the transverse component exhibited the most significant disparity between calibrated and uncalibrated data attributes (Figures 2.21 and 2.22). The gaussian plot of the calibrated data velocity estimates between 29.5 and 30 seconds is statistically different from those we obtained from the associated uncalibrated records within the same time window. Furthermore, the calibrated data mean azimuth estimate is within 0.1° of the catalog-derived azimuth of the local event, while the mean azimuth of related uncalibrated records disperse by over 1° from the catalog-derived azimuth.

On the radial front, the distribution of computed S wave azimuth from the calibrated data over a window of 0.3 seconds includes the catalog azimuth; the catalog azimuth clearly sits outside the uncalibrated data azimuth bell curve (Figure 2.21). The resulting distributions of apparent velocity estimates for the calibrated and uncalibrated radial records effectively share the same statistics (Figure 2.21). Their mean velocities only vary by 0.01 km/s with error (standard deviation) differences of a similar order of magnitude. The Z component data attributes exhibited the least disparity between calibrated and uncalibrated data attributes.
Figure 2.20: (a) Continuous wavelet transform (CWT) plot that shows absolute wavelet coefficient amplitude as a function of scale and time and (b) corresponding calibrated vertical component waveform for the magnitude 4.4 local event recorded at station 4115 of the gradiometer array. The prominent phases are as annotated by the arrows in the CWT domain.
Figure 2.21: Gaussian models computed using gradiometry derived mean and standard deviations of (a) horizontal apparent velocity and (b) azimuth estimates distributions within time windows of 2, 0.3 and 0.5 seconds for the gradiometer vertical, radial and transverse components’ records of the magnitude 4.4 local event respectively. Extracted P wave phase from the vertical record was analyzed, while S wave phases were extracted from the horizontal components’ records and analyzed. The calibrated data distributions are shown in solid lines and their uncalibrated analogs are displayed in dashed lines. The catalog-obtained azimuths are plotted in dotted lines.
Figure 2.22: (a) Extracted S wave phase from calibrated gradiometer transverse records of the magnitude 4.4 local event used to evaluate the effect of array calibration on computed wave attributes. (b) Computed ground displacement for the gradiometer reference location and corresponding wave azimuth and slowness as a function of time. The horizontal velocities and azimuths were obtained within the outlined window (between 29.5 and 30 seconds).
The mean P wave azimuth estimates we obtained over a 2 second time window for the two scenarios only differ by 0.1°, and both distributions share same error of 0.3° over the time window we considered. The P wave azimuths we determined are about 6 degrees off the catalog azimuth. It is possible the wave did not travel on a great circle path due to structure heterogeneity; however, we are not certain about the true wave propagation direction. Associated mean apparent velocity estimates differ by 0.05 km/s: the difference in the uncertainties of these distributions is 0.01 km/s.

Summari ly, this high-frequency instrument calibration exercise appears to improve relative amplitudes, which in turn enhanced the accuracy of computed wave attributes via gradiometry technique: especially on the horizontal components. These favorable results were aided by signal coherence and excellent (in the case of the vertical component) to acceptable (for horizontal components) SNR of the teleseismic records used to calibrate the instruments. As an aside, the improvement in wave attributes’ accuracy after calibration also lends credence to the verity of the estimated calibration parameters. Detailed comparisons of gradiometer and phased array data analyses are being prepared for a future manuscript.

2.7 Conclusions

The aim of this study is to mitigate the effects of instrument and site errors on seismic records so that the accuracy of wave attributes computed from these records using gradiometry can be improved. We used Teleseismic P and S waveforms recorded by the 2016 IRIS community experiment in Northern Oklahoma to calibrate the 3-component nodal instruments deployed during the experiment. This calibration procedure assumes that a common wavefield from a teleseismic event is recorded over a small-aperture array. We obtained the most favorable calibration results at frequency bands where SNR and/or signal coherence are highest. Thus,
SNR and signal coherence should be duly considered in formulating data quality control measures for future instrument calibration exercises prior to gradiometry analyses. In addition, shifting all teleseismic record relative to the master station before inversion to remove wave propagation effect provided the optimal instrument calibration parameters in this study.

The improved relative amplitude coupled with the attendant boost in accuracy of wave attributes estimates after calibration for the horizontal component data suggest that instrument calibration exercise as this should be encouraged as a data quality control endeavor before embarking on future gradiometry analyses, especially on high-frequency instrument records. The marginal differences in computed wave attributes for the calibrated and uncalibrated vertical component data indicate that amplitude statics error likely had a minimal influence on these vertical component data, and, by extension, shows that the structure beneath the gradiometer array may be largely homogeneous. Looking into the future, it will be interesting to see the effectiveness of the vertical component inversion technique used in this study on less coherent vertical component array records from another region.

2.8 Data and resources

The data used for this work were acquired during the IRIS Community Wavefield Experiment conducted in the summer of 2016 near Enid Oklahoma. The waveforms and associated metadata are archived and available from the Incorporated Research Institutions for Seismology Data Management Center (http://service.iris.edu/ph5ws/dataselect/1/): last accessed July 2019. There are three supplemental materials in the .txt format that comes along with this article. They are the tables of correction factors for the gradiometer, E-array and D-array stations. Seismic Analysis Code and ArcGIS were used to analyze and display data in the course of this work and are acknowledged.
2.9 Acknowledgments

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References


Chapter 3  
Partitioning Local Seismogram Wavefields Using Continuous Wavelet Transform Methods for IRIS Wavefield Experiment Arrays

3.1 Introduction

Mousavi & Langston (2016) presented a seismic denoising procedure that employs a hybrid block thresholding technique in the CWT domain to separate event noise from seismic signals. This thresholding method uses automated means to find signal models that select a batch of only essential variables (leaving out the insignificant variables) to fit seismic data. These variables are wavelet coefficients which are complex amplitudes obtained from CWT of seismic records on the scale-time plane. The denoising procedure of Mousavi & Langston (2016) can be computationally intensive and may not be optimal in terms of algorithm speed for a large array dataset. Langston & Mousavi (2019) developed a technique to remove noise from seismic signal that is computationally faster and gives an intuitive outlook of the signal and noise on the scale-time plane. This procedure assumes that the presignal noise (or postsignal noise) is stationary: the statistics of the presignal noise is used to compute thresholding parameters that are applied to the seismic data to remove noise. More than getting rid of event noise, this technique offers simple but elegant means to select signal segments on the scale-time plane that potentially represents seismic phases that may be of interest to an analyst.

In this article, we applied the denoising and signal partitioning techniques developed by Langston & Mousavi (2019) to local earthquake and explosion datasets to delineate the seismic phases that are contained in these records, with further emphasis on concurrent arrivals that could be missed in the time series. In particular, we intend to answer the following questions: first, what seismic phases are contained in the seismograms of various local earthquakes? While the time domain representation of a seismic signal may show apparent seismic phases that are
distinct in amplitude and arrival time, it is difficult to visually pick out phases that share a common arrival time at the recording station. Furthermore, wave codas from earlier-arriving phases can interact in time with later-arriving phases that can be of interest to an analyst. The CWT offers an alternative means to represent seismic signals on the scale-time plane where phases with different spectral content superposed in time are individually distinct in scale (or period) and can be separated from each other on the scale-time plane. We will employ the scale-time gating technique of Langston & Mousavi (2019) to partition seismic phases of interest in the CWT domain using data recorded by two seismic arrays.

Another question that is of interest to us dwells on how seismic noise might be suppressed before embarking on earthquake signal partitioning in the CWT domain. Langston (2021) observed that denoising explosion records from an array through nonlinear thresholding improved SNR by more than two orders of magnitude: can this result be replicated on earthquake signals using same technique?

The final question we will explore in this work has to do with the semblance of the denoised waveforms across an array. In attempting to answer this question, we will show the effect of denoising on beamforming results obtained from analyzing seismic records from a phased array.

We will address the three scientific questions using seismic data acquired during the 2016 Incorporated Research Institutions for Seismology (IRIS) Wavefield Experiment carried out in northern Oklahoma (Sweet et al. 2018). Using data from two of the arrays deployed during the experiment, we will first evaluate the effectiveness of the nonlinear thresholding technique in separating signal of interest from its attendant noise; then, we will partition and extract distinct seismic phases on the scale-time plane. We will use the beam forming technique to investigate
change in signal similarity across array stations after removing event noise from the array records. Finally, we apply CWT techniques to the problem of wave gradiometry using data recorded by the 112-element geodetic array deployed during the IRIS experiment. Denoising and partitioning the wavefield before computing wave attributes represents a new result that improves gradiometric analyses.

3.2 Data

In the summer of 2016, IRIS embarked on a community wavefield experiment in the vicinity of Enid, Oklahoma. The field experiment entailed four array geometries that were put out to densely sample the wavefield induced by largely man-made seismicity in the northern Oklahoma area (Sweet et al. 2018). The largest of the four configurations was a 13 km aperture array that was composed of 254 FairfieldNodal Zland 3 component 5Hz seismometers. The widest arm of this array runs east-west, and two 5 km long arms run north-south, crossing the longest arm at about 4 and 7 km from its western end (Figure 3.1). Going forward, we will refer to this array as the H array. This array was deployed from 20 June to 20 July 2016. A 1 km aperture gradiometer array composed of 112 elements was put out alongside the largest array of the 2016 experiment (Figures 3.1 and 3.2). The gradiometer consisted of 7 square concentric rings with each ring being made up of 16 sensors. Every ring was four times the size of its immediate smaller ring. The gradiometer shares the same instrument makeup and deployment history as the largest array.

A phased array, called the “Golay” array, composed of 18 broadband Guralp CMG-3T seismometers was also deployed during the Wavefield Experiment. The maximum width of the Golay array is about 7 km, and it was put out over a five-month period beginning June 18, 2016.
Figure 3.1: The spatial distribution of the 2016 IRIS Community Wavefield Experiment arrays. (inset) Map of the continental United States showing the location of the Wavefield Experiment in Northern Oklahoma area. The pushpin markers show the extent of the 13-km aperture configuration that is composed of 254 three-component Fairfield nodal sensors. The circled segment of this array, labelled cross array, consists of 80 nodal elements, and we used it as a regional phased array to characterize seismic signals for this study. The nested squares are designated as the gradiometer: composed of 112 nodal seismometers arranged into 7 concentric squares with 16 elements each. The Golay array, symbolized by triangles, is made up of 18 three-component broadband instruments. The color version of this figure is available only in the electronic edition.
Figure 3.2: The Wavefield Experiment gradiometer array composed of 112 elements. Going outwards from the innermost square, successive squares are four times the area of the immediate smaller square, and each square consists of 16 three-component 5 Hz Fairfield nodal sensors. The first digit of each station number reflects the array code, the second digit expresses the subarray number, with the smallest square designated as 1 and this numbering progressively increases up to the largest square that is labelled 7. The last two digits of the station code, labelled clockwise from the north-west corner of each subarray, provides the position of a sensor within the subarray.
Nine infrasound instruments were collocated with nine of the broadband sensors to serve as a data quality control measure for the broadband data. The seismic data recorded during the Wavefields Experiment was archived under network code “YW” in the IRIS Data Management center.

The nodal sensors of the YW array recorded hundreds of magnitude 2 and greater earthquakes over their one-month deployment. We will use seismic data from three earthquakes and one explosion selected from this data pool to answer the posed scientific questions. The first event, that we will designate as “event 1” going forward, is a magnitude 4.2 earthquake that happened about 100 km from the YW array at a depth of 7.3 km (Table 3.1). As we will later show, records from this event consist of body and surface wave phases typical of a shallow local earthquake in layered Earth structure. The second event occurred 56 km from the YW array at a depth of 3.8 km having a local magnitude of 2.6 (Table 3.1). This earthquake’s prominent pre-event noise makes it an ideal dataset to investigate the effectiveness of nonlinear thresholding in removing noise from seismic signal. We will subsequently refer to this event as “event 2.” The third event we will consider is a magnitude 2.7 earthquake that occurred at a depth of 7 km about 10 km from the array. We picked this event because of its proximity to the YW array. Henceforth, we will call this earthquake “event 3.” Within the first month of YW array deployment, the Airforce Technical Applications Command (AFTAC) detonated a series of small explosions at three locations within a 70 km radius of the YW array. In addition to the three earthquakes, we also used in this study YW array records from a 2000 lb explosion (Table 3.1) that was set off about 15 km south-east of the YW array.
Table 3.1: Parameters of seismic events considered in this study

<table>
<thead>
<tr>
<th>Event</th>
<th>Date/time yyyy/mm/dd hh:mm:ss</th>
<th>Latitude (°)</th>
<th>Longitude (°)</th>
<th>Depth (km)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2016/07/08 21:31:57.600</td>
<td>36.4765</td>
<td>-98.7387</td>
<td>7.315</td>
<td>4.2 (M_w)</td>
</tr>
<tr>
<td>2</td>
<td>2016/07/11 22:19:32.500</td>
<td>36.2200</td>
<td>-97.2807</td>
<td>3.795</td>
<td>2.6 (ml)</td>
</tr>
<tr>
<td>3</td>
<td>2016/07/08 18:41:55.300</td>
<td>36.6485</td>
<td>-97.7445</td>
<td>7.045</td>
<td>2.7 (ml)</td>
</tr>
<tr>
<td>Explosion</td>
<td>2016/07/16 00:37:00.000</td>
<td>36.5516</td>
<td>-97.5180</td>
<td>0</td>
<td>2000 (lb)</td>
</tr>
</tbody>
</table>

For the purpose of this study, we carved out an 80-element subarray from the 13 km long array. This devised geometry is a 3X3 km, cross-shaped array at the intersection of east-west arm of the largest array and its 5 km arm to the east (Figure 3.1). The spatial resolution of the cross array is expressed in its co-array plot, which is a vector space of relative sensor position and azimuth taking into account all cross array sensor pairs (Figure 3.3). The cross array is suited to signal wavelengths from 0.5 to 3.5 km. We used this array to analyze seismic signals within a spectral range of 1 and 9 Hz.

Prior to data analysis, we removed the nominal instrument response of the FairfieldNodal instruments and also applied a phaseless Butterworth filter with corner frequencies 0.5 and 50 Hz within a passband of 0.05-80 Hz. In the course of the Wavefield Experiment, five nodal sensors returned nil data (Sweet et al. 2018). We also took out waveforms with transients and those that have exceptional noise levels. The nodal instruments recorded data at 200 samples per second.

The gradiometer array data required an extra initial processing step to remove amplitude statics before conducting gradiometry analysis on them. Amplitude statics describes errors introduced to recorded seismic wave amplitudes from nonuniform digitizer gains over array sensors and near surface structural variations across sensor locations (Bungum et al. 1971; Cranswick et al. 1985; Ekstrom et al. 2006; Tasič & Runovc 2013; Langston 2018).
Figure 3.3: (a) Cross array geometry and (b) the co-array plot made from the cross array sensor distribution: the co-array plots all possible sensor-sensor distance based on the respective relative azimuth of every sensor pair of the cross array. The co-array reveals the spatial resolution of the cross array.
We corrected the gradiometer records for amplitude statics determined by Bolarinwa & Langston (2021) before conducting gradiometry analysis on the data. The gradiometer can analyze signal wavelengths that are 10 to 100 times the aperture of the gradiometer (Langston 2007a; Langston 2007b).

3.3 CWT and Nonlinear Thresholding

Detailed description of the Nonlinear Thresholding method for removing noise from seismic signal has been given by Langston & Mousavi (2019). However, we will highlight the key concepts of the technique before applying it to seismic data. The mathematical expression for the CWT following Grossmann et al. (1989) and Starck et al. (2010) is

\[ W(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi^* \left( \frac{t - \tau}{a} \right) dt \] (3.1)

Where \( W \) is the wavelet coefficient (which is a complex variable), \( a \) is the scale, \( \tau \) is the time lag and the asterisk denotes complex conjugate. In simpler terms, the CWT is a cross correlation between signal \( f(t) \) and a scaled basis function \( \psi(t) \). The basis function is typically complex and is widely called the “mother wavelet.” The wavelet coefficient is expressed in the Fourier domain as

\[ \hat{W}(a, \omega) = \sqrt{a} f(\omega) \hat{\psi}^*(a \omega) \] (3.2)

Equation (3.1) has an inverse transform given by

\[ f(t) = \frac{1}{c} \int_{0}^{\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{a}} W(a, \tau) \psi \left( \frac{t - \tau}{a} \right) \frac{d\omega d\tau}{a^2} \] (3.3),

and \( c \) is evaluated as below:

\[ c = \int_{0}^{+\infty} \frac{\hat{\psi}^*(\omega) \hat{\psi}(\omega)}{\omega} d\omega \] (3.4)

The basis function is required to have a zero mean for equation (3.4) to be finite at \( \omega = 0 \).
Just like Langston & Mousavi (2019), we used the Morlet’s wavelet as the mother wavelet for all analysis involving CWT in this work.

We took out noise using a soft thresholding procedure that removes wavelet coefficients below a preset noise threshold and reduces those above the threshold by the noise estimate as follows:

$$\tilde{W}(a, \tau) = \begin{cases} 
\operatorname{sign}[W(a, \tau)]\left(|W(a, \tau)| - \beta(a)\right) & \text{if } |W(a, \tau)| \geq \beta(a), \\
0 & \text{otherwise},
\end{cases}$$

(3.5)

Where $$\operatorname{sign}[W(a, \tau)] = \frac{W(a, \tau)}{|W(a, \tau)|}$$.

The noise threshold term, $$\beta(a)$$ is evaluated from the empirical cumulative distribution of the statistics of the wavelet coefficients for a presignal (or postsignal) noise time window such that

$$\beta(a) = \operatorname{ECDF}^{-1}(P = 0.99),$$

(3.6)
at the 99% confidence level.

3.4 Effect of Nonlinear Thresholding on Earthquake data

We applied the soft thresholding denoising procedure to the cross array records of event 2, and we used a 55 second pre-event noise time window to compute the scale-varying $$\operatorname{ECDF}_a$$ and its corresponding thresholding parameter (Figure 3.4). As an extra step after the denoising procedure, we applied a scale band reject filter on the scale-time plane to remove scales above 2, 3, and 4 seconds respectively for the vertical, radial, and transverse components data. These band rejected signals are high-scale (long-period) artifacts that lingered from the soft thresholding operation.

Following the denoising step, the resulting signals showed notable improvements in SNR – which is defined in this article as the ratio of signal maximum absolute amplitude to noise maximum absolute amplitude (Figures 3.4 and 3.5). Using event 2 records at the central station
of the cross array (station 3017) as examples, the transverse data SNR improved from 3 to 213 after denoising while SNR of the radial component record saw a boost from 2 to 170 following noise removal. The vertical component data at this station had the least raw data SNR of 1, which notably increased to 47 after soft thresholding its event noise. Further, the cross array records of event 2 also show marked improvement in SNR across array waveforms on all three components (Figure 3.5). Besides the reduced noise level following denoising, there is a clear improvement in waveform similarity across the cross array three component data. Also, the P wave onsets that were buried within noise are clearer even on the horizontal components data after the denoising procedure (Figure 3.4).

3.5 Scale-time Gating and Seismic Phase Extraction

The 2D representation of a signal in scale and time in the CWT space offers a more comprehensive view of a 1-D time series. The added scale dimension in the wavelet domain may allow concurrent arrivals to be viewed as separate energy packets if they are separated in scale on the scale-time plane. As the CWT is a linear operation, the CWT coefficients of distinct wave packets can be delimited using a polygon on the scale-time plane, and inverse-transformed back to the time domain to obtain a temporal realization of the gated segment (Langston & Mousavi 2019; Langston 2021). For example, Figure 3.6 shows the process for the fundamental mode Rayleigh wave observed from an AFTAC explosion. We also use an automated scale-time gating method to process the data from many array elements (Langston 2021). The idea is to designate a station close to the center of an array as the reference station, use a polygon to delimit a seismic phase of interest on the CWT scalogram of the reference station record, and afterward do a 2D correlation of this block of CWT with scalograms from other array stations.
Figure 3.4: (a) Radial component data for event 2 (Table 3.1) at station 3017 of the cross array. The top panel shows the continuous wavelet transform (CWT) scalogram that shows the magnitude of the wavelet coefficient amplitude as a function of scale and time. The annotated window on the scalogram, between 5 and 60 s, shows the noise window used to threshold the data. The noise is composed of high scale and relatively low scale components, while the desired signal seats over the high scale noise segment. The lower panel displays the raw time series at the station. The plot on the top left corner shows the average noise amplitude within 5 and 60s noise window. The estimated amplitude threshold obtained through the empirical cumulative distribution function (ECDF) method (Equation 3.6) is shown as well. (b) The results from applying the soft thresholding procedure to remove noise from the data in (a). Top panel reveal the scalogram of the denoised data, and the lower panel displays the time domain realization of the denoised data.
Figure 3.5: Raw and denoised cross array records of event 2 (Table 3.1) for (a) vertical, and (b) radial components. There are 75 waveforms plotted over each other on all eight plots. The denoised waveforms show significant improvement in signal to noise ratio relative to their respective raw records.
Figure 3.6: (a) Raw vertical component record of the explosion (Table 3.1) at cross array station 3017. Top panel displays the scale-time representation of the data in the lower panel. (b) Scalogram of the denoised data showing body and surface wave phases, and the time domain realization of the denoised data. The polygon around the fundamental Rayleigh wave mode will be used to separate out this phase from other wave packets. (c) Extracted fundamental mode phase after zeroing out CWT coefficients that are outside the polygon.
Using station 3017 as the cross array reference station, we employed the 2D correlation method to extract the surface wave modes from all cross array vertical records of the explosion (Figure 3.7).

Not all seismic phases are as discrete in the wavelet domain as seen with the AFTAC explosion data. A “rule of thumb” in categorizing a block in the CWT plane is to put forward a hypothesis as to the phase(s) contained in a CWT block, perform an inverse CWT on the coefficients within the block, then analyze the ensuing time series to confirm or reject the hypothesis. We will use the gradiometer records of events 1 and 3 as cases in point for visually identifying seismic phases on the scale-time plane (in other words, making hypotheses), and later characterizing those phases.

The scale-time representation of the event 1 vertical record at station 4115 reveals three distinct segments: there is the low scale P and possibly S wave segment (“Body wave and coda”, Figure 3.8), the fundamental mode Rayleigh wave arrival and a mid-scale segment that appears to contain the “high scale S wave and higher mode Rayleigh wave” (Figure 3.8). We clipped out the “high scale S wave and higher mode Rayleigh wave” block in the CWT domain and transformed the block’s CWT coefficients back to the time domain; the resulting time series shows at least two discrete phases (Figure 3.8). We will later analyze this wave packet across all gradiometer elements to verify our hypothesis that it contains high scale S and higher mode Rayleigh waves. Figure 3.9 shows the result of applying the scale-time gating technique to partition 97 gradiometer array vertical waveforms of event 1 utilizing station 4115 as the reference station. The extracted phases are coherent in phase and amplitude over the gradiometer array. Also, the high-frequency body wave coda that is buried in the raw data time series is visible following signal partitioning.
Figure 3.7: Extracted vertical component (a) body wave, (b) first higher mode, and (c) fundamental mode phases across all cross array data from the explosion. We extracted each surface wave phase’s waveforms by first encompassing the continuous wavelet transform (CWT) block of the phase at reference station 3017 with a polygon, then we did a 2D correlation of this CWT block with CWT realizations of other array elements data to get the required time shift needed to clip out similar CWT block from individual array element. For the body waves, we made a super CWT block from the reference station data by tracing a polygon around the two surface wave phases, then we cross correlated this super block with CWT scalograms of other station elements, to obtained appropriate time shifts and employed this time lags to remove the CWT coefficients of individual stations superblocks leaving behind the body wave phase. There are 78 waveforms plotted over each other on each of the three plots.
Figure 3.8: (a) Continuous wavelet transform (CWT) scalogram of Raw vertical-component waveform of event 1 (Table 3.1) at gradiometer station 4115, and its seismogram. (b) The CWT scalogram of the denoised data with much of its upper segment revealing low scale body waves; the polygon in the middle appears to encompass a mix of high scale S and higher mode Rayleigh waves, while the fundamental mode lies in the highest scale region on the scale-time plane. The waveform in (b) presents the denoised data in time. (c) The gated High scale S and first higher mode Rayleigh waves in the wavelet domain, and its time domain realization.
Figure 3.9: Extracted seismic phases from the vertical-component gradiometer records of event 1 (Table 3.1). (a) Body wave phases, (b) a mix of high scale S and first higher mode Rayleigh waves, and (c) the fundamental mode Rayleigh waves. Each of the three plots shows 97 waveforms plotted atop each other. These waveforms were extracted by the same procedures described in Figure 3.7 caption.
Less like the first event, event 3 is smaller in magnitude and happened close to the YW array so that its waveforms are relatively richer in short period (low scale) waves (Figure 3.10). There are three largely distinct blocks on the scale-time plane view of the vertical component data of event 3 at station 4101 of the gradiometer array. The low scale P and S wave segment exhibited peak power on the entire scalogram, and signal power seems to wane with increasing scale down to the high scale S wave. We have designated the mid-scale block as a “mystery” phase because we are not clear about its characteristics based on its scale-time position on the scalogram. Therefore, we separated out the mystery phase across the gradiometer vertical records using station 4101 as the reference station (Figure 3.10). In due course, we will characterize the mystery phase using gradiometry. The High scale S wave phase appears to have the least signal power of the three phases contained in the gradiometer records (Figure 3.10).

3.6 Characterizing Partitioned Signals Using Beamforming and Gradiometry

So far, by means of the scale-time gating technique, we are able to make some hypotheses about the seismic phases that are contained in the local earthquakes considered. While we could identify many of the seismic phases in the wavelet space, a few phases have only been either provisionally identified or could not be named at all based on their time and scale-time representations in the time and wavelet domain, respectively; we will conduct array analysis on array records of these phases in a bid to characterize these phases. In addition, we will also verify by array analysis that the characteristics of the selected CWT blocks of the explosion data matches their nomenclature that was based on arrival times. We have also shown by means of the nonlinear thresholding procedure that seismic noise can be effectively suppressed in earthquake signals. It remains to be seen what the effect of denoising is on the beam formed from the noise-free phased array data of event 2.
Figure 3.10: (a) Scale-time plot of vertical-component raw data of event 3 (Table 3.1) at gradiometer station 4101, and the temporal plot of the data. (b) CWT scalogram of the denoised data showing body and surface wave phases, and a puzzling phase we tentatively call the “mystery” phase. Next is the time series of the denoised data. (c) Plot of the gated mystery phase CWT coefficients in the wavelet space, and the time plot of the inverse CWT transform of the mystery phase coefficients to the time domain. (d) Ninety-four extracted mystery phase waveforms from gradiometer array elements data of event 3, using the scale-time gating procedure (and nonlinear thresholding) with station 4101 as the reference station.
3.6.1 Array Analysis of the AFTAC Explosion

We examined the extracted seismic phases from the vertical component data of the AFTAC explosion by means of gradiometry and frequency-wavenumber (f-k) methods to validate phase interpretations. The explosion is too close to use the plane wave assumption in beamforming; therefore, we used the 1-D f-k method, in conjunction with the signal great circle path (GCP), to characterize the explosion. We utilized the 1-D f-k procedure to analyze the extracted cross array body and surface wave records of the explosion. The two surface wave phases exhibited characteristic dispersion over 1-4 Hz spectral band on the f-k plot (Figure 3.11). The resulting higher mode Rayleigh wave phase velocities range from 1.5 to 3 km/s, while those of the fundamental mode phase varies from 0.9 to 1.6 km/s. The f-k plot of the P wave phase revealed a longitudinal wave rolling across the phased array at a speed of 5 km/s.

We also used the gradiometry technique to analyze the extracted surface wave phases of the explosion vertical records over a frequency band between 1 and 4 Hz. If the innermost square subarray of the gradiometer is designated as square 1, and other subarrays are progressively labelled in the order of size up to the largest - square 7, we used data from squares 1, 2, 3, 4 and squares 2, 3, 4 to characterize the higher mode and fundamental mode surface waves, respectively. The higher mode Rayleigh wave shows characteristic dispersion as its slowness plot from gradiometry reveal a progressive increase in slowness through time (Figure 3.12). The corresponding phase velocity starts out at 3.6 km/s and tapers to 1.5 km/s around the 13 second mark - the velocity estimates show a wider variability beyond 13 seconds. The spike around 10 second on the slowness plot is due to the interaction between the two major arrivals on the reference position waveform.
Figure 3.11: Data and 1-D f-k result from analyzing gated body and surface wave phases from vertical records of the explosion at 78 cross array stations. (a) The P wave window of the gated body wave data bandpass filtered between 8.5 and 9 Hz (b) The 1-D f-k plot for the P wave showing consistent wave speed over the 0.5 Hz frequency band considered. (c) First higher mode Rayleigh wave phase filtered between 1 and 4 Hz. (d) The f-k plot expressing the dispersive nature of the first higher mode. (e) The fundamental mode Rayleigh wave records filtered between 1 and 4 Hz. (f) The broad band f-k plot for the fundamental mode that also shows the variability of the wave’s speed across a range of frequencies.
Figure 3.12: Gated body and surface wave phases (Figure 3.7) from vertical-component waveforms of the explosion as recorded by the gradiometer. (a) Higher mode Rayleigh phase bandpass filtered between 1 and 4 Hz. (b) Top panel shows the computed higher mode ground motion at the reference position (center) of the gradiometer. The middle panel displays wave azimuth through time at the reference position, and the bottom panel expresses the corresponding wave slowness as a function of time. (c) The fundamental mode Rayleigh wave record filtered between 1 and 4 Hz. (d) Top plot displays the ground displacement due to the fundamental mode at the gradiometer’s reference position. The middle and bottom panels respectively show the azimuth and slowness of the fundamental mode through time. (e) P wave record section filtered between 8.5 and 9 Hz. (f) P wave ground motion at reference position (top panel), wave azimuth and slowness through time are shown in the middle and bottom panels, respectively. The means of the wave attributes within the annotated windows in (b) (d) and (f) were discussed in the body of this article.
The higher mode Rayleigh wave appears to be traveling towards the gradiometer from an azimuth of 308° going by the average wave properties within the 6.5 and 7.5 second time window of its reference position time series (Figure 3.12). The slower fundamental mode Rayleigh wave analyzed between 1 and 4 Hz also exhibited dispersion as it begins with a phase velocity of 1.6 km/s, then it slows to 1.2 km/s just before the 18 second mark on the slowness plot (Figure 3.12). Its average azimuth over a 2 second time window is 304°. The least velocity estimate of the higher mode surface wave is the same for the 1-D f-k and gradiometry techniques, while the upper bound velocity estimates from the two methods differ by 0.6 km/s. The fundamental mode phase velocities computed from the two methods share the same upper bound; however, the least velocities from gradiometry are greater than that from the 1-D f-k technique by 0.3 km/s. The estimated surface wave azimuths are within 4° of the signal GCP.

We also examined the P wave phase on the vertical records of the explosion by means of gradiometry. In a bid to characterize this phase in the same frequency band as the 1-D f-k result, we used waveforms from the three innermost squares of the gradiometer. Mean attribute estimates over a 1 second time window reveal that the phase is coming from an azimuth of 306° at a speed of 6.4 km/s. The gradiometry-derived P wave azimuth is within 1° of the signal GCP, while corresponding phase velocity varies by 1.4 km/s from the f-k-derived phase velocity estimate. The gradiometer appears to be more useful than the 1-D f-k method in characterizing close-in sources, since signal azimuth can be determined.

3.6.2 Array Analysis of Events 1 and 3

In order to identify the mystery phase from event 3 data and also validate the “high scale S and higher mode Rayleigh waves” block from event 1, we conducted gradiometry analysis on
the extracted gradiometer records of the two wave packets. The result from analyzing the mystery phase data revealed two discrete phases traveling with different velocities from separate azimuths (Figure 3.13a). The velocity of the first phase starts out at 6.2 km/s, which is typical of a P wave. The later phase had a phase velocity of 3.4 km/s that puts it in the ballpark of S waves. Also, there is a difference of 15° in the azimuths of these two phases. Therefore, the mystery phase from event 3 is most likely a mix of direct P and S wave phases based on the array analysis results.

The gradiometry analysis of the “high scale S and higher mode Rayleigh waves” data shows that there are more than two seismic phases in the extracted signal (Figure 3.13b). Phase velocity and azimuth plots show an arrival with moderately varying azimuth that starts out with a phase velocity of 6.5 km/s, characteristic of a P wave, then around the 33 s mark there is a transition to a phase traveling at around 3.6 km/s, that is suggestive of a S wave, from a relatively stable azimuth of 100°. The abrupt spike about the 45 s mark on the wave attribute plots is an indication of phase interference between the S wave and the slower surface wave that is emanating from the same general direction of the first two phases (Figure 3.13).

3.6.3 Evaluating the Beam Quality of the Denoised Records of Event 2

The joint application of nonlinear thresholding and scale-time gating to event 2 data improved individual waveform SNR, which in turn improved waveform semblance across the entire cross array for the event. We will go a step further to do phased array analyses on the denoised and raw radial data of event 2 to look for variation in the quality of array beams that will be generated for the two instances. Specifically, we will generate these beams by means of the 2D f-k technique.
Figure 3.13: (a) Top panel shows the extracted mystery phase from 94 vertical-component gradiometer waveforms of event 3 (Table 3.1). The records have been scale band filtered between 1.02 and 1.16 s. The bottom panel consists of the estimate of the ground motion at the reference position of the gradiometer, and the mystery phase’s velocity and azimuth through time at the reference position. (b) Top panel displays the “high scale S and first higher mode” wave packet extracted from 97 vertical-component gradiometer records of event 1 (Table 3.1). These records were filtered between scales of 1.25 and 1.48 s. The bottom panel display the “high scale S and first higher mode” ground motion at the gradiometer reference position, and the other two plots shows the phase velocity and azimuth of this wave packet at the reference position through time. The wave attributes within the annotated time windows in (a) and (b) were discussed in the body of this article.
We conducted the f-k analysis on the H array radial data of event 2 filtered between 0.3 and 1 Hz. Figure 3.14 shows the f-k results for the raw and denoised radial data. The beam from the denoised data shows improved resolution when compared to that from the raw data. The BAZ estimate of 141° from the denoised data is within 1° of the GCP, while the BAZ derived from the raw data deviates by more than 3° from the GCP. The phase velocities from the denoised and raw data are 3.7 km/s and 3.9 km/s, respectively. The results from analyzing the denoised data further shows that denoising does not reduce signal correlation across an array.

3.7 Discussion

The scale-time representation of local earthquake signals in the CWT space has offered us the latitude to partition these signals into energy packets dictated by the scale-time distribution of their CWT coefficients in the wavelet domain. All explored events in this article exhibited both body and surface wave phases on the scale-time plane – with a possible exception of event 2 that is largely made up of body waves. We were able to successfully confirm the names assigned to some of the seismic phases by further doing array analyses on their respective array records. Additionally, the dispersive property of the extracted surface wave phases from the earthquakes and explosion data is another indication of the success of the signal partitioning exercise. In practice, the scale-time gating method may be applied in surface wave studies which particularly focuses on analyzing seismic signals within the surface wave time window.

While we were able to identify several seismic phases through signal partitioning, we observed a puzzling energy packet on the scalogram of event 3 vertical records (Figure 3.10). Tentatively, we called this strange phase mystery phase. The gradiometry analysis we conducted to demystify the content of the mystery block shows it is composed of P and S waves traveling across the gradiometer from two distinct azimuths that are about 15° apart.
Figure 3.14: Results from conducting 2D f-k analyses on raw and denoised H array radial records of event 2. The raw and denoised data were bandpass filtered between 0.3 and 1 Hz before beamforming. (a) Raw data record section is shown on the left panel, and the right panel is the f-k plot from analyzing the raw data. (b) Denoised data record section (left panel), and its corresponding f-k plot (right panel).
This finding buttresses the assertion that the act of gating out a block of CWT coefficients in the wavelet domain is a hypothesis in itself, and the content within the gated block may be verified by doing array analysis on the extracted block for all array element data.

Nonlinear thresholding harnesses spectral disparity between signal and noise on the scale-time plane to suppress noise. Essentially, this procedure removes noise in the low SNR areas of the scale-time plane and preserves the signals of interest in adjacent areas. But the signal properties in the low SNR segment will be forfeited in the process (Langston & Mousavi 2019). We observed that the nonlinear thresholding procedure significantly improved the SNR of event 2 on all three component records. This remarkable SNR boost is consistent with the result of Langston & Mousavi (2019), who used nonlinear thresholding to denoise explosion data. They noted an SNR improvement from less than 1 to approximately 200 after denoising, which is close to an SNR boost from 3 to 213 that we observed after removing noise from the transverse record of event 2 at the cross array reference station. In addition to the improved SNR, the clearer phase onset and improved signal coherence of the array records following noise removal will aid seismic phase picking across array records and potentially improve locating noisy events.

We took yet another step to further appraise the improvement in coherence of the denoised cross array records of event 2 in the light of corresponding undenoised records as we generated beams from the two datasets in the f-k domain. Phased array analysis of the denoised records of this event showed improvement in beam precision, and accuracy - with respect to the event BAZ – when compared to results from the undenoised data. These array analysis results further shows that denoising array data does not degrade signal coherence across an array. Furthermore, the concept of gradiometry assumes the generalized form of a single propagating
wave over time, and multiple arrivals in time can introduce artifacts into the wave attributes computed using gradiometry (Langston 2007a; Langston 2007b). These artifacts are expressed as abrupt spikes and gaps in the gradiometry results. CWT denoising and signal Partitioning techniques helped to improve gradiometric analysis as waves were separated before estimating wave attributes.

### 3.8 Conclusion

We used the nonlinear thresholding and scale-time gating techniques to understand the seismic phases that are contained in local event seismograms. We partitioned waveforms from three local earthquakes, and one local explosion, recorded by the 2016 IRIS community experiment nodal instruments deployed in northern Oklahoma, to identify and characterize the seismic phases that make up individual seismograms. Nonlinear thresholding assumes presignal (or postsignal) noise stationarity, and it uses ECDF to compute the representative statistics of the presignal noise, and afterwards applies these statistics to threshold the entire signal. Scale-time gating capitalizes on likely dichotomy between signal and noise segments on the scale-time plane to separate signal from noise. This signal partitioning procedure successfully separated local earthquake and explosion seismograms into their respective constituent phases. Further array analyses on a number of extracted phases affirmed the tentative names given to them on account of their position in scale and time in the CWT space.

The SNR of a noisy event records improve markedly after we applied the nonlinear thresholding procedure to the event data. We saw SNR improve by as much as two orders of magnitudes after taking out the noise. In addition, P wave onset that was lost in presignal noise became clearer on the horizontal component data of the event after denoising. We also observed improvement in signal beam quality following noise removal from the noisy records. Overall, the
findings from this study show that CWT thresholding and wave partitioning is not detrimental to f-k and gradiometry analyses.

3.9 Data Availability Statement

The waveforms used for this study were recorded during the 2016 IRIS Wavefields Experiment conducted near Enid, Oklahoma. The data is archived at the Incorporated Research Institutions for Seismology Data Management Center (http://service.iris.edu/ph5ws/dataselect/1/): last accessed March 2022. Arc GIS and Seismic Analysis Code were used to analyze and display data during this study, and they are acknowledged.

3.10 Acknowledgements

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References


Langston, C. A. (2021) Phased Array Analysis Incorporating the Continuous Wavelet


Chapter 4  

Comparison of Regional Gradiometer and Phased Array performances from the 2016 IRIS Wavefields Experiment

4.1  Introduction

The Incorporated Research Institutions for Seismology (IRIS) embarked on a large-N community driven experiment in the summer of 2016 around the town of Enid, Oklahoma. The objectives of the scientific campaign were to show the field use of the nodal instruments, acquire an interesting dataset that can be processed via innovative methods, and appraise the utility of new array designs and sensors (Sweet et al., 2018). This study carries on with the third objective of the 2016 Wavefields Experiment as we used data from the experiment to do a novel comparison of the performance of the experimental gradiometer fielded during the geophysical exercise to a phased array that was deployed in the same experiment. We assessed the usefulness of the gradiometer by using the phased array and the gradiometer to characterize signals emanating from the same seismic source.

The beam forming technique characterizes seismic waves in the time or frequency domain by the delay-and-sum process: it employs the differential travel times of phased array elements data to find suitable slowness and azimuth that best focus seismic energy, emanating from a source (Aki and Richards, 1980; Hinich, 1981; Rost and Thomas, 2002). In addition to the standard wave attributes obtained from the phased array method, the method also enhances the signal-to-noise ratio (SNR) of the array data as it concurrently muffles noise while preserving the desired signal (Husebye et al., 1984; Rost and Thomas, 2002; Schweitzer et al., 2012). For adequate performance, the phased array method requires that the array aperture is at least equal to the longest wavelength of interest (Asten and Henstridge, 1984). This size constraint can prevent successful phased array deployment over an area with limited areal extent. The
The gradiometry technique offers an alternative array analysis method that can analyze records from an array whose aperture is at most one-tenth of the suitable signal wavelengths. Less like the standard phased array technique that requires a number of signal cycles to accurately compute wave attributes, gradiometry employs a fraction of the signal cycle to characterize seismic waves. This quality dictates the compactness of the gradiometer and makes it deployable in areas inaccessible to larger aperture arrays.

The gradiometry technique incorporates ground motion, its spatial gradient and particle velocity into a system of equation that is solved to characterize seismic waves. Besides horizontal phase velocity and signal azimuth, gradiometry analysis also reveal radial and azimuthal amplitude variations across the gradiometer (Langston, 2007a, 2007c). These attributes can be utilized to understand local structure, wave attenuation, scattering and anisotropy in a region, which can potentially help illuminate the tectonic framework of that region (Liang and Langston, 2009). The compactness of the gradiometer, coupled with the complimentary wave attributes derived from its records could make it preferred for certain geophysical campaigns.

This study incorporates continuous wavelet transform (CWT) denoising and block manipulation technique (Langston and Mousavi, 2019) into both gradiometry and phased array processing pipelines in a bid to mitigate noise and isolate prominent seismic phases in the wavelet space prior to array analysis. The end goal of these steps is to be able to compare seismic wave attributes, particularly wave azimuth and horizontal phase velocity, derived from applying gradiometry technique on 2D array records from an explosion and local earthquakes with those attributes obtained from analyzing seismic signals recorded from same events by a phased array.

The gradiometry method yields wave properties from analyzing time series records; these results
themselves are time series that provide point-wise attributes of the analyzed signal as a function of time. Thus, to faithfully appraise the gradiometry results, it is necessary to check them against a method that equally provides time varying wave properties. Langston (2021) applied the continuous wavelet transform (CWT) to produce beams of extracted seismic phases in the wavelet domain. The outputs of this method are plots of wave horizontal phase velocities and back azimuths (BAZ) as functions of time. We compared the results from gradiometry with those obtained from conducting phased array analysis in the wavelet domain.

We analyzed seismic data from an explosion and local earthquakes that were recorded during the 2016 Incorporated Research Institutions of Seismology IRIS Wavefields Experiment in northern Oklahoma (Sweet et al., 2018). The experimental gradiometer of the wavefields experiment is composed of a dense sensor arrangement that can be dissected into a myriad of subarrays for array optimization studies. Consequently, we used this compact property of the gradiometer to find the sub-configuration that gives the most desirable wave attributes for a selected spectral band, within the context of the phased array results.

4.2 Data

IRIS conducted a large-N geophysical experiment around Enid, Oklahoma in the summer of 2016. The seismic experiment fielded diverse seismic instruments ranging from nodal seismometers to broadband, and infrasound sensors, in an effort to acquire a range of temporal and spatial datasets of local induced seismicity (Sweet et al., 2018). The deployment consisted of a dense arrangement of identical 3-component, 5 Hz Fairfield nodal sensors that was 13 km along its single east-west segment; the two 5 km segments of this array ran north-south, and they intersected the east-west arm at about 1.5 km away from its center on either side (Figure 4.1). This array is the largest of the 2016 IRIS experiment arrays and it consists of 254 nodal
elements. Next in aperture is the “Golay” array composed of 18 broadband Guralp CMG-3T seismometers with an aperture of about 7 km (Figure 4.1). Nine infrasound sensors were collocated with 9 of the broadband elements to quality-check the broadband records for possible acoustic signals.

The smallest array from the 2016 campaign was the gradiometer, which is an eponym that stems from how its records are analyzed via the gradiometry method: since gradiometry characterizes seismic waves using the wavefield and its spatial gradient (Langston, 2007a, 2007c). The fielded gradiometer array consisted of 112, 3-component, 5 Hz Fairfield nodal instruments (Figures 4.1 and 4.2). Each square is four times the area of the immediate smaller square; the essence of this geometry is to achieve a broadband spatial sampling of the ambient wavefield over time. The widest intersensor distance of the gradiometer was about 1 km. The 2016 Wavefield Experiment data is archived under network code “YW” at the IRIS Data Management Center.

To gauge the performance of the gradiometer in characterizing seismic waves, we contrived a 3X3 km, cross-shaped, phased array from the largest array of the 2016 deployment (Figure 4.1). This phased array contains 80 nodal seismometers that are largely 100 m apart, with a denser spacing of 33 m about the center of the array where it crosses the east segment of the EW line. Going forward, we will refer to this 80-element phased array as the “cross array.” The range of spatial frequencies the cross array can analyze is expressed in its co-array, which is a plot of bearing and distance between every element pair of the array (Figure 4.3). The cross array is amenable to resolve signal wavelengths from 0.5 to 3.5 km.
Figure 4.1: The 2016 IRIS Community Wavefields Experiment nodal and broadband sensors deployed near Enid, Oklahoma. The inset shows the location of the experiment over a background of the continental United States map. The circles in the main figure show the sensor distribution of the largest array of the deployment, with an aperture of 13 km. The Golay array elements expressed as pentagons are broadband instruments; the array was composed of 6 subarrays with 3 sensors each. The widest distance between a sensor pair in the Golay array is about 7 km. The Gradiometer was made up of 112 nodal instruments arranged into 7 concentric square rings. Each ring has 16 elements. We carved out the 3X3 km cross array from the largest array of the deployment to serve as a regional phased array. It is composed of 80 nodal seismometers.
Figure 4.2: A close-up view of the gradiometer array from Figure 4.1. The array was made up of 7 square subarrays in which each subarray is four times the area of the immediate smaller square. Each subarray is labelled clockwise from its north-west corner such that the first digit is the array code, the second the subarray code, and the last two digits define the location of each element in its respective subarray.
Figure 4.3: (a) The regional cross array, which is a subset of the largest array of the Wavefields Experiment and (b) the co-array plot that captures the spatial resolution of the cross array. It is a plot showing the distance and azimuth of every sensor pair of the cross array.
For this study, we used the cross array to analyze signal bands between 1 to 10 seconds. We will also use the 13-km long east-west arm of the largest array to characterize longer period signals, and we will henceforth refer to this array as the “linear array”.

To evaluate the effectiveness of the gradiometer array in characterizing seismic signals, we will examine records of an explosion and four local earthquakes. The explosion, composed of 2000 lb of explosives, was detonated about 13 km south-east of the YW array (Table 4.1). One of the earthquakes we will consider is a magnitude 4.2 earthquake that happened 100 km westward of the array at a depth of 7.3 km (Table 4.1). For the rest of this article, we will refer to this event as “event A.” Next in size is a magnitude 3.6 earthquake that occurred 92 km away from the array in the vicinity of Langston, Oklahoma; we will call this earthquake “event B” going forward. The third earthquake we will consider happened 33 km north-west of the array and has a magnitude of 2.8. In line with the first two events, we will subsequently refer to this earthquake as “event C.” The final earthquake we will examine has a magnitude of 2.6 and happened 55 km from the array at a focal depth of 3.8 km (Table 4.1). Henceforth, this earthquake will be referred to as “event D.”

**Table 4.1: Parameters of seismic events used for this work**

<table>
<thead>
<tr>
<th>Event</th>
<th>Date/time yyy/mm/dd hh:mm:ss.sss</th>
<th>Latitude (°)</th>
<th>Longitude (°)</th>
<th>Depth (km)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2016/07/08 21:31:57.600</td>
<td>36.4765</td>
<td>-98.7387</td>
<td>7.315</td>
<td>4.2 (Mw)</td>
</tr>
<tr>
<td>B</td>
<td>2016/06/28 19:57:56.000</td>
<td>35.8562</td>
<td>-97.2272</td>
<td>5.641</td>
<td>3.6 (Mw)</td>
</tr>
<tr>
<td>C</td>
<td>2016/07/15 11:58:50.800</td>
<td>36.8541</td>
<td>-97.8607</td>
<td>6.201</td>
<td>2.8 (ml)</td>
</tr>
<tr>
<td>D</td>
<td>2016/07/11 22:19:32.500</td>
<td>36.2200</td>
<td>-97.2807</td>
<td>3.795</td>
<td>2.6 (ml)</td>
</tr>
<tr>
<td>Explosion</td>
<td>2016/07/16 00:37:00.000</td>
<td>36.5516</td>
<td>-97.5180</td>
<td>0</td>
<td>2000 (lb)</td>
</tr>
</tbody>
</table>
The gradiometer, linear array, and cross array instruments were sampled at 200 sps. We took out the theoretical instrument response of individual nodal instrument from the recorded data and bandpass filtered the data using a zero-phase trapezoidal Butterworth filter with corner frequencies 0.5 and 50 Hz over a passband of 0.05-80 Hz. Over the duration of the 2016 experiment, five of the nodal seismometers returned no data (Sweet et al., 2018). We also discarded undesirable waveforms: for example, waveforms with transients or high levels of noise compared to adjacent stations.

Following the pre-processing steps, we corrected the gradiometer records for amplitude statics and instrument orientation errors (Bolarinwa & Langston, 2021). These errors stem from nonstandard digitizer gains across array seismometers, near-surface heterogeneities along instrument sites, and instrument misorientations (Bungum et al., 1971; Cranswick et al., 1985; William and Larson, 2000; Larson, 2002; Hutt and Ringler, 2011; Langston, 2018).

4.3 Gradiometry in 2D: Theoretical background

Gradiometry uses a wavefield and its time and spatial derivatives to characterize seismic waves (Langston, 2007a). The theoretical formulations of gradiometry have been described in a series of articles: Langston (2007c, 2007b, 2007a); Langston and Liang (2008); we will give a succinct description of the theory to clarify succeeding sections of this paper.

In 2D, gradiometry models a wavefield as a generalized form of a single propagating wave from a point source as follows:

\[ u(t, x, y) = G(x, y)f \left( t - p_x(x - x_0) - p_y(y - y_0) \right) \]  \hspace{1cm} (4.1).
Where $G(x, y)$ encodes both radial and azimuthal amplitude variations, and the phase term contains the ray parameters $p_x$ and $p_y$ in the $x$ and $y$ directions, respectively. The analytical spatial derivatives of the ground motion in equation (4.1) are expressed as

$$\frac{\partial u(t, x, y)}{\partial x} = A_x(x)u(t, x, y) + B_x(x)\frac{\partial u(t, x, y)}{\partial t}$$  \hspace{1cm} (4.2a)$$

$$\frac{\partial u(t, x, y)}{\partial y} = A_y(y)u(t, x, y) + B_y(y)\frac{\partial u(t, x, y)}{\partial t}$$  \hspace{1cm} (4.2b),

in which,

$$A_x(x) = \frac{1}{G(x, y)} \frac{\partial G(x, y)}{\partial x}$$  \hspace{1cm} (4.3a)$$

$$A_y(y) = \frac{1}{G(x, y)} \frac{\partial G(x, y)}{\partial y}$$  \hspace{1cm} (4.3b)$$

$$B_x(x) = -\left(p_x(x) + \frac{\partial p_x}{\partial x}(x - x_0)\right)$$ \hspace{1cm} (4.3c)$$

$$B_y(y) = -\left(p_y(y) + \frac{\partial p_y}{\partial y}(y - y_0)\right)$$ \hspace{1cm} (4.3d).$$

The amplitude and slowness coefficients ($A_x(x), A_y(y)$ and $B_x(x), B_y(y)$, respectively) in equations (4.2a) and (4.2b) serves to mediate the relationship between the wavefield and its associated derivatives, and evaluating these coefficients is a necessary step in gradiometry analysis. Langston (2007b) developed a theoretical procedure to evaluate these coefficients using the analytic forms of the time series in equations (4.2a) and (4.2b). Having obtained the four
coefficients, and assuming \( p_x \) and \( p_y \) are constant over the small-aperture gradiometer, equations (4.3c) and (4.3d) reduce to

\[
B_x(x_o) = -p_x \quad \quad (4.4a)
\]

\[
B_y(y_o) = -p_y \quad \quad (4.4b).
\]

The fidelity of computed wavefield spatial gradient is cardinal to the accuracy of the wave attributes evaluated from gradiometry method (Langston, 2007a; Bolarinwa and Langston, 2021). It has been proposed that the spatial derivatives and the wave displacement in equations (4.2a) and (4.2b) can be reliably estimated through a Taylor-series expansion of the wavefield record at individual sensor position about a reference position (preferably the array’s geometric center) of the gradiometer as expressed by equation (4.5) (e.g., Langston, 2018). The ground displacement, \( u(x_0, y_0) \) and its spatial derivatives up to the second order emerge as the model parameters of the matrix equation (4.6). This inverse problem is inherently linear and can be solved by the least-square technique as expressed in equation (4.7).

\[
\begin{align*}
  u(x_i, y_i) &\approx u(x_0, y_0) + \frac{\partial u}{\partial x} \bigg|_0 (x_i - x_0) + \frac{\partial u}{\partial y} \bigg|_0 (y_i - y_0) + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \bigg|_0 (x_i - x_0)^2 \\
  &+ \frac{1}{2} \frac{\partial^2 u}{\partial y^2} \bigg|_0 (y_i - y_0)^2 + \frac{\partial^2 u}{\partial x \partial y} \bigg|_0 (x_i - x_0)(y_i - y_0)
\end{align*}
\]

(4.5).
\[ \begin{bmatrix} 1 (x_1 - x_0) (y_1 - y_0) & \frac{1}{2} (x_1 - x_0)^2 & \frac{1}{2} (y_1 - y_0)^2 & (x_1 - x_0)(y_1 - y_0) \\ \vdots & \vdots & \vdots & \vdots \\ 1 (x_n - x_0) (y_n - y_0) & \frac{1}{2} (x_n - x_0)^2 & \frac{1}{2} (y_n - y_0)^2 & (x_n - x_0)(y_n - y_0) \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} (x_0, y_0) \\ \frac{\partial u}{\partial y} (x_0, y_0) \\ \frac{\partial^2 u}{\partial x^2} (x_0, y_0) \\ \frac{\partial^2 u}{\partial y^2} (x_0, y_0) \\ \frac{\partial^2 u}{\partial x \partial y} (x_0, y_0) \end{bmatrix} \]

\[ G \quad m = d. \]

\[ m = (G^T G)^{-1} G^T d \] (4.7).

With the three time series and the corresponding amplitude and slowness coefficients at hand, going from the Cartesian to the cylindrical coordinate system, the wave slowness, \( p \) and wave azimuth, \( \theta \) are respectively given by

\[ p = \sqrt{p_x^2 + p_y^2} \] (4.8),

\[ \theta = \tan^{-1} \left( \frac{p_x}{p_y} \right) \] (4.9).
4.4 Array Analysis in the CWT domain

4.4.1 Denoising Seismic Signals

The Nonlinear thresholding method developed by Langston and Mousavi (2019) is an effective procedure for removing noise from desired signal if the signal and noise spectra are distinct in CWT space. The CWT as defined by Grossmann et al. (1989) is expressed as

\[ W(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi^* \left( \frac{t-\tau}{a} \right) dt \]  

(4.10).

\( a \) is the scale, \( \tau \) is the time lag, \( f(t) \) is the signal, and \( \psi(t) \) is a scaled basis function. The asterisk represents the complex conjugate of the function. The basis function is complex and is called the “mother wavelet.” Being a linear operation, the CWT has an inverse transform expressed as

\[ f(t) = \frac{1}{C} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{a}} W(a, \tau) \psi \left( \frac{t-\tau}{a} \right) \frac{d\alpha d\tau}{a^2} , \]  

(4.11)

where \( C \) has the form

\[ C = \int_{0}^{+\infty} \frac{\hat{\psi}^*(\omega) \hat{\psi}(\omega)}{\omega} d\omega \]  

(4.12).

We used the “Morlet” wavelet as the mother wavelet for all CWT related analysis in this article.

As part of the processing steps, we removed noise from the data by a soft thresholding technique that expels wavelet coefficients smaller than a predetermined noise threshold and shrinks those coefficients above the threshold by the noise estimate as below:

\[ \tilde{W}(a, \tau) = \begin{cases} 
\text{sign}[W(a, \tau)] (|W(a, \tau)| - \beta(a)) & \text{if } |W(a, \tau)| \geq \beta(a), \\
0 & \text{otherwise}, 
\end{cases} \]  

(4.13)

in which \( \text{sign}[W(a, \tau)] = \frac{W(a, \tau)}{|W(a, \tau)|} \).
\( \beta(a) \) is the noise threshold term and is obtained from the empirical cumulative distribution of the wavelet coefficient statistics for a pre-event (or post-event) noise time window so that

\[
\beta(a) = ECDF^{-1}(P = 0.99),
\]

where \( P = 0.99 \) denotes the 99\% confidence level.

Figure 4.4 shows plots of the time- and CWT domain representations of the vertical component record of event B at station 4115 of the gradiometer. The pre-signal noise on the time series is expressed by high scale (low frequency) CWT coefficients from 0 up to around the 60 second mark on the scale-time plane. The signal energy packets are mostly separated from the noise. We applied the soft thresholding procedure on this waveform using the 0 to 60 second time window to generate the required noise model. In addition to denoising, we applied a band reject filter to remove scales larger than 8 seconds, which may contain high-scale artifacts that made it through the soft thresholding step.

The denoising step improved the signal to noise ratio (SNR) of the signal by an order of magnitude – if the SNR is estimated as the ratio of the maximum absolute amplitude within the signal window to the value of same parameter within the noise window. The raw data had an SNR of 18, that went up to 189 after denoising. The waveform we selected to demonstrate the denoising procedure has a good SNR that was even made better through soft thresholding; Bolarinwa and Langston (2022) reported close to two orders of magnitude increase in SNR when they applied the soft thresholding technique to a smaller earthquake with an initial SNR of 2.

### 4.4.2 Partitioning Seismic Signals

The theory of gradiometry models a wavefield as a generalized single propagating wave from a point source (Langston, 2007a).
Figure 4.4: (a) Vertical component data of event B (Table 4.1) at station 4115 of the gradiometer array. The top panel shows the time-frequency representation of the data in which the magnitude of the wavelet coefficients is plotted as a function of scale and time. The presignal noise manifest as the high scale, low energy strip preceding the signal. The top plot to the left captures the mean noise amplitude between 0 and 60 seconds. The noise amplitude threshold that was computed from the empirical cumulative distribution function (ECDF) method is also shown on the same plot. The annotated time window from 0 to 60 seconds was used to compute the noise model employed to denoise the data. (b) The top panel shows the denoised data obtained from applying the soft thresholding procedure on the raw data in (a), while the second panel is the time domain representation of the denoised.
However, in reality, the earth is not homogeneous and more than one seismic phase can concurrently arrive at a seismic station and interact with each other. The interference of these simultaneous arrivals can engender abrupt variations in wave attributes (Langston, 2007b; Langston and Liang, 2008; Langston et al., 2009; Liang and Langston, 2009; Poppeliers, 2011).

In a bid to minimize this effect, we adopted a procedure that uses apparent scale-time properties of seismic phases in the CWT space to partition these phases.

The time series in Figure 4.4 shows a distinct P wave arrival; the time-frequency representation of the time series on the scale-time plane in the same figure reveal that the supposed “distinct” P wave may be composed of more than one phase. The 2D representation of this time-series in the CWT space provides an extra dimension that offers more information about the signal than the 1D time domain representation. Langston and Mousavi (2019) developed a method that can be used to separate seismic wave phases based on their spectral content in scale and their duration in time. This method takes advantage of the linearity of the CWT by encompassing a wave packet of interest with a polygon in the CWT space, then doing and inverse CWT transform of the selected coefficients to obtain a time realization of this block.

The top scalogram in Figure 4.5 shows annotated phases of the denoised seismogram from Figure 4.4. As indicated, the body wave segment is made up of a low-scale component we refer to as body wave and coda, and the localized high-scale PL wave (Oliver, 1964). The dispersive nature of the fundamental mode surface wave is expressed in its gentle descent in scale through time (note that the scale axis is inverted). The less prominent higher mode surface wave coefficients run parallel with those of the fundamental mode on the scale-time plane (Figure 4.5).
Figure 4.5: Signal partitioning in the continuous wavelet transform (CWT) domain. (a) The same denoised data from Figure 4.4(b) showing a polygon encompassing CWT coefficients of the fundamental mode surface wave on the top panel. In addition, the figure highlights prominent body and surface waves. The lower panel is the time representation of the scalogram above it. (b) A solo plot of the fundamental mode CWT coefficients on the scale time plane after the coefficients of other phases have been zeroed out is shown on the top panel. The panel below is derived from doing inverse CWT of the gated fundamental mode coefficients.
To illustrate the signal partitioning procedure, we picked a polygon around the fundamental mode coefficients in Figure 4.5, then inverse transformed these selected coefficients to exclusively view the fundamental mode phase in the time domain. The gated phase exhibited the characteristic dispersion expected of a surface wave, as progressively higher frequencies are delayed in time. Still following Langston and Mousavi (2019), we did a 2D correlation of the selected fundamental mode coefficients with scalograms of vertical records of event B at other gradiometer stations to extract similar phase from these stations. Figure 4.6a captures all the body- and fundamental mode waveforms extracted from the vertical component gradiometer data of event B. Each collection of traces in the figure is made up of 98 seismograms, and both phases show notable coherence across the gradiometer array. For a comprehensive insight on partitioning of local seismograms of this sort in the CWT domain, we refer the reader to a companion article: Bolarinwa and Langston (2022).

4.5 Gradiometry and Phased Array Analyses

The central objective of this article is to show how gradiometry-derived wave attributes compare to those attribute estimates from the phased array method. In building up to this comparison, we denoised the gradiometer and cross array data of events A and B and extracted body and surface wave phases from individual station records of the two arrays. The surface wave phases of event C are not as developed as surface waves in events A and B; we denoised event C records used for this study, but we did not apply the signal partitioning technique on its data before conducting array analysis. Following denoising and signal partitioning, we did the phased array analysis for a number of scale bands to find the scale band where the phased array result shows a “good beam” over time. The idea of what makes a good beam will be discussed in the next section.
Figure 4.6: (a) Extracted seismic phases from the vertical component gradiometer records of Event B (Table 4.1). Top panel shows body wave phases from 98 gradiometer elements. A plot composed of 98 fundamental mode Rayleigh waveforms is shown on the lower panel. (b) Top panel shows plot of extracted body wave signals (using same technique expressed in Figures 4.5 and 4.6) from the cross array vertical component records of event A (Table 4.1). There are 79 waveforms on the plot. Bottom panel shows 97 body wave signals partitioned from the gradiometer vertical component data of event A. These two records will be examined later.
Once we have a handle on the signal scale band that provides a good beam through time for a given event, we conducted gradiometry analysis on gradiometer record of the event using same scale band as the phased array case.

### 4.5.1 Phased array Analysis

In order to make a proper comparison between the phased array and gradiometer results, we employed a phased array technique that yields wave attributes as a function of time in the same manner as gradiometry. This method capitalizes on the fact that time shift operations in time directly translates to identical time shifts in the wavelet coefficients, which ultimately allows one to directly beamform on the wavelet coefficients (Langston, 2021).

The cardinal themes of the phased array results error estimates are to minimize the area of the peak power of the array beam to get a “good peak”, and simultaneously maximize the low power area on the scale-slowness plane. With this in mind, one can estimate the pixels within 90% of the peak power, and also find all the pixels in the low amplitude area of the scale-slowness plane that are below 2.5% of the peak power. A measure of beam “peakness,” called the “R factor” is defined such that a scale to quantify beam “peakness” as expressed below:

\[
R = \frac{n_{\text{low}}}{n_{\text{peak}}} \times \frac{n_{\text{low}}}{n_{\text{Total}}} = \frac{n_{\text{low}}^2}{n_{\text{peak}} n_{\text{Total}}} \quad (4.15)
\]

Where \(n_{\text{low}}\) is the number of pixels below 2.5% of the peak power, \(n_{\text{peak}}\) is the number of pixels above 90% of peak power, and \(n_{\text{Total}}\) is the total number of pixels in the entire scale-slowness plane. It is important to keep in mind that we set R to zero if the peak is near the edge of the scale-slowness plot or if the amplitude of the reference station data point in question is a hundred thousandth of the absolute maximum of the entire reference station trace. We observed that a threshold of 0.4 for R works well for the cross-array data. For this phased array, we estimated
wave slowness (inverse of horizontal phase velocity) and signal BAZ errors as a measure of the peak width at the 90% power level on the scale-slowness plane.

We conducted phased array analysis over several scale bands that span from 0.1 to 1 second. In particular, for the P wave attribute estimates, we looked at scale bands 0.1-0.5, 0.2-0.5, 0.13-0.2, 0.25-0.5, 0.17-0.25, 0.13-0.17, and 0.1-0.13 seconds. For the slower S waves, we looked scale bands 0.14-1, 0.25-1, 0.14-0.25, 0.33-1, 0.25-0.5, 0.2-0.33, 0.17-0.25, 0.14-0.2 seconds. Table 4.2 has the details of the scale bands that offered good beam “peakness” over time and the signal length that we considered for the P and S wave attributes on the vertical, radial, and transverse components data of individual event. We will compare the selected phases’ wave attributes obtained within this scale bands and time window lengths indicated in the table with those from identical phases in the gradiometer data.

**Table 4.2:** Lists of time window lengths, scales bands where we observed focused phased array beams through time and the optimal gradiometer subgeometry from analyzing vertical, radial and transverse body wave data of events A, B and C. The numbering of the squares is proportional to the square size with the smallest square being 1 and the largest 7.

<table>
<thead>
<tr>
<th>Event</th>
<th>Component</th>
<th>Time window width (seconds)</th>
<th>Scale band (seconds)</th>
<th>Optimal gradiometer geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Vertical</td>
<td>0.5</td>
<td>0.13-0.17</td>
<td>Squares[1,3,5]</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>0.25</td>
<td>0.25-1.00</td>
<td>Squares[4,6,7]</td>
</tr>
<tr>
<td></td>
<td>Transverse</td>
<td>0.5</td>
<td>0.25-0.50</td>
<td>Squares[4,5]</td>
</tr>
<tr>
<td>B</td>
<td>Vertical</td>
<td>0.4</td>
<td>0.13-0.17</td>
<td>Squares[4,5]</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>0.2</td>
<td>0.14-0.20</td>
<td>Square[5]</td>
</tr>
<tr>
<td></td>
<td>Transverse</td>
<td>0.4</td>
<td>0.25-0.50</td>
<td>Squares[3,4]</td>
</tr>
<tr>
<td>C</td>
<td>Vertical</td>
<td>0.4</td>
<td>0.17-0.25</td>
<td>Squares[1,5]</td>
</tr>
<tr>
<td></td>
<td>Radial</td>
<td>0.1</td>
<td>0.25-0.50</td>
<td>Square[6]</td>
</tr>
<tr>
<td></td>
<td>Transverse</td>
<td>0.6</td>
<td>0.20-0.33</td>
<td>Squares[145]</td>
</tr>
</tbody>
</table>

The spectral content of the surface wave we considered is higher in scale (lower frequency) than the scales the cross array can resolve with reasonable fidelity. Therefore, we will
compare gradiometer results of the surface waves with results from 1-D f-k analysis of the linear array surface wave data, and the great circle path of the signal between the source and the gradiometer array reference position.

### 4.5.2 Gradiometry Analysis

The wavefield’s spatial gradient is an important parameter in estimating wave attributes from gradiometry, as the accuracy of estimated signal slowness and azimuth depends on the exactness of the computed reference wavefield and its spatial gradient. The accuracy of the reference ground motion and its spatial derivative in turn rely on the gradiometer aperture and sensor spacing for a given signal wavelength. Thus, we investigated the experimental gradiometer precision (within the same scale band as the phased array result) by boot strap methods in which we computed spatial gradients and resulting wave attributes for all possible combinations (127 in all) of the gradiometer square subarrays – examples of these combinations are square 1, square 7, squares 236, squares 3467, squares 134567, just to name a few. Then we compared all 127 horizontal phase velocity estimates with the phased array-derived horizontal phase velocity for identical phases. We compared the corresponding gradiometer-derived azimuths with the signal great circle path from the event epicenter to the reference point of the gradiometer. We then selected the subgeometry that gives the best simultaneous estimate of wave phase velocity and azimuth as the optimal gradiometer geometry. The final column of table 4.2 contains the optimal gradiometer subconfigurations that provides phase velocity and azimuth that are jointly optimal for the given scale bands.
Figure 4.7: Phased array and gradiometry results from analyzing the vertical component body wave data of event A (Table 4.1; Figure 4.6b) within a scale band of 0.13-0.17 seconds. The gradiometer attributes have been shifted in time to remove wave propagation effect due to the spatial difference in the locations of the two arrays. The rectangular box that goes across the figure reveal the 0.5 second time window over which we estimated the mean attribute values from the two methods. The three dashed lines showed the time points at which the highlighted beams were computed. The circles with error bars are phases array results, and the red circles for the colored version of this article represent beams with sharp peaks, while those circles in black show diffused beams or beams with multiple peaks. The solid blue lines are the attributes derived from conducting gradiometry on the gradiometer subgeometry composed of squares 1 (smallest), 3 and 5. (a) Phased array reference station (station 3017) waveform filtered within a scale band of 0.13-0.17 seconds. (b) Horizontal phase velocity plot at every other 10 time points, and (c) associated signal back azimuth, also plotted at every other 10 time points.

4.5.3 Comparing Gradiometry and Phased array Results

The optimal phase velocity and azimuth estimates from gradiometry mostly compare well with the phased Array-derived wave properties. For instance, Figure 4.7 captures the wave phase velocity and BAZ through time as estimated from the phased array and gradiometer data filtered within a scale band of 0.13-0.17 seconds. The gradiometer attributes are plotted in solid blue lines, while the phased array attributes are in red and black circles, with the enumerated colors only available in the electronic edition of this article. The red circles represent scale slowness beams with single prominent peaks like the first and third beams highlighted in Figure 4.7, while the black circles portray diffused beams (e.g., third beam in Figure 4.7) or scale slowness images with multiple peaks. The gradiometer attributes have been shifted to align in time with the phased array attributes by using the time lag between a phase that is prominent on the cross array center station and the gradiometer reference station waveforms. In addition, the azimuth estimates from gradiometry were scaled to respective BAZ for easy comparison with phased array-derived BAZ. We compared wave attribute estimated from the two arrays over the 0.5
second time window outlined by the rectangular box in Figure 4.7. The average cross array-derived phase velocity within this time window is \(6.20 \pm 0.19 \text{ km/s}\), and the average of same attribute from gradiometry is \(6.17 \pm 0.04 \text{ km/s}\). The corresponding average BAZ from the cross array is \(262.9 \pm 1.8 \text{ degrees}\), while that from gradiometry is \(263.4 \pm 1.0 \text{ degrees}\). The errors in the average gradiometry results are the standard deviation of the attributes in the 0.5 second time window considered. Looking on in time from the time window we examined, the signal BAZ from the two methods agree fairly well with each other, revealing later arriving phases from the same general direction. The series of drawdowns and spikes on the wave attribute plots from gradiometry are due to interfering seismic phases that arrived concurrently at the gradiometer.

Figure 4.8 extends the results to show the optimal phased velocities and BAZ we obtained over designated time windows (Table 4.2) for the body wave phases on the vertical, radial and transverse component records of events A, B and C. We examined the P wave phase on the vertical component and considered S waves on the two horizontal components. If the phase velocities and BAZs derived from the two array analysis techniques are identical, they should plot on the straight line of each plot; therefore, the proximity of each data point to the straight line is a measure of the precision of the gradiometer result relative to the phased array results. Thus, by precision, we refer to closeness of the mean of gradiometry-derived attribute to corresponding mean attribute estimate from the phased array method. The P wave BAZ estimates from the vertical data of the three events reveal the best precision of the three components. The vertical component BAZ estimates from the two methods are within 1 degree of each other for events A and C, while those from event B differ by about 5 degrees.
Figure 4.8: Plots of gradiometry-derived mean wave attributes over a length of time (Table 4.2) against those attributes obtained from doing phased array analysis over the same length of time. These plots are from analyzing body wave data of events A, B and C (Table 4.1). The proportionality line on each plot shows the ideal spots individual results should plot if the attributes from the two methods are identical. (a) BAZ estimates from analyzing vertical component P wave, and transverse and radial components S waves. (b) corresponding horizontal phase velocity estimates.
The S wave BAZ estimates from gradiometry and the phased array method also show good correspondence for events A, B and C (Figure 4.8). Of the three events, event A shows the best precision in BAZ; the BAZ obtained from the two array techniques vary by less than 1 degree for the two horizontal components of the event. The S wave BAZ computed from the radial gradiometer records of event C exhibited the greatest variability with a deviation of about 21 degrees from its corresponding phased array-derived BAZ.

The mean horizontal phase velocity estimates from gradiometry compare well with those determined from the phased array. The transverse component result offered the most precise comparison between the phase velocity estimates from the two techniques. For instance, the transverse component S wave velocity estimates from the phased array and gradiometer records of Event C are identical to two decimal places, while those from event B showed the largest variability of 0.06 km/s (Figure 4.8). On the vertical component plot, the difference in velocity estimates from the two methods range from 0.03 km/s, for event A, to 0.11 km/s, for event B. A careful look at the axes on the phase velocity plots in Figure 4.8 accentuates the variability within the radial component plot relative to the other two components. With the exception of S wave velocity of event B on the radial component plot, other phase velocity results plot in the vicinity of the proportionality line; in fact, the average radial S wave velocities from analyzing event A are the same to two decimal places for the two methods.

So far, we have carried out all phased array analysis using data from the cross array. The CWT coefficients of the prominent surface waves of events A and B (e.g., Figure 4.5) mostly lie outside the cross array resolution limit. To appraise the utility of the gradiometer in characterizing surface waves, we did a 1-D f-k analysis of the gated vertical fundamental mode surface wave records of event A on the linear array, to shed light on the corresponding
gradiometer result. The 1-D f-k plot in Figure 4.9 shows the fundamental mode Rayleigh wave slowness distribution for frequency band between 0.25 and 0.55 Hz. The inverse of the peak slowness at 0.3 Hz frequency mark amounts to a phase velocity of $2.86 \text{ km/s}$. Figure 4.10 shows the fundamental mode Rayleigh wave extracted from the gradiometer record of event A. Gradiometry analysis of the Rayleigh wave within 0.25 and 0.35 Hz reveal an average phase velocity of $2.82 \text{ km/s}$ over the 9 second time window shown in Figure 4.10.

**Figure 4.9:** Filtered waveforms and 1-D f-k plot from analyzing (denoised and) extracted fundamental mode phase from event A vertical records on the linear array composed of all sensors along the east-west segment of the largest array in Figure 4.1. (a) Record section made up of 139 waveforms that have been filtered between 0.25-0.55 Hz. (b) 1-D f-k plot that reveal increase in slowness with frequency, capturing the dispersive property of the analyzed surface wave.

The associated azimuth estimate over the same time window is 83.1 degrees. The gradiometry-derived velocity of the Rayleigh wave only differ by 0.04 km/s from that obtained from the 1-D f-k analysis, while azimuth estimate from gradiometry is 2.6 degrees off the signal great circle path between the source and the reference position of the gradiometer.
Figure 4.10: (a) Denoised and gated fundamental mode Rayleigh wave from the vertical component gradiometer data of event A. The plot is made up of 97 waveforms. (b) The ground displacement estimates at the reference position (geometric center of the gradiometer) of the gradiometer filtered between 0.25 and 0.35 Hz. (c) Wave horizontal phase velocity at the reference position. (d) Corresponding wave azimuth at the reference position. The results in (b), (c) and (d) were obtained from doing gradiometry on the 97 waveforms from (a) filtered between 0.25 and 0.35 Hz. The time window bounded by the rectangular box shows the 9 second time window over which we estimated phase velocity and azimuth.
4.6 Discussion

In all, the high SNR data generally gave consistent results between the phased array and the gradiometer wave analysis models. As an illustration, the vertical component P wave BAZs obtained from gradiometry are within 1 degree of those BAZ estimates from the phased array for two of the three events considered (Figure 4.11). The BAZ estimates from applying the two techniques on the horizontal data of these events show more pronounced variability, with an exception of the two horizontal components of event A, that exhibited a precision of less than 1 degree in BAZ from the two methods (Figure 4.11). The reduced precision in the horizontal components BAZ estimates from gradiometry may not be a concern in event location applications, as the more accurate BAZ estimates from the vertical records could be used to orient the unrotated horizontal data to respective particle motion directions, before estimating, for example, S wave phase velocity from the data. These results suggest that, if signal SNR is high, the gradiometer could be used to locate seismic events.

The phase velocity estimates from gradiometry show good precision with respect to those obtained from the phased array. P and S wave velocities from all three components of event A offered the most precise set of velocities from the two array methods, with variability ranging from 0 (if attributes are considered to two decimal place) to 0.03 km/s (Figures 4.9 and 4.12). On the other extreme, body wave phase velocities computed from gradiometer records of event B exhibited the most dispersion from their phased array equivalents, with variability ranging from 0.06 to 1.45 km/s on all three components.

The boot strap technique employed to examine the precision of the gradiometer attributes by testing different subconfiguartions of the gradiometer reveal that all the optimal attributes were derived from combining at most three gradiometer square subarrays (Table 4.2).
Figure 4.11: Plots showing absolute difference between wave properties obtained through gradiometry and corresponding wave properties derived from the beamforming method – see Figure 4.8 for actual values of the wave properties. (a) Phase Velocity difference. (b) BAZ difference. The circles, diamonds and squares in the plots show that the associated gradiometry-derived attributes were computed from gradiometer subarray composed of 1 square, 2 squares and 3 squares of the gradiometer array, respectively (Table 4.2). The plots reveal that a gradiometer subarray composed of three gradiometer squares gives the most desirable attributes in the light of the beamforming results.
Out of the nine sets of optimal attributes computed from the three-component body wave data of events A, B and C, three attribute sets were estimated using a combination of three gradiometer squares, four attribute sets were computed using two gradiometer squares, and two attribute sets were obtained using a single gradiometer square. The attribute estimates from analyzing subarrays made up of three gradiometer squares offer the most precise gradiometer attributes in relation to their phased array equivalents (Figure 4.8 and 4.11). This suggest that a 48-element gradiometer may be desirable for future gradiometer deployments.

The reference position ground motion and its spatial gradient are products of an inversion estimate (equation 4.6), and the presence on noise in the data that the model operates on can reduce the accuracy of the resulting ground displacement and its spatial gradient, which will also negatively impact the wave properties that will be derived from these parameters. Therefore, we investigated the impact of data noise on gradiometry results by comparing gradiometry result from undenoised radial data of Event D with that from denoised radial records of the same event. For reference, the gradiometer is at an azimuth of 324.4 degrees from the epicenter of event D. Figure 4.12 shows the wave attributes estimates from conducting gradiometry on both denoised and undenoised radial component data of event D using a scale band of 1.8-2.2 seconds. The effect of noise is evident in the variability of the signal azimuth through time for the undenoised data, which makes it difficult to distinguish between the different phases. We looked at the average BAZ estimate between 15 and 21 seconds and found BAZs of 321.6 degrees for the denoised case, whereas the BAZ from the undenoised data is 299.1 degrees. While the BAZ estimate from the denoised records is within 3 degrees of the events great circle path (GCP), the undenoised data BAZ estimate deviates by over 25 degrees from the event’s GCP.
Figure 4.12: (a) Top panel shows raw (undenoised) radial component gradiometer data of event D composed of 95 waveforms, while the lower panel shows the same waveforms after denoising the records using the soft thresholding procedure. (b) Gradiometry result from analyzing the undenoised (raw) data. (c) Results from doing gradiometry on the denoised data. The bounding box shows the 6 second time window over which the mean wave attribute was computed. The results from the denoised records showed improved stability through time compared to the raw, noisy data.
This significant variability suggests that denoising operations should carried out prior to gradiometry analysis, especially for noisy event records.

The gradiometry model assumes a cylindrical wave propagating outwards from the source (Langston, 2007a, 2007c), and this theoretical premise could make gradiometry an efficient array technique in characterizing signals from close sources. We explored the usefulness of the gradiometer and the standard array in characterizing sources close to the arrays by analyzing the vertical component data recorded by the two arrays from an explosion that was detonated about 13 km from the arrays. We appraised the performance of the two arrays against a 1-D f-k analysis performed over a 5 second time window of the P wave record of the explosion filtered between 8 and 10 Hz (Figure 4.13). The 1-D f-k result reveal that the P wave phase from the explosion is traveling across the cross array with a phase velocity of 5 km/s. Phase velocity estimates from doing CWT beamforming on the cross array data within a scale band between 0.1 and 0.13 seconds reveal a phase velocity of 4.86 km/s, while gradiometry analysis of the explosion within the same scale band shows a P wave phase velocity of 5.11 km/s (Figure 4.14). The gradiometry-derived azimuth is within 4 degrees of the GCP, while BAZ from the phased array is less than 3 degrees off the GCP. The phased array method modeled the signal as a plane wave, and for a source this close, the phased array, and gradiometry, methods show good correspondence with the reference results. These results suggests that the denoised seismic data works well for both phased and gradiometer arrays.

There are two sources uncertainty in our findings that are worth some discussion. The first is the reality that a number of the wavefields experiment sensors returned no data, while some station data could not be used for this study due to the effect of noise and transients.
Figure 4.13: (a) Waveform and results from conducting 1-D f-k analysis on cross array vertical component body wave data of the explosion. (a) Cross array records of the explosion filtered between 8 and 10 Hz. There are 78 waveforms in the record section. (b) The 1-D f-k result from analyzing the explosion data in (a).

For example, of the 99 calibrated sensors of the gradiometer, we were able to analyze vertical records of event B from 98 of these sensors, while we could only use event C radial component data from 89 elements due to enumerated problems. Furthermore, while we were able to use vertical component data from 79 of the 80-element cross-array for the phased array analysis of event C, we could only use this event data from 75 of the horizontal component sensors for this study. Quantifying these uncertainties is beyond the scope of this article; however, we recommend a future work in which equation 4.6 can be harnessed to extrapolate the wavefields to stations with missing data for a particular event, prior to doing array analysis on the data.
Figure 4.14: Gradiometer and phased array vertical component data from the explosion (Table 4.1), and results from conducting gradiometry and phased array analysis on data from the two arrays. (a) Denoised cross array body wave data composed of 78 waveforms. (b) Denoised body wave data from gradiometer subgeometry composed of squares 1 (smallest), 2 and 3. The plot contains 41 waveforms from the subgeometry. (c) The reference station waveform of the phased array filtered between a scale band of 0.1-0.13 seconds. (d) The symbol system in this panel and the panel below is the same as those of corresponding attribute plots on Figure 4.7 for the colored version of this article. The circles with error bars represent beamforming estimates of the horizontal phase velocity at every 10th time point in the time series, while the solid blue lines are the corresponding gradiometry-derived horizontal phase velocity estimates through time. (e) The associated BAZ to the horizontal phase velocity estimates in the panel above. The phase velocity and BAZ estimates from gradiometry have been shifted in time to remove wave propagation effect resulting from the spatial difference in the locations of the two arrays. The rectangular box that runs through Figure 4.14 (c), (d) and (e) shows the time window over which mean phase velocity and azimuth estimates were computed for the two array processing techniques. The wave properties in this figure were estimated with a scale band of 0.1-0.13 seconds.
The second source of uncertainty has to do with the way the two array methods we have considered estimates wave attributes. While the phase velocity computed from the phased array method is some sort of average over the phased array, the gradiometer-derived phase velocity is closer to a point measurement, and this nuance will introduce some level of uncertainty into our results. In addition, the center of the cross array is 2 km away from the center of the gradiometer, and possible differences (even if subtle) in the structure below the two arrays may have also introduced uncertainty in our results. Also, we have made a one-to-one comparison of BAZ estimates from the gradiometer with phased array-derived BAZ, and the fact that the two arrays are not at the same location would have introduced uncertainties into this comparison.

4.7 Conclusion

The 2016 IRIS Community Wavefields experiment afforded us a novel Large-N data set to thoroughly explore the effectiveness of the gradiometer in estimating wave attributes in the form of wave BAZ and horizontal phase velocity. We compared wave properties derived from gradiometry with those obtained from a cross phased array that was carved out of the largest array fielded during the wavefields experiment. Prior to array analyses, we removed noise from the data via the nonlinear thresholding technique in the CWT domain. This step was necessary to mitigate the potential impact of noise on gradiometry result. Moreover, we gated out seismic phases that are evident on the scale-time plane in a bid to reduce the effect of concurrent arrivals on gradiometry results. There is good correspondence between the wave attributes computed from gradiometry and the phased array estimates, especially on the vertical components. We observed BAZ difference as little as less than a degree from BAZ estimates from the two array methods, and we also saw phase velocities that are identical to two decimal places from applying
gradiometry and the phased array technique to three earthquakes that occurred during the wavefield experiment. These results suggest that the gradiometry technique can be used to estimate wave properties and also locate seismic events.

We also examined the effect of noise on the wave properties estimated from gradiometry. To quantify this effect, we carried out gradiometry analysis on both denoised and raw radial records of a low SNR event that happened south-east of the YW array. The BAZ estimate from the denoised data is within 3 degrees of the GCP between the event source and the gradiometer, while the BAZ computed from the raw noisy data deviates by more than 25 degrees from the GCP. Thus, we recommend the denosing procedure as a data pre-processing step before doing gradiometry on array data, especially for noisy records.

Some station data were discarded during this work due to noise and transients. The missing data could have introduced some uncertainty into our results. Quantifying these uncertainties is beyond the purview of this article. However, we recommend a future work where the wavefields we have studied are extrapolated to stations missing data by means of truncated Taylor series approximation of the wavefield (e.g., equation 4.6), before conducting gradiometer and phased array analyses on the resulting data set. The good precision of the wave properties estimated from gradiometry in relation to those from the phased array shows that the gradiometry technique can be applied in detecting, locating, and characterizing seismic and anthropogenic events.

4.8 Data and Resources

The data analyzed in this study were acquired during the IRIS Community Wavefields Experiment carried out in northern Oklahoma in 2016. The data lives at the Incorporated
Research Institutions for Seismology Data Management center page

http://service.iris.edu/ph5ws/dataselect/1/. This page was last visited in December 2021. We used Seismic Analysis Code and Arc GIS to analyze and display data in this work, and they are acknowledged.

4.9 Acknowledgements

This research was supported by the National Science Foundation grant EAR-1723067 and by the Center for Earthquake Research and Information, University of Memphis, and is very much appreciated. The authors thank students, faculty, PASSCAL and IRIS staffs who acquired the data used for this work during the 2016 Community Wavefields Experiment in northern Oklahoma.

References

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Chapter 5  Conclusions

The 2016 IRIS Community Wavefields Experiment conducted in northern Oklahoma offers a rich dataset to investigate a range of geophysical questions that includes the relative performance of different array geometries and array processing techniques. We examined the performance of a gradiometer array, that uses the wavefield and its spatial gradient to characterize seismic waves, against a phased array method that generates wave attributes by beamforming on wavelet coefficients in the CWT domain. Before conducting these analyses, we used Teleseismic P and S waveforms recorded by the gradiometer to calibrate its elements in an effort to mitigate instrument and site errors in the recorded gradiometer data. The premise of the calibration exercise is that the small-aperture gradiometer array sensors will record a common wavefield from a teleseismic source. We found out that the most desirable calibration parameters were associated with frequency bands where SNR and/or signal coherence are highest. More than the improvement in the teleseismic signal relative amplitudes after calibration, we saw a boost in the accuracy of wave attributes estimated from a local earthquake following calibration. These suggests that calibration procedures such as this should be adopted into the data processing pipelines in future gradiometry analysis.

We also harnessed the nonlinear thresholding and signal partitioning techniques to denoise and separate out seismic phases that are contained in the phased array and gradiometer waveforms of local earthquakes and explosion. We observed up to two orders of magnitude improvement in data SNR after applying the nonlinear thresholding procedure on noisy records of a particular event. Also, we successfully identified many seismic phases by reason of their position on the scale-time plane. In addition, the denoising and signal partitioning procedures gave rise to a boost in signal coherence across array records. These results shows that CWT
thresholding and signal partitioning are not counterproductive to gradiometry and phased array analyses.

Ultimately, we examined the utility of the experimental gradiometer in characterizing seismic signals within the purview of a phased array that was fielded alongside the gradiometer during the wavefields experiment. In particular, we applied gradiometry and phased array technique to array records from the same earthquake and compared wave attribute estimates obtained from gradiometry with those attributes derived from phased array analysis. The gradiometry analysis reveal several signal BAZ within one degrees of their analogous phased array counterparts. We also observed in some cases gradiometry-derived phase velocities that are identical to corresponding phase velocities estimates from the phased array method. The good correspondence between the wave attribute estimates from gradiometry and those from the phased array method indicate that the gradiometer can be used to detect, locate and characterize seismic events.
# Appendix A  
Supporting Information for “Calibrating the 2016 IRIS Wavefields Experiment Nodal Sensors for Amplitude Statics and Orientation Errors”

## Table A1: Gradiometer array correction factors obtained at 0.25-0.5 Hz.

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Table A2: E-array correction factors obtained at 0.25-0.5 Hz.

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<th>OCF (deg)</th>
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Table A3: D-array correction factors obtained at 0.1-0.5 Hz for horizontal components and 0.25-0.5 Hz for associated Z component.

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<th>OCF (deg)</th>
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